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THE

PHYSICAL REVIEW

A JOURNAL OF EXPERIMENTAL AND
THEORETICAL PHYSICS

CONDUCTED WITH THE CO-OPERATION
OF THE
AMERICAN PHYSICAL SOCIETY
BY
EDWARD L. NICHOLS, ERNEST MERRITT,
AND FREDERICK BEDELL

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THE

PHYSICAL REVIEW.

SOME ELECTRICAL PROPERTIES OF SELENIUM. I.

By Louise S. McDowell.

IN the thirty-five years since the discovery was made that the resistance of selenium diminishes under the action of light, many investigators have studied the phenomenon and several theories have been developed to account for the action. Little attention, however, has been paid to the form of the recovery curves, *i. e.*, the curves which show the rate of return to the normal dark resistance after exposure to the light has ceased, although it has been frequently noted that the lowering of resistance under the action of light is rapid, while the return to the dark resistance is much slower, occupying from a few minutes to hours or even days according to the intensity and duration of the excitation.

In the hope that a study of the form of the recovery curves as affected by a variation in the initial conditions might throw some light upon the general problem, the present experiments were undertaken at the suggestion of Professor Merritt, and to him and to Professor Nichols the author wishes to express her grateful appreciation for their unfailing interest and inspiring suggestions. The variations in initial conditions studied were of three kinds: (1) a variation in the wave-length of the exciting light; (2) a variation in the intensity of excitation for constant time of exposure; (3) a variation in the duration of exposure at constant intensity. Since the excitation as well as the recovery is a process requiring time, a further study was made of the effect upon the saturation curves of (1) change in the intensity of the exciting light; (2) change in the wave-length of the exciting light.

The first experiments were concerned with the variation in the form of the recovery curve produced by a variation in the wave-length and in the intensity of the exciting light, the time of exposure remaining constant. A few earlier investigators have shown such curves. Athanasiadis,[1] for example, allowed the selenium cell to be excited a long time and then suddenly brought into darkness. The time of recovery for a cell thus excited to saturation, he finds to be approximately fifty or sixty hours. The curves are plotted as time resistance curves and show in a general way the initial rapid recovery gradually diminishing in rapidity as the dark resistance is approached. Ruhmer[2] studied the excitation and recovery of five cells which he divides into two kinds, "hard" and "soft." Soft cells were those annealed at a temperature of 200°; hard cells those annealed at a lower temperature and cooled more quickly. The soft cells he found to be relatively more sensitive to weak excitations, while the hard cells showed a more rapid recovery than the soft. The curves as plotted show the relation of resistance to time, both for five minute excitation and for the ensuing recovery. The data when replotted with the reciprocal square roots of the changes in conductivity as the ordinates, give curves similar in form to those obtained by the writer.

The general method of the experiments carried on by the writer was to measure the dark resistance of the cell, then expose it for a definite number of seconds to the light, note the resistance at the instant when the light was shut off and at frequent intervals during the return to the normal dark resistance. The cells used were both of the Bidwell[3] and the Ruhmer types,[4] while for the excitation there were used the light of the arc lamp dispersed by a flint-glass prism and the light of a 16-c.p. incandescent lamp screened by red and green glasses. As a current source a single storage cell was used throughout the experiments. The resistance was measured by means of a Wheatstone bridge of which the cell formed one arm. The bridge was first balanced for a known resistance approximately equal to that of the cell and calibrated by substituting known resistances in place of the cell and noting the deflections produced upon the graduated scale by a beam of light reflected from the galvanometer mirror. A curve was plotted with scale deflections as ordinates and the corresponding resistances above and below equilibrium resistance as abscissæ. From this curve the resistance of the cell for any observed deflection could be determined. The method, while less accurate than the zero method, possessed the obvious advantage of rapidity, thus making possible the obtaining of any desired number of points upon the recovery curves.

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[1] G. Athanasiadis, Annalen der Physik, Vol. 25, p. 1.

[2] Ernst Ruhmer, Phys. Zs., Vol. 3, p. 468.

[3] In the Bidwell cell the electrodes are two fine platinum or copper wires perhaps a meter in length, wound in alternate notches upon a rectangular plate of mica or slate, at distances apart varying from .06 to .1 cm. The thin coating of selenium is melted upon one side and the cell sensitized by prolonged heating.

[4] In the Ruhmer type the wires are wound upon a porcelain cylinder enclosed in a glass vacuum tube.

In the early experiments with the Bidwell cell the dark resistance of the cell was found to be far from steady, especially on dark days. The probable sources of error seemed to be inconstancy of galvanometer, variation in temperature of the cell, and the presence of a film of moisture. The first of these errors was found upon testing the galvanometer with a standard cell and known resistance to be negligible. The second and especially the third were the more serious. To obviate these difficulties the cell was placed in a water-bath with a glass window and with the cell there was included a calcium chloride tube. With these precautions the dark resistance remained comparatively steady. After each run the cell was allowed to remain in the dark, on open circuit, approximately twenty-four hours although no attempt was made to bring the initial resistance exactly to the same point before each experiment. To avoid errors due to a change in the strength of the current the galvanometer was frequently tested and if necessary recalibrated. As a final precaution, to prevent the admission of stray light to the cell, the dispersed light was allowed to pass through a light-tight box against the farther end of which the cell was placed. A metal shutter protected the face of the cell when not exposed. (See Fig. 1.)

In the first set of observations taken the exciting light was a 16-c.p. incandescent lamp, the light from which passed through a small circular opening in a tin screen and fell upon the colored glass window of a light-tight box. In this case a 10 cm. × 10 cm. water-cell was placed in the box to prevent any effect due to change in temperature, and the intensity of the light was varied by changing the size of the opening. The cell used was of the Bidwell type with a dark resistance of approximately 60,000 ohms. Observations were made for light passing through red glass; through a combination of two glasses, green and blue, since neither alone was found sufficient entirely to shut out the red rays; and finally through thin rubber. The uniform time of excitation was 20 seconds.

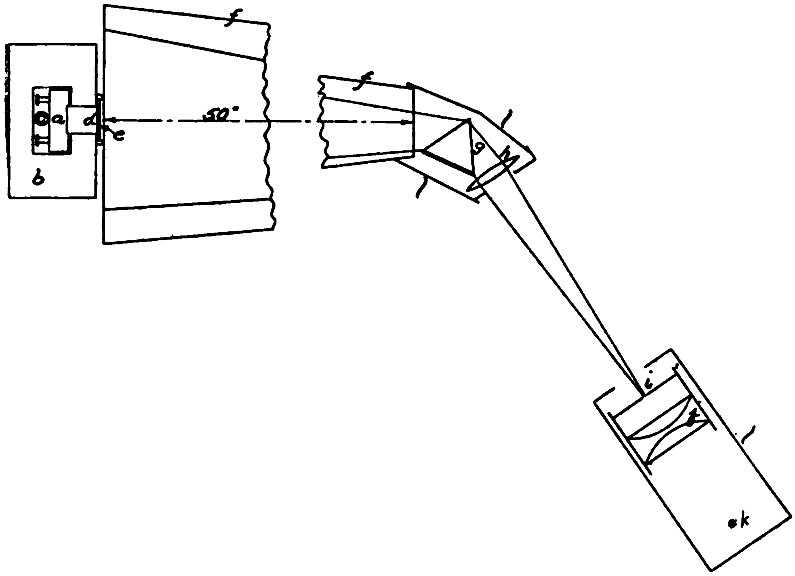

Fig. 1. *a*, Cell; *b*, tin water-bath; *c*, calcium chloride tube; *d*, glass window; *e*, metal shutter; *f*, box to prevent admission of stray light; *g*, flint glass prism; *h*. focusing lens; *i*, adjustable slit; *j*, condensing lenses; *k*, arc; *l*, screens.

In the remaining experiments the arc lamp was used for excitation as described above. In the second of the two sets there was used a cell of the Bidwell type manufactured by Max Kohl, with a dark resistance of approximately 500,000 ohms at room temperatures, and exceedingly sensitive. The third set of data was obtained with a cell of the Ruhmer type also made by Max Kohl, with a dark resistance of about 45,000 ohms. The cell was exhausted by a Gaede mercury pump and freed from moisture by repeated exhaustion and admission of dry air. It was then placed in a light-tight wooden box closed on one side by a brass shutter so that one half the selenium surface was exposed to the exciting light. In both these sets the time of excitation was 5 seconds. Readings were taken for light of three wave-lengths, blue-violet, red and infra-red. In the case of the infra-red the possibility of stray light was avoided by passing the dispersed light through a thin sheet of rubber. For convenience the essential facts concerning each set of observations are given below in tabular form.

Set.	No. of Cell.	Type.	Approximate Dark Resistance.	Source of Light.	Time of Excitation.	Figs.
I.	I.	Bidwell	60,000	16 c.p. lamp	20 sec.	4- 7
II.	*A*	"	500,000	Carbon arc	5 "	8-11
III.	*B*	Ruhmer	45,000	" "	5 "	12-15
IV.	*C*	Bidwell	9,000	" "	5 "	16-19

RESULTS OF OBSERVATIONS.

As was to be expected the time of recovery was longer when the intensity was greater, but the form of the curve was found to be essentially the same for each color and for each intensity. The curves were plotted with the reciprocal square roots of the change in conductivity, $1/\sqrt{C - C_0}$, as ordinates and the corresponding times as abscissæ. The form of the curve so plotted is similar to the curves obtained for the decay of phosphorescence in Sidot blende by Nichols and Merritt, whose results suggested this method of plotting. The curve consists of two parts nearly straight, but of decidedly different slope, connected by a gradual bend. Usually neither part is exactly straight but bends somewhat towards the axis of abscissæ. Fig. 2 shows two curves, however, in which the lines are approximately straight. These curves suggest that after prolonged rest and at absolutely constant temperature the curves might be of that form. The first of these curves was obtained with Cell I., which had been kept in darkness, on open circuit, five days, and was then excited three minutes by red light and the recovery studied for three hours. The cell used requires thirty minutes for even approximate saturation. The second of the two curves was obtained by a very brief iron spark excitation of possibly not more than a single spark. In this case the straight line form was assumed

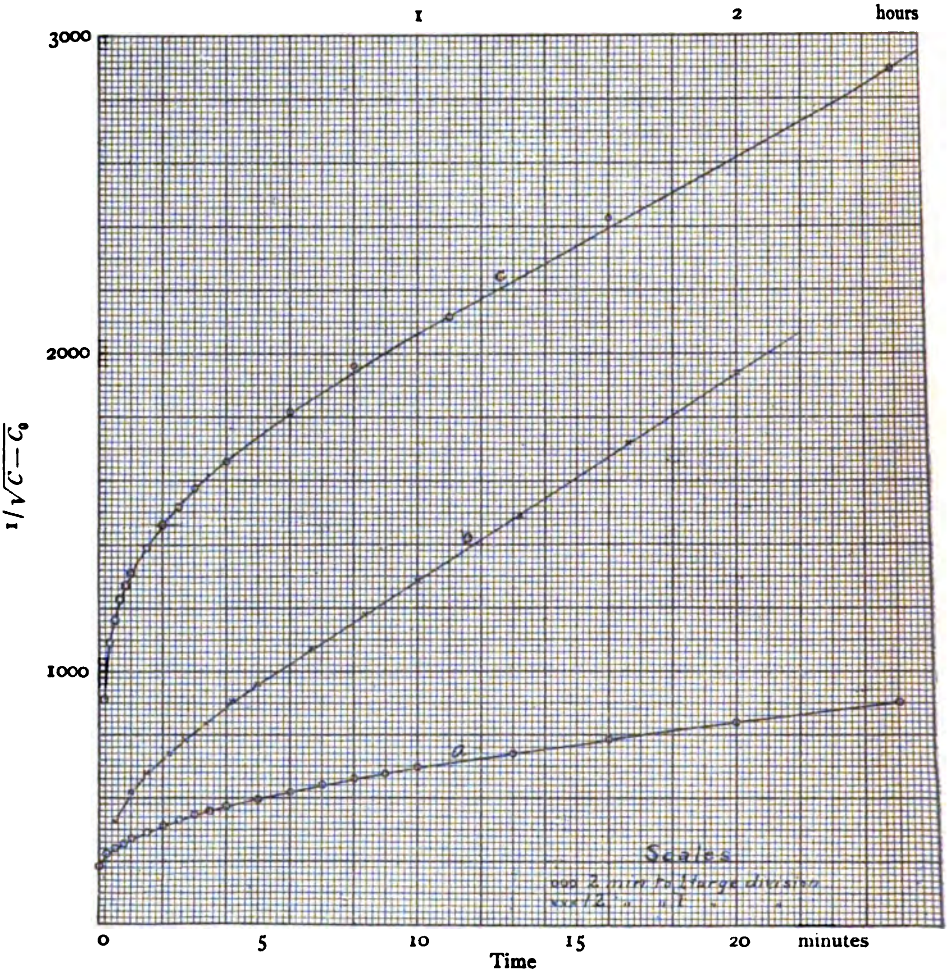

Fig. 2. Curves to show general type of recovery after prolonged rest. Cell I. *a*, *b*, Three minutes excitation by red light after five days in darkness; *c*, brief iron spark excitation.

after about ten minutes, whereas with the longer excitation there was a gradual bending during the first hour of decay.

The relation of the plotted curves to the actual observations is shown in Fig. 3, where the same data are plotted in three different ways. In curve *a* the ordinates are the observed resistances and the abscissæ the times. This curve approaches asymptotically the line of dark resistance *b*. In *c* the ordinates are the conductivities. This curve in like manner approaches the line of constant dark conductivity *d*. In *e* the data are plotted by the usual method. The quantities used as ordinates are the reciprocal square roots of the differences between the ordinates *c* and *d* $(1/\sqrt{C-C_0})$. As the values of $C-C_0$ become small, *i. e.*, as the resistance approaches

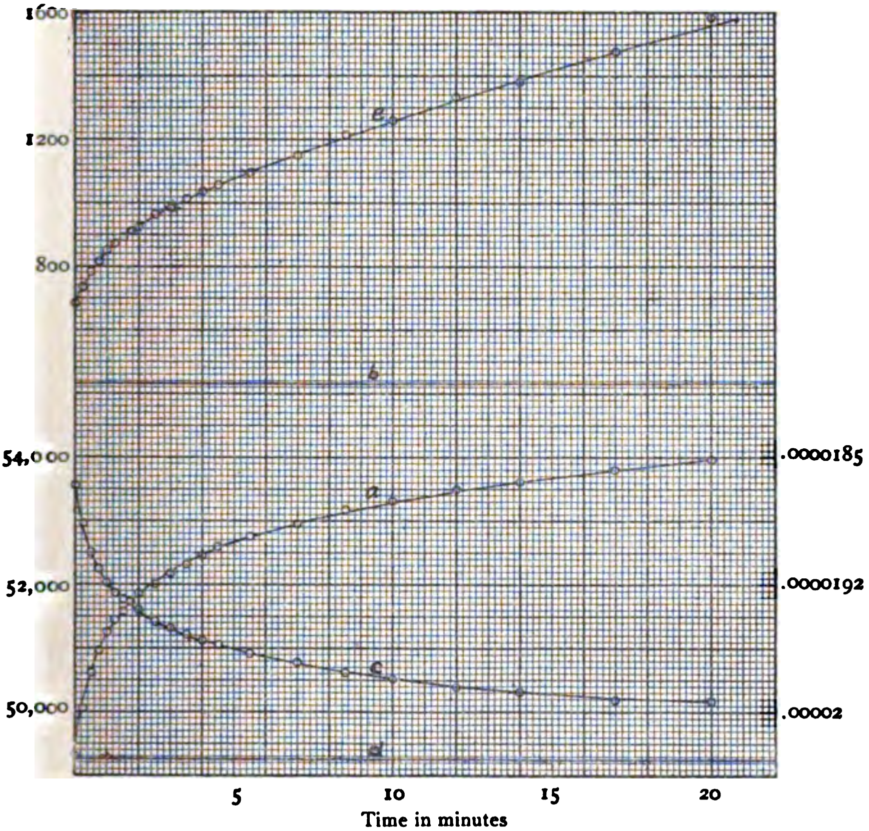

Fig. 3. Curves showing the relation of the plotted curves to the observations. *a*, Observed resistances as ordinates; *b*, line of constant dark resistance; *c*, conductivities as ordinates; *d*, line of constant dark conductivity; *e*, reciprocal square roots of the change in conductivity as ordinates.

the initial dark resistance, evidently slight errors in either C or C_0 will produce relatively large errors in the corresponding values of the ordinates of the derived curve. Hence slight variations in the form of the latter part of the curve are of less importance.

In order accurately to compare the form of the curves for the different colors, it was necessary to obtain curves for each color which should start from the same initial value of the reciprocal, $1/\sqrt{C - C_0}$. To accomplish the reduction from the curves plotted from the observed data to these comparison curves the following

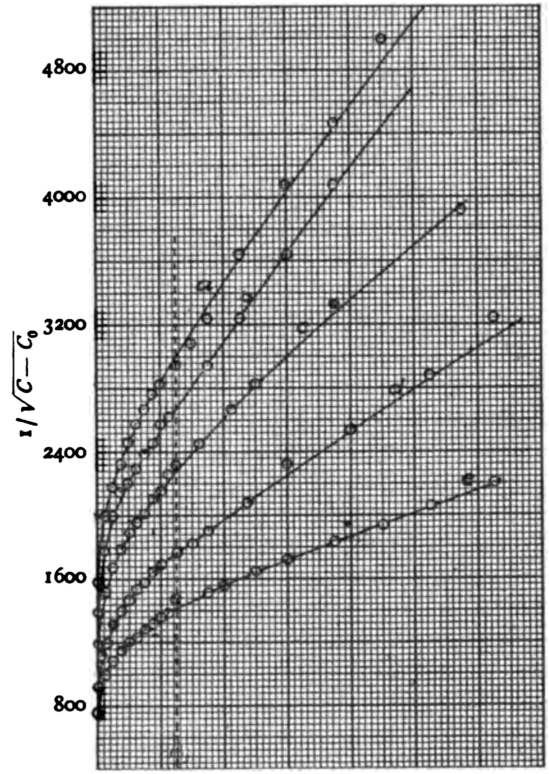

Fig. 4. Variation in the rate of recovery with intensity of excitation. Cell I. Twenty seconds excitation by green light. Initial dark resistances: *a*, 56,200; *b*, 51,090; *c*, 52,520; *d*, 52,660; *e*, 55,290.

method was used. Through the original curves for any one color lines were drawn parallel to the axis of ordinates through different points on the axis of abscissæ. (See dotted line, Fig. 4.) Secondary curves were then plotted by taking the points of intersection of a single line with the various curves as the ordinates of points the abscissæ of which were the ordinates of the initial points of the curves so intersected. These points served to determine a curve which gave the value of the ordinate, $1/\sqrt{C - C_0}$, at the particular time chosen, for any desired value of the ordinate of the initial point. For example, the curve for 30 seconds for red light gave the value of $1/\sqrt{C - C_0}$ for each of the curves of Fig. 4, 30 seconds after recovery began. By constructing a series of secondary curves for different times sufficient points were obtained to plot the curves of Fig. 7.

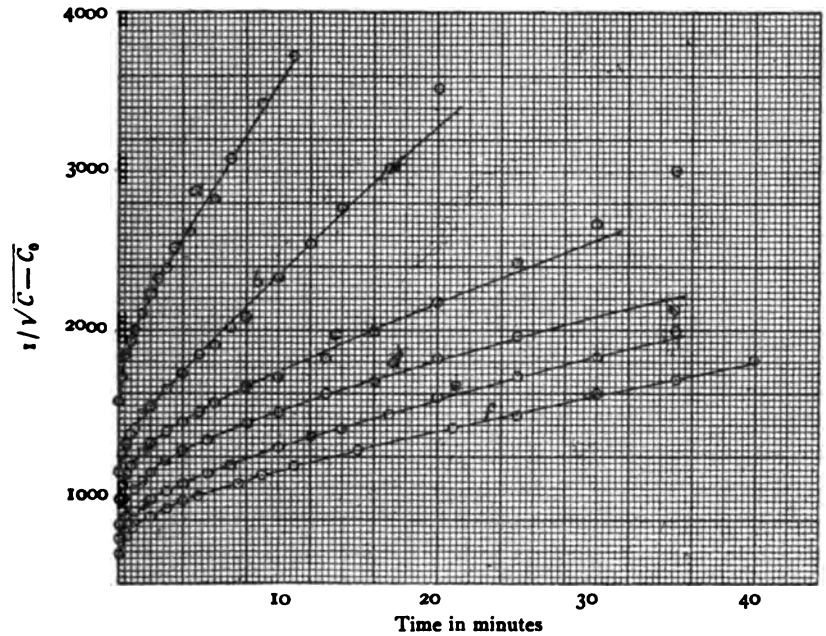

Fig. 5. Variation in the rate of recovery with intensity of excitation. Cell I. Twenty seconds excitation by red light. Initial dark resistances; *a*, 53,920; *b*, 51,040; *c*, 51,800; *d*, 52,070; *e*, 55,160; *f*, 54,010.

The sets of color curves for the four cells used are shown in Figs.

4–19. From these curves it will be noted that in each case the recovery after infra-red excitation is much slower than after red or green or violet. The difference between the two colors is not so marked. Cells A and C of the same type and make, but of widely differing sensibility, show the most rapid recovery for the red light while Cells I. and B show the quickest recovery for the shortest wave-length.

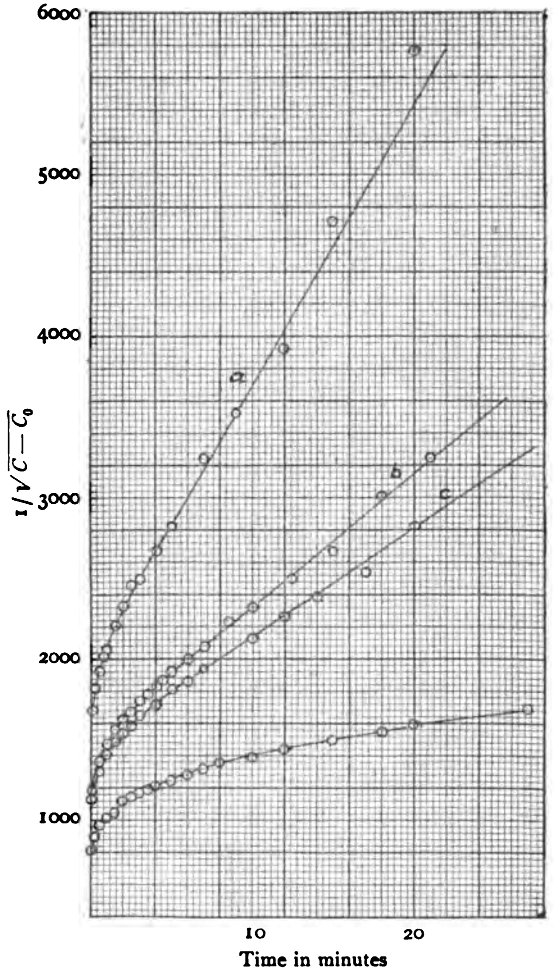

Fig. 6. Variation in the rate of recovery with intensity of excitation. Cell I. Twenty seconds excitation by infra-red light. Initial dark resistances: *a*, 53,920; *b*, 54,050; *c*, 52,210; *d*, 54,290.

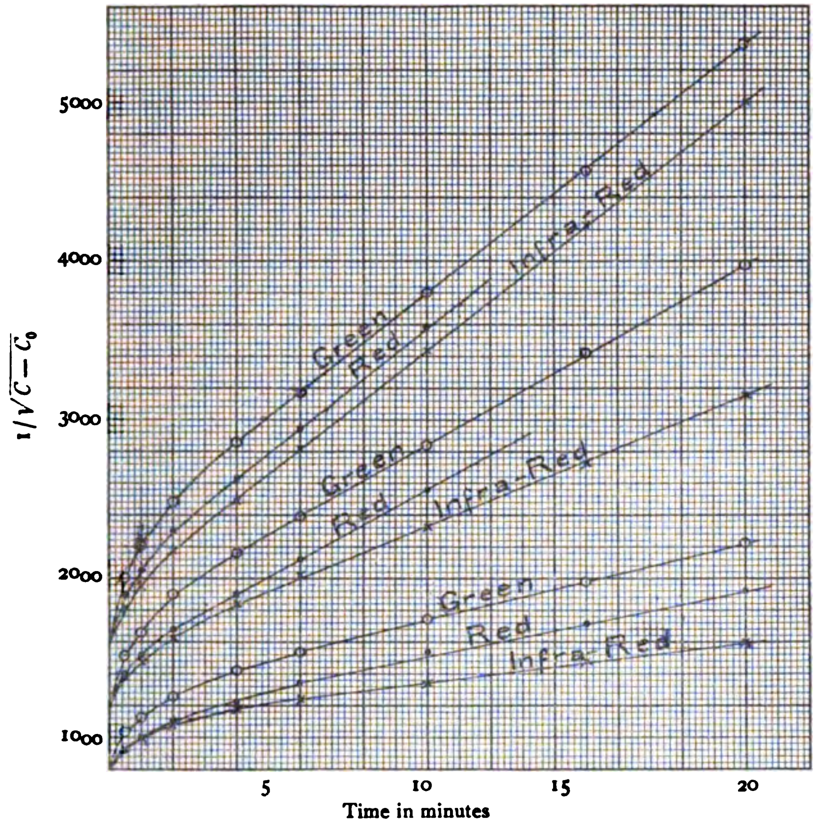

Fig. 7. Variation in rate of recovery with change in wave-length of light. Cell I.

VARIATION WITH TIME OF EXCITATION.

To determine the effect of time of excitation upon the form of the recovery curve, another set of observations was taken with the same apparatus as in the first set for study of intensity. For excitation both red and green lights were used. Curves were taken for times ranging from ten seconds to a time which produced saturation. The cells chosen were Cells I. and II., a home-made cell of the Bidwell type with a dark resistance of approximately 60,000 ohms. Both cells were extremely slow in responding to excitation and in returning to the dark resistance. Figs. 20–22 show the results plotted in the usual manner. The effect of longer time is similar to the effect of increased intensity in making the recovery slower. The curves for Cell II., especially, show for very slight differences in the initial values a rapid decrease in the slope of the curve as the time of excitation is increased. It will be noted also that all the long-time curves show a decided tendency to bend towards the axis of abscissæ.

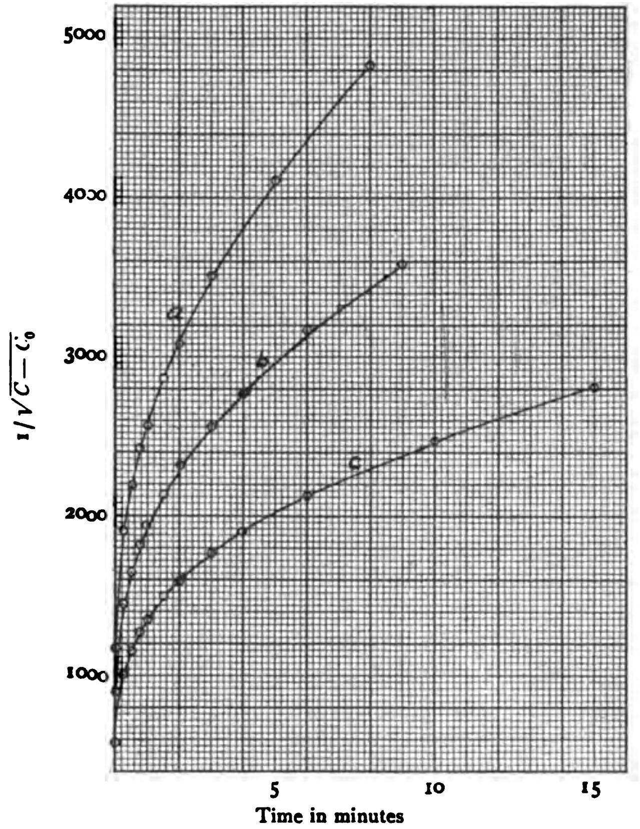

Fig. 8. Variation in rate of recovery with intensity of excitation. Cell A. Five seconds excitation by violet light. Initial dark resistances: *a*, 435,000; *b*, 459,000; *c*, 475,000.

SATURATION CURVES.

The inertia of selenium, *i. e.*, the gradual change in resistance produced by continued lighting, has been frequently noted. Marc[1] says that there is no inertia of selenium itself, but the appearance is due to the fact that the action of light is not limited to the exposed surface but spreads over the entire cell and presses into the interior. In the following study of the relation of the time of saturation to the intensity and wave-length of the exciting light, two cells were used: Cell I., which had shown the most rapid recovery for green light and Cell A, which had shown the most rapid recovery for red light. The apparatus and the precautions taken were essentially the same as in the first set of color observations with two exceptions: the incandescent lamp was replaced by an acetylene flame; and for the first experiments the colored glass screen was omitted. The source of light was placed at distances of twenty-five, fifty, and one hundred centimeters and a saturation curve taken for each

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[1] Zs. für Anorgan. Chemie, Vol. 37, p. 459.

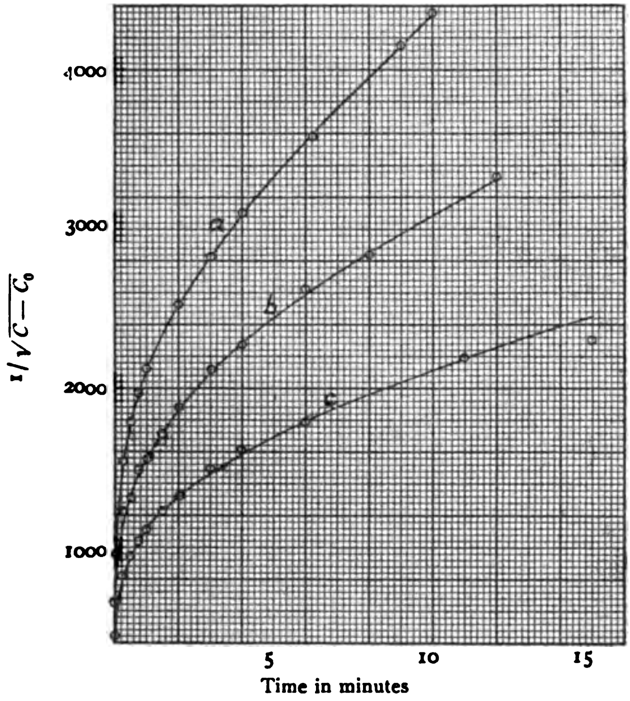


Fig. 9. Variation in rate of recovery with intensity of excitation. Cell A. Five seconds excitation by red light. Initial dark resistances: *a*, 461,000; *b*, 401,000; *c*, 440,000.

distance. In Fig. 23 three resulting curves for white light excitation are plotted with the changes in conductivity $(C - C_0)$ as ordinates and the times as abscissæ. The curves will be seen to rise rapidly at first and then bend until they become approximately parallel to the axis of abscissæ as saturation is approached. While complete saturation was not attained in four hours, 80 per cent. of the change took place in the first five minutes of excitation. In Table I. are given the ratios of the changes in conductivity at different times, *e. g.*, the change in 45 seconds to the change in 4 hours. The results indicate that the change takes place more rapidly at first in the case of the more intense light but that the rate of excitation also diminishes more rapidly with intense light so that within five minutes the percentage changes have become very nearly equal.

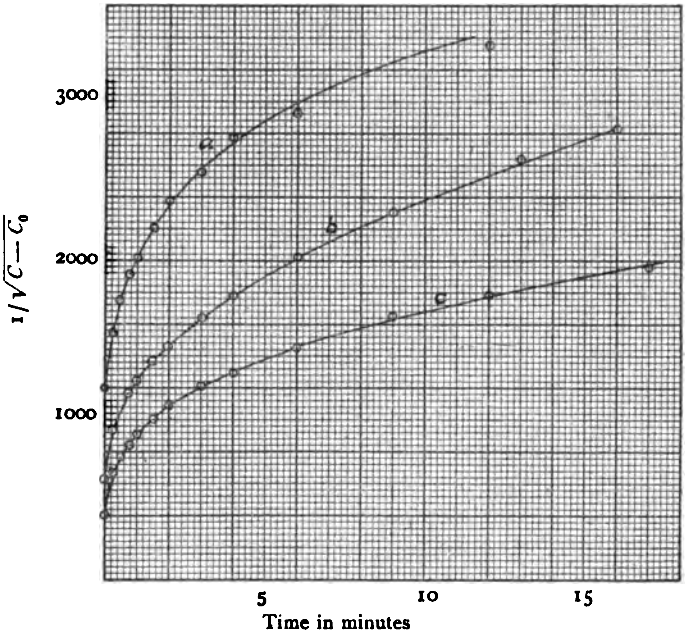


Fig. 10. Variation in rate of recovery with intensity of excitation. Cell A. Five seconds excitation by infra-red light. Initial dark resistances: *a*, 496,000; *b*, 488,000; *c*, 411,000.

The next attempt was to trace the dependence of time of saturation upon the color of the exciting light. For this purpose the

Table I.

Values of the Ratios, $(C_1 - C_0)/(C_2 - C_0)$, for Different Times during Excitation.
(Cell I.)

	Curve I. (100 cm.)	Curve II. (50 cm.)	Curve III. (25 cm.)
45 sec. / 4 hrs.	.515	.663	.666
5 min. / 4 hrs.	.773	.848	.779
30 min. / 4 hrs.	.871	.933	.882
1 hr. / 4 hrs.	.927	.960	.928
1 min. / 1 hr.	.614	.734	.740
5 min. / 1 hr.	.834	.884	.840
30 min. / 1 hr.	.964	.974	.950

The ratios of the final resistance to the initial resistance were: 100 cm., 57.4 per cent.; 50 cm., 51.6 per cent.; 25 cm., 32.8 per cent.

colored glass screens were replaced in the apparatus. In Table II. are given for Cell I. the ratios for red and green light compared with similar ratios for white light of varying intensity.

It will be noted that although the final percentage change due to

Table II.

Values of the Ratios, $(C_1 - C_0)/(C_2 - C_0)$, for Different Colors. (Cell I.)

	Green.	Red.	White.	
			(100 cm.)	(50 cm.)
30 sec. / 45 min.	.310	.306	.481	.631
1 min. / 45 min.	.401	.459	.624	.745
5 min. / 45 min.	.741	.695	.846	.896
10 min. / 45 min.	.857	.802	.922	.935
30 min. / 45 min.	.979	.967	.976	.986

The ratios of the resistance attained in a 45 minute excitation to the initial resistance were: green, 76.4 per cent.; red, 70.5 per cent.; white, 100 cm., 59.7 per cent.; 50 cm., 54.6 per cent.

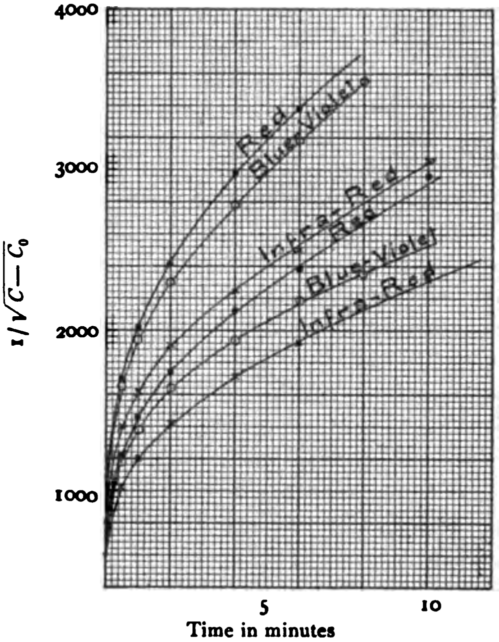


Fig. 11. Variation in rate of recovery with change in wave-length of light. Cell A.

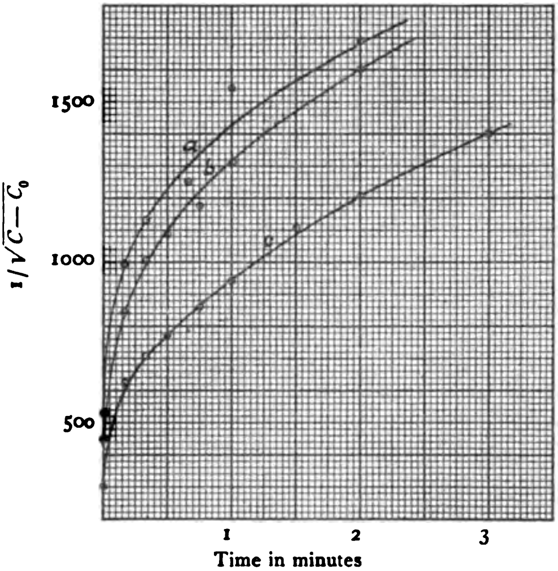


Fig. 12. Variation in rate of recovery with intensity of excitation. Cell B. Five seconds excitation by violet light. Initial dark resistances: *a*, 37,900; *b*, 39,700; *c*, 34,600.

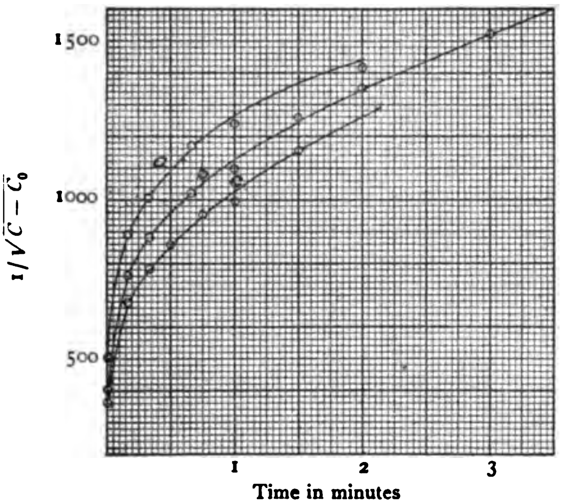


Fig. 13. Variation in rate of recovery with intensity of excitation. Cell B. Five seconds excitation by red light. Initial dark resistances : *a*, 37,500 ; *b*, 36,000 ; *c*, 35,100.

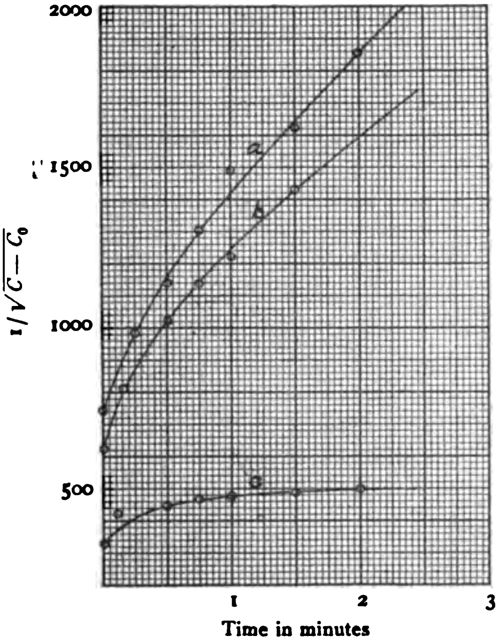
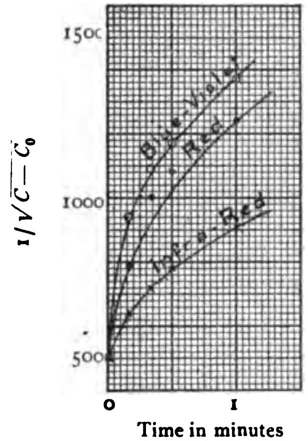


Fig. 14. Variation in rate of recovery with intensity of excitation. Cell B. Five seconds excitation by infra-red light. Initial dark resistances : *a*, 34,700 ; *b*, 33,750 ; *c*, 40,700.

the red was greater than that due to the green, the rate of saturation in the case of the green was greater than in the case of the red. Had the intensities been so chosen as to produce the same final effect we should expect to find this difference yet more marked, since, as was



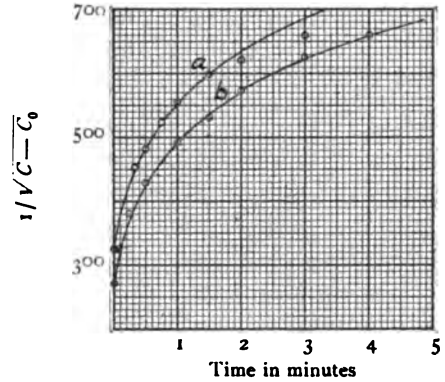


Fig. 15. Variation in rate of recovery with change in wave-length of light. Cell B.

Fig. 16. Variation in rate of recovery with intensity of excitation. Cell C. Five seconds excitation by violet light. Initial dark resistances: *a*, 8,595; *b*, 8,505.

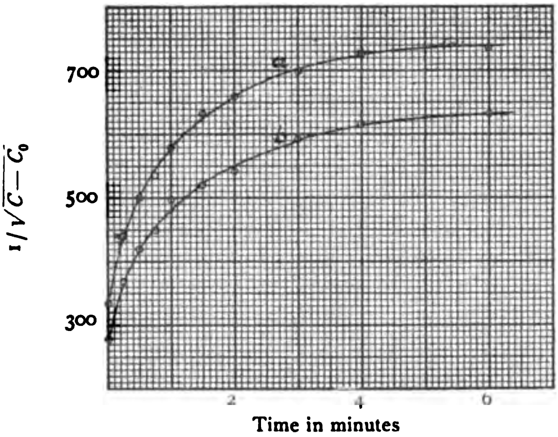
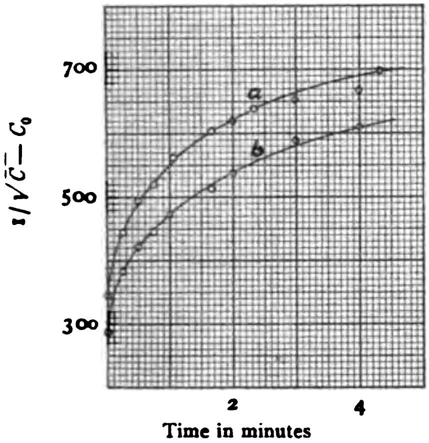


Fig. 17. Variation in rate of recovery with intensity of excitation. Cell C. Five seconds excitation by red light. Initial dark resistances: *a*, 9,760; *b*, 9,705.

shown in the case of white light, the effect of increasing intensity is to increase the rate of saturation for the first few minutes. This result corresponds to the fact earlier observed that for the cell in question the recovery for green light is more rapid than for red. (See Fig. 7.) A comparison of the time of saturation for red and infra-red light of varying intensity, for the same cell, with the same apparatus, gave the results indicated in Table III.



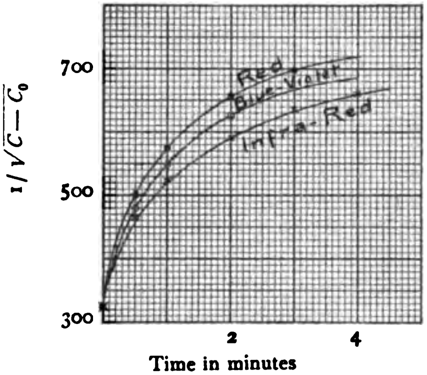


Fig. 18. Variation in rate of recovery with intensity of excitation. Cell C. Five seconds excitation by infra-red light. Initial dark resistances: *a*, 10,125; *b*, 8,040.

Fig. 19. Variation in rate of recovery with change in wave-length of light. Cell C.

For each color saturation was reached more quickly for the less intense light. A study of the percentages shows, however, that the rate of excitation for the more intense light is greater at first. This result is shown graphically for red light in the curves of Fig. 24, where the values of these ratios are plotted against the times. For exact results the denominator of each ratio should have been the change $(C - C_0)$ when complete saturation was attained but, as can be seen from the table, the error introduced by taking the 2 ½ hour ratio is slight and does not seriously modify the results. For the least intense light complete saturation was attained in one hour, hence the one hour ratio was used.

A comparison of the results for excitation by red and infra-red light at a distance of 50 centimeters, where the final effects produced

TABLE III.

Values of the Ratios, $(C_1 - C_0)/(C_2 - C_0)$, *for Different Times during Excitation.*
(Cell I.)

	Red.			Infra-red.	
	(100 cm.)	(50 cm.)	(25 cm.)	(50 cm.)	(25 cm.)
$\frac{\text{10 sec.}}{2\frac{1}{2}\text{ hrs.}}$		.340	.449	.192[1]	.282
$\frac{\text{30 sec.}}{2\frac{1}{2}\text{ hrs.}}$		.559	.601	.371	.457
$\frac{\text{1 min.}}{2\frac{1}{2}\text{ hrs.}}$		.687	.655	.493	.572
$\frac{\text{5 min.}}{2\frac{1}{2}\text{ hrs.}}$		.847	.766	.746	.741
$\frac{\text{10 min.}}{2\frac{1}{2}\text{ hrs.}}$		.889	.813	.828	.807
$\frac{\text{30 min.}}{2\frac{1}{2}\text{ hrs.}}$		.945	.894	.949	.906
$\frac{\text{1 hr.}}{2\frac{1}{2}\text{ hrs.}}$		.968	.946	.992	.942
$\frac{\text{2 hrs.}}{2\frac{1}{2}\text{ hrs.}}$		.992	.985	1.000	.986
$\frac{\text{10 sec.}}{\text{1 hr.}}$	.197	.351	.475	.194	.299
$\frac{\text{30 sec.}}{\text{1 hr.}}$	.372	.577	.636	.374	.486
$\frac{\text{1 min.}}{\text{1 hr.}}$	.533	.710	.693	.497	.606
$\frac{\text{5 min.}}{\text{1 hr.}}$	.860	.875	.809	.751	.787
$\frac{\text{10 min.}}{\text{1 hr.}}$	.929	.918	.859	.834	.858
$\frac{\text{30 min.}}{\text{1 hr.}}$	.987	.975	.945	.957	.962

The ratios of the final resistance to the initial resistance were for red light: 100 cm., 72.8 per cent.; 50 cm., 58.2 per cent.; 25 cm., 35.2 per cent. For infra-red: 50 cm., 59.8 per cent.; 25 cm., 41 per cent.

were approximately the same, shows that infra-red excitation proceeds more slowly than red. This again confirms the fact that recovery for infra-red light was found to be slower than for red. (See Figs. 7, 11, 15, 19.)

A rather curious result was obtained by dividing the ratios by the initial conductivity, which varied somewhat from time to time.

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[1] The denominator in this case was the value of $C - C_0$ for two hours, the time at which the cell appeared to be saturated.

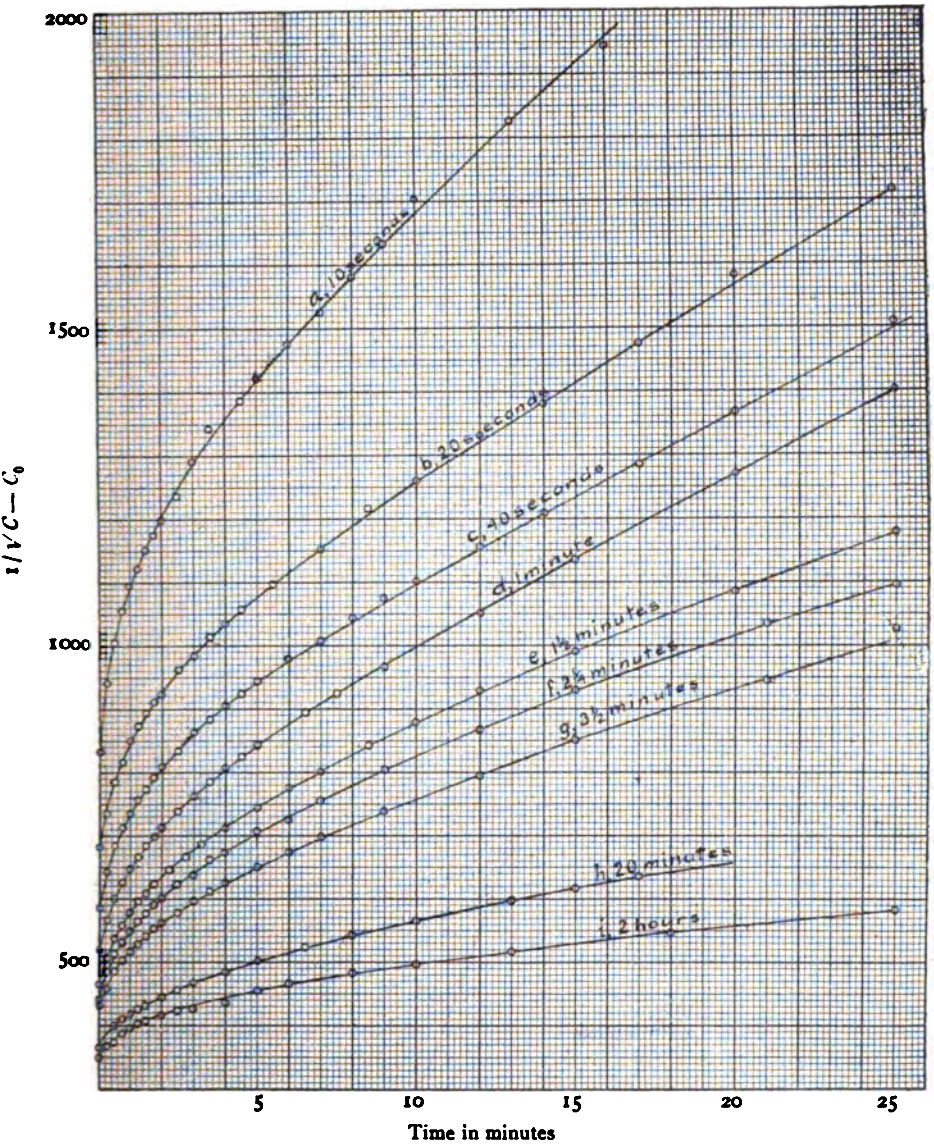

Fig. 20. Variation in rate of recovery with time of excitation. Cell I. Red light excitation. Initial dark resistances: *a*, 49,760; *b*, 55,160; *c*, 52,810; *d*, 52,870; *e*, 52,350; *f*, 53,120; *g*, 52,380; *h*, 51,480; *i*, 52,200.

These ratios, given in Table IV., will be seen to be almost identical for infra-red excitation which reduced the resistance to 41 per cent. and red which reduced it to 58 per cent. of the dark resistance.

Table IV.

Values of the Ratio, $\dfrac{\dfrac{C_1 - C_0}{C_2 - C_0}}{C_0}$*, for Different Times during Excitation by Red and Infra-red Light.* (*Cell I.*)

	Red.			**Infra-red.**	
	(100 cm.)	**(50 cm.)**	**(25 cm.)**	**(50 cm.)**	**(25 cm.)**
$\frac{10 \text{ sec.}}{2\frac{1}{2} \text{ hrs.}}$	10,500	17,050	26,600	9,250	16,950
$\frac{30 \text{ sec.}}{2\frac{1}{2} \text{ hrs.}}$	19,600	28,000	35,500	17,800	27,500
$\frac{1 \text{ min.}}{2\frac{1}{2} \text{ hrs.}}$	28,400	34,400	38,800	23,700	34,400
$\frac{5 \text{ min.}}{2\frac{1}{2} \text{ hrs.}}$	45,800	42,500	45,300	35,900	44,600
$\frac{10 \text{ min.}}{2\frac{1}{2} \text{ hrs.}}$	49,500	44,600	48,100	39,900	48,600
$\frac{30 \text{ min.}}{2\frac{1}{2} \text{ hrs.}}$	52,500	47,400	52,900	45,700	54,500

The initial values of the conductivity were: red, 100 cm., 1/53,250; 50 cm., 1/50,100; 25 cm., 1/59,150; infra-red, 50 cm., 1/48,100; 25 cm., 1/60,150.

The experiments with Cell A were with the same apparatus except that for the green excitation the metal screen was removed and the light of the acetylene flame allowed to fall directly on the green glass window in order to obtain an effect comparable with that for the red. The results are shown in Table V. and Fig. 25.

These results differ from the results for Cell I. much as the recovery curves for the two cells differ. Apparently the time in which saturation is attained is very nearly independent of the intensity of excitation, but, as in the case of Cell I., the rate of excitation for the first minute increases rapidly with the intensity. A comparison of the results for green, and for red excitation at 100 centimeters, which gave approximately the same reduction in resistance indicates that the excitation by red light is more rapid than that by green light except for the first few seconds. These results agree fairly closely with the results obtained for recovery (see Fig. 11).

TABLE V.

Values of the Ratio, $(C_1 - C_0)/(C_2 - C_0)$, for Different Times during Excitation.
(Cell A.)

	Green.	Red.			Infra-red.		
	(25 cm.)	(100 cm.)	(50 cm.)	(25 cm.)	(100 cm.)	(50 cm.)	(25 cm.)
$\frac{10 \text{ sec.}}{2\frac{1}{2} \text{ hrs.}}$	.325	.208	.453	.569	.165	.318[1]	.423
$\frac{30 \text{ sec.}}{2\frac{1}{2} \text{ hrs.}}$	.427	.371	.620	.691	.294	.449	.553
$\frac{1 \text{ min.}}{2\frac{1}{2} \text{ hrs.}}$	.494	.490	.722	.752	.382	.558	.629
$\frac{5 \text{ min.}}{2\frac{1}{2} \text{ hrs.}}$	.643	.726	.856	.863	.632	.737	.779
$\frac{10 \text{ min.}}{2\frac{1}{2} \text{ hrs.}}$	.705	.795	.886	.896	.729	.794	.835
$\frac{30 \text{ min.}}{2\frac{1}{2} \text{ hrs.}}$	.793	.881	.928	.934	.840	.898	.910
$\frac{1 \text{ hr.}}{2\frac{1}{2} \text{ hrs.}}$	.853	.929	.963	.962	.914	.949	.951
$\frac{2 \text{ hrs.}}{2\frac{1}{2} \text{ hrs.}}$	.951	.985	.988	.988	.983	1.000	.992
$\frac{10 \text{ sec.}}{3\frac{1}{2} \text{ hrs.}}$	.303	.203	.443		.159		
$\frac{30 \text{ sec.}}{3\frac{1}{2} \text{ hrs.}}$	.397	.359	.612		.283		
$\frac{1 \text{ min.}}{3\frac{1}{2} \text{ hrs.}}$	.460	.478	.711		.368		
$\frac{5 \text{ min.}}{3\frac{1}{2} \text{ hrs.}}$	.598	.709	.838		.608		
$\frac{10 \text{ min.}}{3\frac{1}{2} \text{ hrs.}}$	.657	.777	.866		.701		
$\frac{30 \text{ min.}}{3\frac{1}{2} \text{ hrs.}}$	.740	.859	.907		.808		
$\frac{1 \text{ hr.}}{3\frac{1}{2} \text{ hrs.}}$	.794	.907	.941		.879		
$\frac{2 \text{ hrs.}}{3\frac{1}{2} \text{ hrs.}}$	.885	.963	.967		.945		
$\frac{2\frac{1}{2} \text{ hrs.}}{3\frac{1}{2} \text{ hrs.}}$	.931	.975	.978		.962		
$\frac{3 \text{ hrs.}}{3\frac{1}{2} \text{ hrs.}}$	.965	.991	.992		.980		

The ratios of the resistance attained in 2½ hours to the initial resistance were: green, 45.8 per cent.; red, 100 cm., 46.9 per cent.; 50 cm., 26.4 per cent.; 25 cm., 17.5 per cent.; infra-red, 100 cm., 51.2 per cent.; 50 cm., 27.7 per cent.; 25 cm., 23.5 per cent. The ratios for 3½ hours were: green, 43.2 per cent.; red, 100 cm., 46.3 per cent.; 50 cm., 25.9 per cent.; infra-red, 100 cm., 50.2 per cent.

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[1] The denominator for the ratios of this column was $C - C_0$ for two hours.

While the form of the saturation curve was not determined for other cells it was noted in connection with the experiments upon sensibility and wave-length that the rate of saturation of Cell B for infra-red light was decidedly slower than for red, green, or violet light. Whereas in the exposure to visible light the greatest effect was obtained during the first three seconds, when the cell was exposed to infra-red light the resistance continued to fall with fair regularity during the entire thirty seconds.

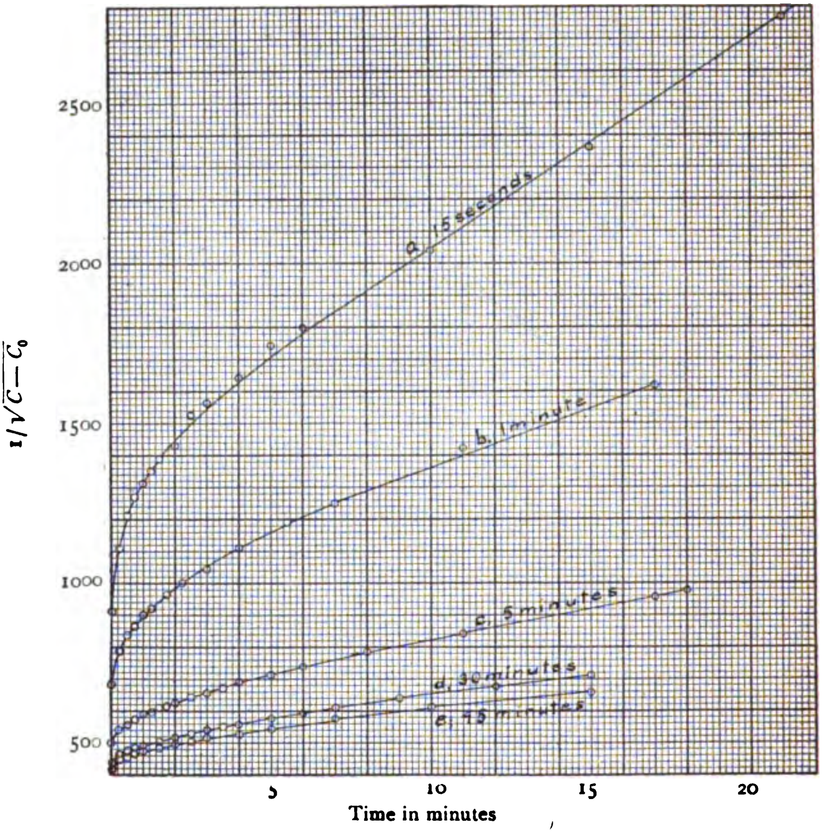

Fig. 21. Variation in rate of recovery with time of excitation. Cell I. Green light excitation. Initial dark resistances: *a*, 54,080; *b*, 52,450; *c*, 52,140; *d*, 51,710; *e*, 54,020.

From the results of the experiments upon recovery and excitation it is evident that there is a marked difference in the behavior of selenium towards light of different wave-lengths and that this behavior varies with the individual cell. May this difference not be due to the difference in the penetrating powers of the light of varying intensities and wave-lengths?

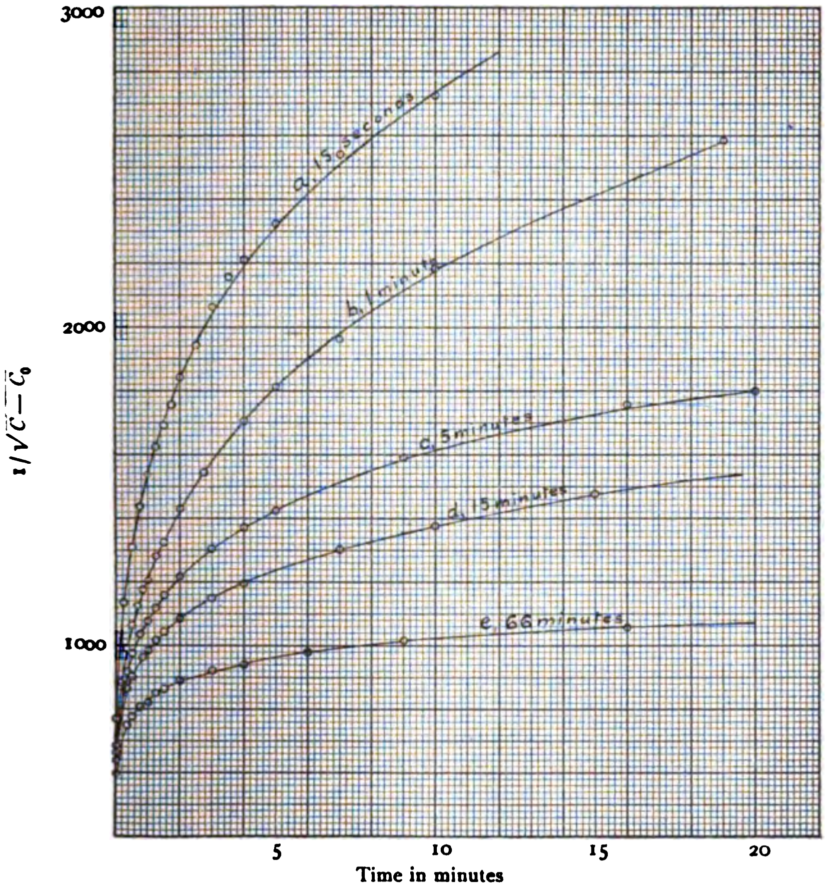

Fig. 22. Variation in rate of recovery with time of excitation. Cell II. Red light excitation. Initial dark resistances: *a*, 58,630; *b*, 58,500; *c*, 61,320; *d*, 60,160; *e*, 62,840.

RESISTANCE AND E.M.F.

In order to compare the relative behavior of the cells studied in other respects than towards color the dark resistance of the cells

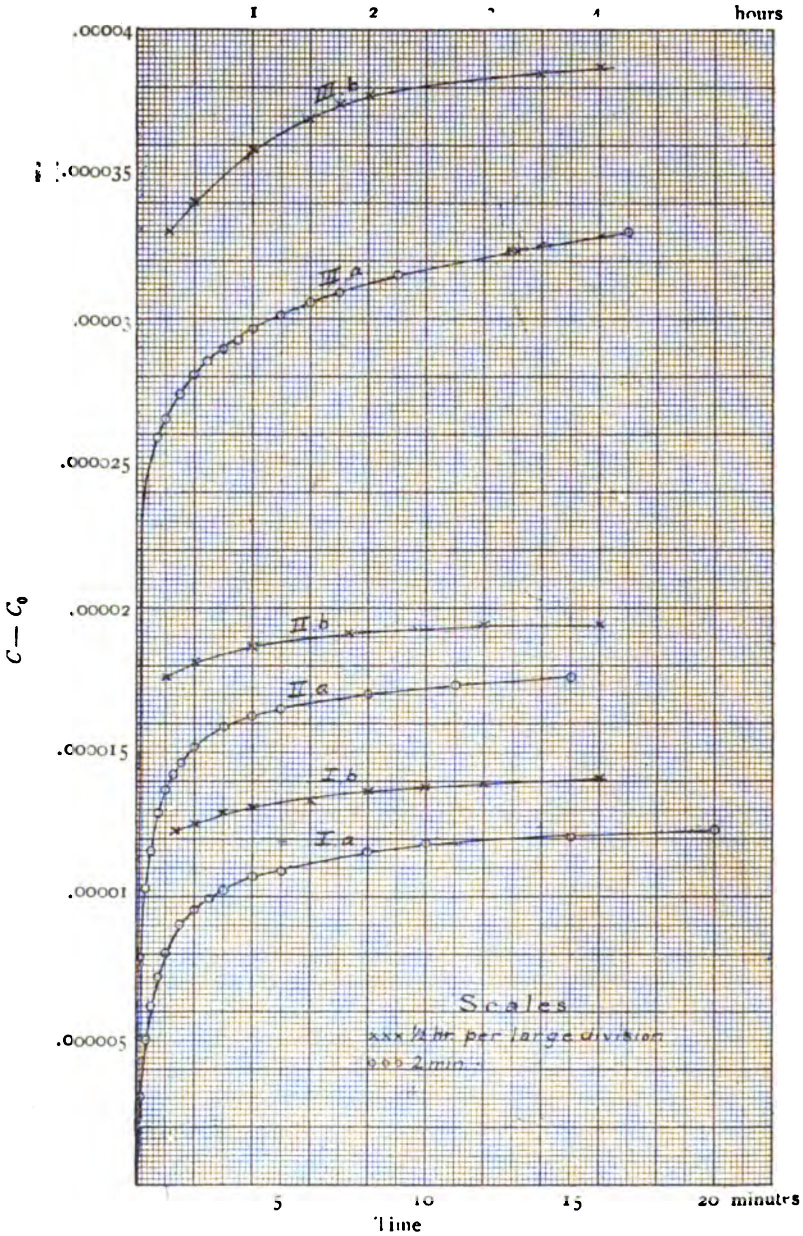

Fig. 23. Saturation curves for different intensities of excitation. Cell I. Distance of exciting light. I., 100 cm.; II., 50 cm.; III., 25 cm. Initial dark conductivities: I., .0000190; II., .0000205; III., .0000198.

was measured when currents of different strengths were sent through them. Currents ranging from that due to a single storage cell to that due to five cells in series were passed through each cell and the corresponding resistances noted. To compensate for any errors due to increase of temperature or permanent changes produced in the cells by the stronger currents, measurements were taken for both increasing and decreasing E.M.F.'s. Curves plotted with measured resistances as ordinates and E.M.F.'s as abscissæ (see Fig. 26) are almost straight lines of similar slope. The two return curves which are of decidedly different slope may probably be explained by a change in temperature as both were for sensitive cells. These results lead to no conclusion regarding variation in cell behavior.

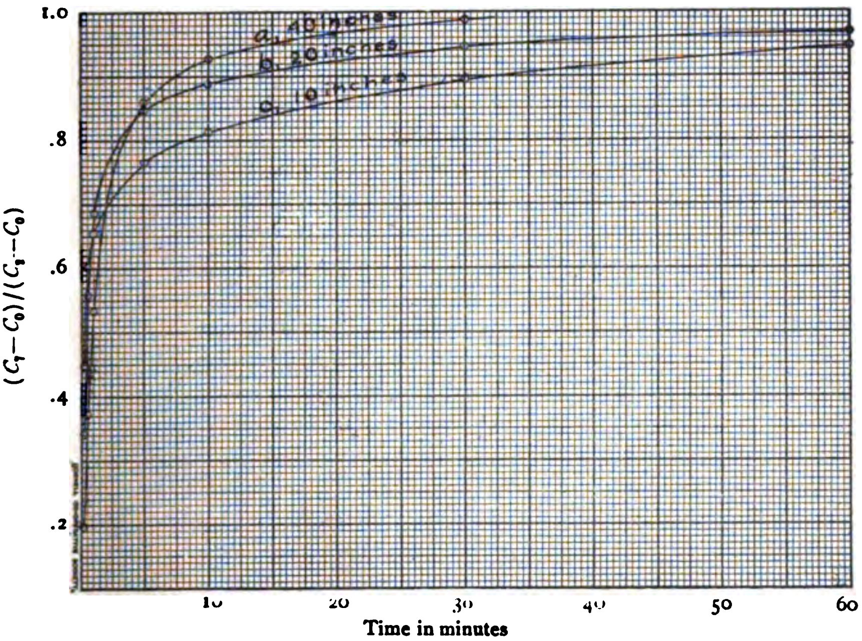

Fig. 24. Relation of rate of excitation to intensity of exciting light. Cell I. Red light excitation.

SENSIBILITY AND WAVE-LENGTH.

As a test for the variation in the sensibility of the cells for the different wave-lengths, the spectrum formed by a Nernst glower

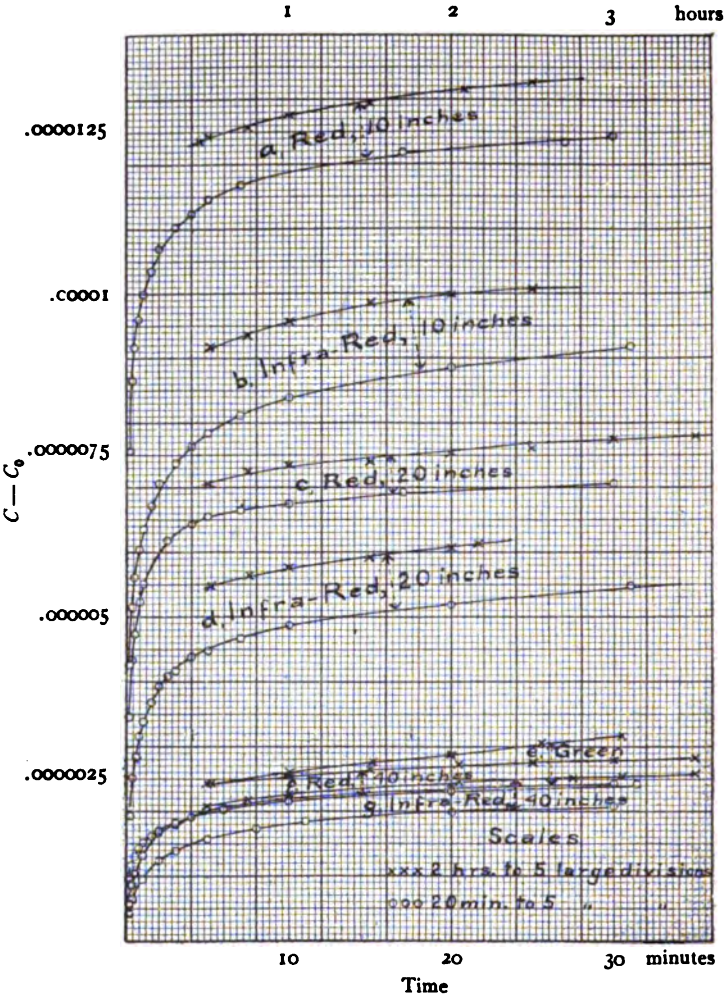

Fig. 25. Saturation curves. Cell A. Initial dark conductivities: *a*, .00000281; *b*, .00000310; *c*, .00000274; *d*, .00000233; *e*, .00000259; *f*, .00000244; *g*, .00000261.

was used. With five-minute intervals for rest, the cell was excited successively by violet, green, red and infra-red light and the resistance before and after excitation measured by the usual Wheatstone bridge method. Then after a ten-minute interval the process was repeated in the reverse direction from infra-red to violet. This method is of course open to the objection that the cell does not have time to recover between readings and therefore has its sensibility altered for the later readings. Furthermore since the intensity of excitation is different for the different wave-lengths, the stage of recovery depends upon the order of the colors used. Partially, at least, to obviate these difficulties a second set of observations was taken, after a rest of a day or more, in the reverse direction proceeding from infra-red to violet and back again to infra-red. Both sets of observations are plotted in Fig. 27. Since the spectrum of the Nernst glower is much stronger in red rays, only the relative sensibility of the different cells is shown. Below are given briefly the results for each cell.

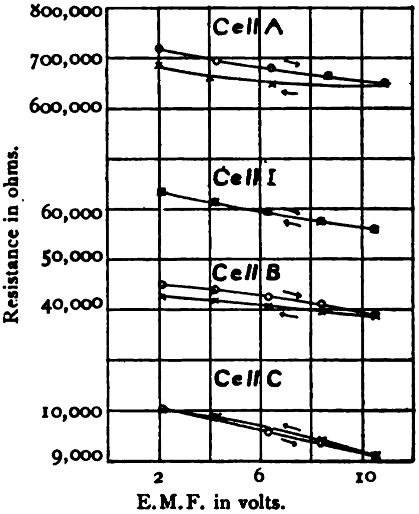

Fig. 26. Relation of resistance of cell to E.M.F.

Cell A is far more sensitive to red light and infra-red than to violet and green and relatively more sensitive to infra-red than to red.

Cell I. gives similar results.

Cell B, on the contrary, is most sensitive for red and shows towards infra-red a sensibility little greater than towards green, while the sensibility towards green is much greater than in the two cells A and I.

Cell C displays the same order of sensibility as cells A and I., but is in general less sensitive towards all colors. Moreover, there is not the wide difference between the sensibility towards red and green found in those cells.

Although these results show a difference in the behavior of the cells, the difference is not such as to explain the difference in the color curves obtained with the different cells. Incidentally it may be noted that the results contradict the statement of Marc[1] to the

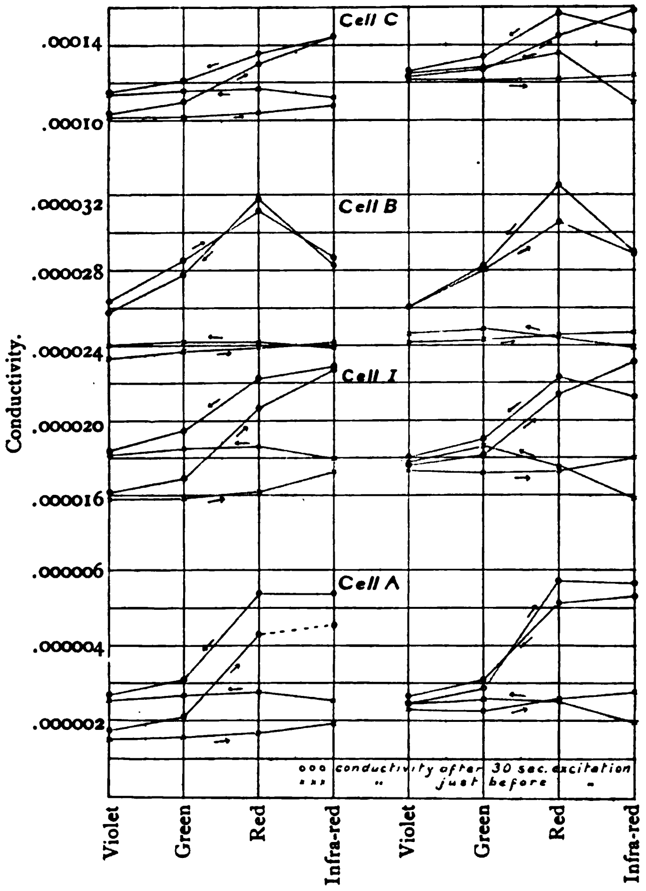

Fig. 27. Relative sensibility for light of different wave-lengths. Excitation by spectrum of Nernst lamp.

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[1] R. Marc, Zs. für anorg. Chemie, Vol. 37, p. 459.

effect that whereas continued blue lighting has a strong influence upon the sensitiveness towards red light, exposure to red light is without influence on the behavior towards blue light. Here previous exposure either to blue or red light is seen to affect the sensibility towards the other.

The Effect of Electric Oscillations.

In the study of Cell I. it was found that the recovery curves were markedly affected by the time of rest. A cell after three days in the dark showed a much more rapid recovery than after two days in the dark and that again a more rapid recovery than after one day in the dark, so that complete recovery was not effected in less than three days, perhaps not then. As a possible means of restoring the cell more quickly to standard condition, the effect of electric oscillations was tried. Previous experimenters who had studied the behavior of selenium towards electric oscillations had reached contradictory conclusions. Berndt[1] says that powerful electric waves lessen the resistance. Perreau[2] who tried the effect of electric oscil lations on a selenium cell obtained only negative results. Massini[3] concludes that electric waves do not have a sensible influence on the resistance of selenium. Any effect observable may be attributed to imperfect contacts and a coherer effect.

Since the conclusions were so uncertain it seemed worth while to test the effect upon different cells. For this purpose a circuit was set up as in Fig. 28. The primary circuit consisted of a medium-sized Leyden jar, a quarter-inch zinc spark gap, and an inductance of three turns of heavy copper wire; the secondary, of a similar

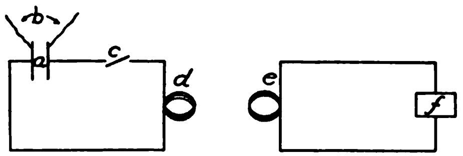

Fig. 28. *a*, Leyden jar; *b*, wires to induction coil; *c*, zinc spark gap; *d*, *e*, coils of three turns of large copper wire; *f*, selenium cell.

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[1] G. Berndt, Beiblätter, Vol. 29, p. 288.

[2] M. Perreau, Comptes Rendus, Vol. 129, p. 956.

[3] A. Massini, Nuova Cimento (5), Vol. 1, p. 358.

inductance connected directly to the cell. The distance between primary and secondary coils was varied from three to twelve inches. Cell I. was first excited strongly for one minute by red light, then placed in the dark and the resistance noted after a definite interval, say 5 minutes. Again the cell was excited, disconnected from the bridge and subjected to oscillations for two minutes. After the same interval the resistance was measured and found to be greater than in the first case. The recovery had apparently been hastened by the oscillations, but the resistance was not at once restored to the initial value. Further trials with varying distances between primary and secondary gave the following results. Very strong oscillations applied after the recovery had been in progress for some minutes caused the resistance at first to increase almost to the initial value and then to decrease by sudden leaps until in a few minutes it had attained very nearly the value given it by exposure to red light. In this condition the cell was comparatively insensitive to light. Subjection to weaker oscillations again hastened recovery as in the first instance. When the cell after subjection to this treatment was left in the dark twenty-four hours it showed a decrease in resistance greater than that due to red light and was again relatively insensitive to light. Subjection to oscillations again increased the resistance. By repeated trials a strength of oscillations was found such that the resistance was restored to its initial value and kept comparatively constant. Exposure for fifteen seconds was sufficient to produce this result. When this strength had been found, a set of color curves was taken in which the cell was subjected to oscillations between each two exposures to light. This set was found to agree fairly closely with curves taken in the usual manner with a twenty-four hour rest between exposures.

The high resistance cell, A, of great sensibility, was next tried and the oscillations found to produce upon it no immediately apparent effect. Cell B, of the Ruhmer type, which had earlier given some trouble because of a slight inconstancy in the dark resistance such as might be due to poor contacts, had its dark resistance markedly increased and rendered constant by subjection to oscillations. After treatment the cell was also somewhat more sensitive than before. After this initial effect the oscillations exerted no further observable influence, but the dark resistance maintained its higher value. Tests of three other cells showed either no effect or only a very slight hastening of the earlier stages of decay.

After these experiments were concluded the cells chanced not to be used for nearly two months. When they were again tested some peculiar results were noted. The dark resistance of Cell I. had fallen from approximately 50,000 ohms to 1,290 ohms. This resistance was unchanged at the end of twenty-four hours, when electric oscillations applied for fifteen seconds raised the resistance approximately to 60,000 ohms. During the succeeding two weeks, when the cell was excited for several hours nearly every day, the dark resistance varied between 48,000 and 60,000 ohms, apparently unaffected by a renewed application of oscillations. Cell A had previously been apparently uninfluenced by the oscillations, but its dark resistance after two months was found to have fallen to 50,000 ohms, and to be extremely unsteady. Fifteen seconds treatment with oscillations brought the resistance at once up to 430,000 ohms, where it remained for several minutes. The cell then responded to light as usual and recovered its normal dark resistance in the ordinary manner. When again tested with oscillations it was apparently uninfluenced. Cell B also showed a diminution of resistance from 50,000 ohms to 25 ohms. As the current from the single storage cell was a second time sent through the cell, the resistance for no apparent reason jumped to 3,130 ohms, but at this value it remained steady despite rapid reversals of the current through the cell. A 16-c.p. incandescent lamp at a distance of four inches reduced this resistance to 2,900 ohms, from which it slowly rose to 4,600 ohms in eighteen hours. Oscillations applied for ten seconds increased the resistance to 51,650 ohms. After six hours its resistance had decreased 2,000 ohms. Cell C when tested after two months of rest showed no influence of the oscillations. Its resistance was 8,650 ohms, a normal dark resistance for the cell.

These results appear rather contradictory. Evidently we have to deal with two effects, the immediate and the deferred. Both actions depend probably upon the condition of the cell. If the cell is in its most sensitive condition it will probably appear unaffected, but if allowed to rest in the dark for any appreciable length of time after treatment will gradually lose resistance and sensibility. A cell which is not in its most sensitive condition may have both resistance and sensibility increased by the action, but this increase may be only temporary. A possible explanation of the immediate results might be that the action is in the nature of a coherer effect; that the extreme slowness of the recovery of Cell I. is due to the massing together of particles which cling together long after the excitation has ceased. The oscillations by causing rapid vibration of the particles break up these aggregations and thus restore the cell to its normal condition. This hypothesis would serve to explain the increased value of resistance of Cell B, which may have had its particles so massed together that the aggregations would only partially break up and immediately reform, thus causing the vibrations in the dark resistance. Under the action of the electric oscillations these aggregations were broken up and the cell brought to its normal dark resistance. This explanation is borne out by the fact that when Cell B was excited by light the vibrations ceased and it was only as the dark resistance was approached that the unsteadiness was manifested. Evidently if this explanation is correct a cell, such as A, upon which no immediate effect was shown would be one in which the particles show little tendency to cohere after the excitation has ceased. A possible objection to this explanation is that sudden jars were not found to produce a similar effect. Other investigators, however, have found such an effect. Hesehus notes that a sudden blow hastens the recovery of the cell after excitation and has exactly the same effect as upon the residual charge in a Leyden jar.[1] This explanation agrees also with the statement sometimes made, to the effect that an alternating current applied to a new cell will increase its sensitiveness. Fritts, for instance, says that the resistance of his cells is raised by sending an alternating current through them and that they immediately become extremely sensitive to light, currents and other influences. Occasionally they drop back.[2]

Just how to account for the deferred effect upon this hypothesis is not so evident. Since it is the more powerful oscillations which produce this effect the more quickly, and since it may be produced in a cell in a highly sensitive condition, possibly the increase of motion due to the oscillations may cause particles already in motion to strike each other with such force as to cohere, and the aggregations once started, although at first too small to have a noticeable effect upon the conductivity, continue to attract other particles until we have a marked coherer effect.

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[1] N. A. Hesehus, Phys. Zs., Vol. 7, p. 163.

[2] C. E. Fritts, Elec. Rev., Vol. 16, p. 208.

SUMMARY.

The results of the investigation may be summarized as follows:

I. The form of the recovery curve depends on the intensity of the excitation and on the wave-length of the exciting light. In general the time of recovery increases with the intensity of the excitation. Recovery is slower for excitation by infra-red light than by light of the visible spectrum. Individual cells differ in the relative time of recovery for different wave-lengths as well as in the absolute time of recovery for light of the same intensity and wave-length.

II. The effect of increased duration of excitation upon the form of the recovery curve is similar to the effect of increased intensity.

III. Excitation requires time. While from 60 per cent. to 80 per cent. of the entire increase in conductivity may be produced in the first five minutes, for complete saturation hours are required. The rate of excitation varies with the intensity and the wave-length of the exciting light. The time required for complete saturation may decrease as the intensity of the light diminishes, but the initial rate of excitation always increases with the intensity. The variation in the rate of excitation of the individual cells for the different wave-lengths corresponds to the variation in their rate of recovery, *e. g.*, those cells which show the most rapid recovery for red light show also the most rapid excitation for red light.

IV. The relation between the resistance and E.M.F. is approximately a straight line relation. The resistance decreases as the E.M.F. increases.

V. Different cells show a variation in the relative sensibility to light of different wave-lengths, but no relation was found between this variation and the variation in the curves of recovery and excitation.

VI. Electric oscillations affect the resistance and hence the sensibility of the selenium cell. These effects are of two kinds, immediate and deferred. Both may be in the nature of coherer effects and both depend upon the condition of the cell at the time when the oscillations are applied.

PHYSICS LABORATORY,
CORNELL UNIVERSITY.

ON THE ROTATION OF CONSTANT DEVIATION PRISMS.

By H. S. Uhler.

IN each of the years 1905 and 1908 the writer has had occasion to set up a specimen of Hilger's constant deviation spectroscope and, in so doing, has experienced much satisfaction because of the excellence of the construction of these pieces of apparatus. A careful study of the various adjustments of this type of spectroscope led to the conclusion that the position of the axis of rotation of the prism table with respect both to the prism and to the axes of the collimator and telescope was a matter of greater importance than might be supposed at first glance. If, for example, the axis of rotation is not correctly placed the emergent beam from the prism will not illuminate the telescope objective symmetrically and hence the waves of light inside the telescope will not converge symmetrically to the focus at the middle of the field of view. When this is the case, the medial lines of the extra-focal images on the two sides of the true focal plane will lie on opposite sides of the optic axis of the telescope, so that, if the observer is not very careful to focus correctly, the settings of the cross-hairs on the centers of these images will cause the readings of the wave-length drum to be a little too large or slightly too small. This inconvenience would practically vanish if the lenses ordinarily furnished with spectroscopes of moderate price were sensibly achromatic for the entire visible spectrum from about 400 $\mu\mu$ to about 800 $\mu\mu$. For then the instrument would remain in perfect adjustment for all visible radiations if it were correctly focused for some one wave-length. However, since such ideal conditions are not realized, in general, it is desirable to place the axis of rotation in the best possible position so as to cause the telescope objective to be illuminated as symmetrically as is consistent with a pure rotation of the prism.

In addition to the preceding remarks, the following arguments may be adduced to show the desirability of having at one's disposal a set of comparatively simple formulæ which give the coördinates of the axis of rotation of the prism table in terms of known physical constants. It would be easier and far more accurate for the manufacturer, who has to deal with different sorts of glass, to substitute his numerical data in the formulæ than to determine anew and experimentally the position of the axis of rotation for each kind of glass and for each prism angle. Again, it is not very difficult to modify an ordinary spectrometer so as to convert the apparatus into a constant deviation spectroscope by replacing the triangular prism by a suitable quadrilateral prism and, if necessary, by displacing the collimator parallel to itself. The resulting spectroscope would not be comparable with Hilger's instruments because of the absence of the drum, but, when the circular scale was once calibrated by the aid of lines of known wave-length, the altered spectrometer would have several important advantages over the unmodified form. In order to properly remodel a piece of apparatus, in the manner just indicated, it would be unquestionably convenient to have the necessary elementary theory worked out in advance in an easily accessible article.

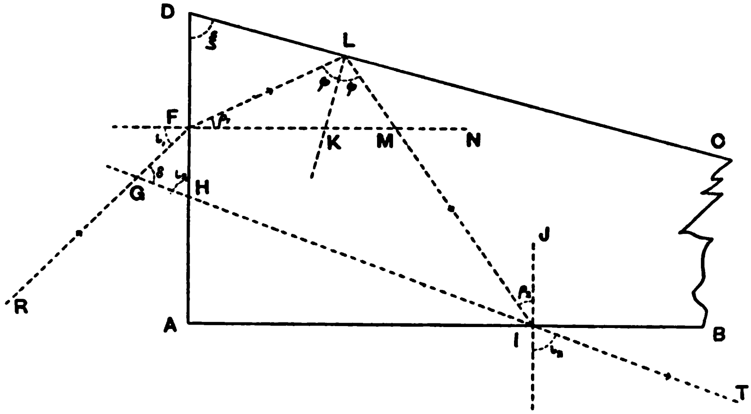

Fig. 1.

Since the writer has sought in vain to find a complete discussion of the quadrilateral prism in any practical journal or in any modern text-book on spectroscopy, and since he has been informed by

Hilger both that the manufacturers are not aware of the existence in print of such a discussion and that it would be desirable to fill up this gap in the literature of prisms, the writer has decided to present formally in the following pages the results of his investigation of the properties of the quadrilateral prism.

In Fig. 1 *ABCD* outlines a portion of a principal section of the type of prism under consideration. The earliest account of a special case of this prism is given by Pellin and Broca.[1] The angle *BAD* equals 90° and the angle *ADC* may have, in general, any value between 45° and 90°. Since we shall only be concerned with the deviation of trains of plane waves and with isotropic media the use of rays is permissible. A general expression for the total deviation of any incident ray, $\overline{RF}$, will now be derived. The path of the light is indicated by the broken line *RFLIT*. The normals to the faces $\overline{AD}$, $\overline{DC}$ and $\overline{BA}$ at the points of deviation *F*, *L* and *I* are denoted respectively by $\overline{FN}$, $\overline{LK}$ and $\overline{IJ}$. The angles to which the various symbols refer may be readily seen by inspecting Fig. 1.

$\angle FHG = \angle JIH = \iota_2$, hence, from the triangle *FGH*, it is evident that the total deviation δ is given by

$$\delta = 90^\circ + \iota_1 - \iota_2. \tag{1}$$

$\angle MKL = \xi$, since the sides of these angles are mutually perpendicular, etc., and also $\angle MKL = \rho_1 + \varphi$, from the triangle *FKL*, therefore

$$\rho_1 = \xi - \varphi. \tag{2}$$

Again, $90^\circ + \rho_2 = \angle BIM = \angle NML = \xi + \varphi$, from the triangle *KML*, hence

$$\rho_2 = \xi + \varphi - 90^\circ. \tag{3}$$

Combining equations (1), (2) and (3) with the relations $\sin \iota_1 = n \sin \rho_1$ and $\sin \iota_2 = n \sin \rho_2$, where n denotes the refractive index of the material of the prism relative to the surrounding medium, it results that

$$\delta = 90^\circ + \sin^{-1} [n \sin (\xi - \varphi)] - \sin^{-1} [n \sin (\xi + \varphi - 90^\circ)]. \tag{4}$$

When $\xi - \varphi = \xi + \varphi - 90^\circ$ or $\varphi = 45^\circ$, $\delta = 90^\circ$ independently of n and ξ. But then $\rho_1 = \xi - 45^\circ$ and $\rho_2 = \xi - 45^\circ$ so that

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[1] Ph. Pellin and André Broca, Jour. de Phys. (3), Vol. 8, p. 314, 1899.

$\rho_1 = \rho_2$. The equality of these angles in the case of a simple triangular prism is, as is well known, the necessary and sufficient condition for minimum deviation and, since the process of total internal reflection at the face $\overline{CD}$ does not alter the focal properties of the waves, it follows that when the wave-normal is deviated by 90° at internal reflection the emergent waves possess all the important properties which they would have acquired on traversing a triangular prism at minimum deviation. But, as we have just seen, when $\varphi = 45°$ the total deviation has the constant value 90° irrespective of n, that is, of the wave-length of the radiations concerned. These last facts make the Pellin and Broca prism more convenient for use in simple spectroscopes than an ordinary triangular prism of equal resolving power since, in the case of the former, it is only necessary to rotate the dispersing system, while the telescope remains fixed at the constant angle 90° with respect to the collimator, in order to bring waves of each wave-length successively to a focus at the cross-hairs under conditions most favorable to good definition.

If 2τ denotes the refracting angle of a triangular prism then, when minimum deviation prevails, $\rho_1 = \rho_2 = \tau$ so that a Pellin and Broca prism of angle ξ is equivalent to a triangular prism of angle 2τ when $\xi - 45° = \tau$. For this reason it is customary in elementary textbooks to analyze a 75° quadrilateral prism into the halves of a 60° triangular prism and a total reflecting prism of angles 45°, 45° and 90°.[1] Obviously, the rays are not literally subjected to a minimum of deviation by the quadrilateral prism and furthermore this prism differs from the equivalent triangular prism in the relatively unimportant matter of absorption. Again, in order to prevent the fourth face of the Pellin and Broca prism from interfering, by reflection or otherwise, with the passage of light whose wave-length differs a little from that of the radiations which traverse the prism at effective minimum deviation, it is desirable in practice to make the faces $\overline{DC}$ and $\overline{AB}$ somewhat longer than would result from the geometrical synthesis of the prism from the three hypothetical triangular prisms mentioned above. For example, in one of Hilger's spectroscopes the face $\overline{AD}$ equals 37.3 mm. and the faces $\overline{DC}$ and $\overline{AB}$ exceed the minimum lengths 44.6 mm. and 60.9 mm. by about 15 mm. and 5.7 mm. respectively.

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[1] See E. C. C. Baly's Spectroscopy, p. 58.

In what precedes, it has been assumed that total internal reflection takes place at the plane $\overline{CD}$ and that a finite fraction of the light emerges from the face $\overline{AB}$. These hypotheses cause restrictions to be placed upon the values of n and τ. At effective minimum deviation $\varphi = 45^\circ$ and hence the index of refraction must be greater than $\sqrt{2}$ ($= 1.4142$), in order that total reflection may occur. Again, for emergence from $\overline{AB}$, when $\varphi = 45^\circ$, the inequality $\tau < 45^\circ$ must be satisfied and from this fact, or from the circumstances which must prevail at incidence at the face $\overline{AD}$, it follows that the index of refraction must not exceed $(\sin \tau)^{-1}$.

Table I. may serve to give in concrete form an idea of the dependence of n and ι_1 upon τ when all the preceding limitations are taken into account and $\varphi = 45^\circ$ or $\rho_1 = \rho_2 = \tau$.

TABLE I.

2τ	τ	Inferior Limit of n, $= \sqrt{2}$.	Superior Limit of n, $= (\sin \tau)^{-1}$.	Inferior Limit of ι_1, $= \sqrt{2} \cdot \sin \tau$.
40°	20°	1.4142	2.9238	28° 55′ 36″
50	25	"	2.3662	36 42 12
60	30	"	2.0000	45 0 0
70	35	"	1.7434	54 12 34
80	40	"	1.5557	65 22 22
85	42.5	"	1.4802	72 49 45
89.8	44.9	"	1.4167	86 36 47

As a practical example, Jena glass number *O.* 165 has an index of refraction 1.7464 for *C* and hence a quadrilateral prism having $\tau = 35^\circ$ and made of this kind of glass could not be used to best advantage in the visible spectrum from a little "below" *C* to the ultra-violet limit. For A', $n = 1.7380$.

Again, Jena glass number *S.* 57 is not suitable for use in a quadrilateral prism having $\tau = 30^\circ$ which angle is, for well-known reasons, most frequently met with in practice. The indices of refraction for A', C, D, F and G' respectively being 1.9349, 1.9493, 1.9626, 1.9981 and 2.0306, while the superior limit is 2.0000.

The inferior limit of n, $\sqrt{2}$, is of little practical importance since the least index of refraction of the media of which prisms designed for visual or photographic purposes are ordinarily made exceeds this value.

As explained above, a suitable rotation of the quadrilateral prism between the objectives of a collimator and telescope, whose optic axes intersect at right angles, will cause the transmitted radiations to come to a focus successively upon the intersection of the cross-hairs under the very desirable conditions pertaining to minimum deviation. However, in order that the axis of the emergent beam from the prism may be made to coincide as nearly as possible with the optic axis of the telescope throughout the entire region of the spectrum, for the study of which interval the instrument is especially designed, it is essential to know the coördinates of the axis of rotation with respect both to the prism and to the optic axes of the collimator and telescope. The reasons why this coincidence should be approximately fulfilled have been suggested in an earlier paragraph.

Of course, the ideal method of procedure would be to actuate the prism by a mechanism which would constrain the center of the face $\overline{DA}$ and the effective center of the face $\overline{AB}$ to remain continuously in the optic axis of the collimator and of the telescope respectively. Theoretically this could be accomplished by any device which would be equivalent to attaching the prism to the movable beam of the type of mounting designed by Rowland for his concave grating spectrograph. The carriage tracks would then be parallel to the optic axes of the collimator and telescope. The practical realization of such a scheme which would produce absolutely perfect accuracy of adjustment would probably entail too great a complication of parts of the apparatus and hence would cause the commercial aspect of the matter to be impracticable. Furthermore, as will be shown later on, a sufficiently high degree of accuracy can be attained by a pure rotation of the prism around a fixed axis. In any particular case, the best position for this axis can be found either by actual trial or by elementary mathematical analysis. Since the substitution in comparatively simple formulæ of constants whose values have to be known in order to design the rest of the system advisedly is a perfectly general process, capable of any required degree of accuracy, the latter plan seems preferable to the former. Hence, we shall now take up the main object of this paper, which is to derive and discuss all formulæ necessary for finding the position, with respect to known lines, of the axis of rotation of any quadrilateral prism of the type under consideration. Moreover, since the problem in hand is a purely geometrical one, no reference will be made to questions of optical and spectroscopic resolving power, of purity, of achromatism, etc. In other words, it will be assumed throughout that all parts of the apparatus are as perfect as possible and that they are in accurate adjustment.

In Fig. 2 the fixed coördinate axes $\overline{OX}$ and $\overline{OY}$ represent the optic axes of the telescope and collimator respectively. The light waves proceed in the direction $\overline{YO}$ before incidence at the first prism face and along $\overline{OX}$ after emergence, that is, they move in the negative direction along the axis of ordinates and in the positive direction

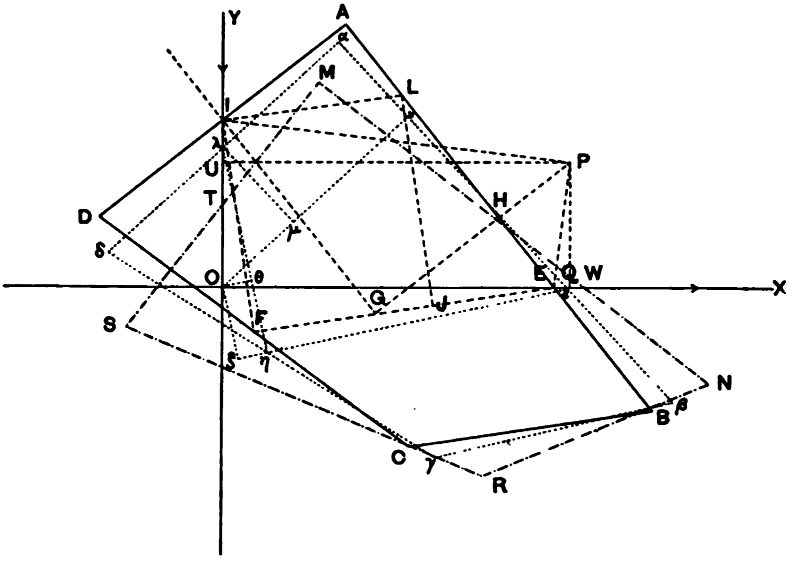

Fig. 2.

along the axis of abscissæ. Let P be the point of the axis of rotation which lies in the plane $\overline{XOY}$. The coördinates $\overline{OQ}$ and $\overline{OU}$ of P, with respect to the fixed axes, will be symbolized by x_0 and y_0. The lines of intersection with the plane $\overline{XOY}$ of the two faces of the prism which are mutually perpendicular will be used as a set of coördinate axes of variable position. The axis lying in the emergence face will be considered as the axis of abscissæ and hence the projection of the incidence face will be the axis of ordinates. The positive direction of the variable axis of abscissæ will be measured from the variable origin along the actual surface of the material prism, whereas the positive direction of the variable axis of ordinates will be reckoned from this origin along the geometrical extension of the incidence face. In one position of the prism A is the variable origin, $\overline{AH} = x'$ and $\overline{HP} = y'$. For sake of brevity the phrase "the center of the emergence face" will be used to denote the point where the axis of a narrow pencil of light meets the emergence face when this axis intersects the incidence face at the geometrical center of the latter and when the light has passed through the prism at effective minimum deviation. Obviously, rotation of the prism about the axis through P can only cause the centers of the incidence and emergence faces to lie simultaneously in the optic axes of the collimator and telescope respectively for two positions of the prism, at most. In Fig. 2 these two orientations of the prism are indicated by $\overline{ABCD}$ and $\overline{MNRS}$. Since the problem in hand is to so determine the four coördinates of the axis of rotation as to cause the axis of the emergent beam to depart as little as possible from coincidence with the optic axis of the telescope, within a given angular interval, it is evident that this condition can be realized best by including the two positions $\overline{ABCD}$ and $\overline{MNRS}$ in the given interval. This angular interval depends upon factors with whose choice we are not here directly concerned, namely, upon the limits of the region of the spectrum for the study of which the instrument is designed and hence upon the indices of refraction of the material of the prism corresponding to the ends of this region.

Therefore, we shall derive formulæ first for x_0, y_0, x' and y' in terms of the width b of the incidence face, of the angle τ, and of the angles of incidence i_1 and i_2 corresponding respectively to the orientations $\overline{MNRS}$ and $\overline{ABCD}$. A later step in the argument will be to find a connection between the angles of incidence pertaining to the ends of the given interval of rotation and the angles i_1 and i_2.

The points T and I represent the first and second positions on the axis OY of the center of the incidence face and likewise the points W and E show the simultaneous locations on the axis $\overline{OX}$

of the center of the emergence face. In order to find the point P around which the line joining the centers of these faces must be rotated so as to move this line from the position $\overline{TW}$ to $\overline{IE}$ it is only necessary to construct the perpendicular bisectors of the segments $\overline{EW}$ and $\overline{TI}$. The middle points of $\overline{EW}$ and $\overline{TI}$ are denoted by Q and U in Fig. 2 and hence the perpendicular bisectors are the lines $\overline{QP}$ and $\overline{UP}$. In the diagram $i_1 > i_2$ and $\overline{MS} = a\delta = \overline{AD} \equiv b$.

It is convenient to commence by finding an expression for the length of $\overline{AE}$, that is, the distance from the edge through A to the center of the emergence face. The broken line $\overline{IFE}$ indicates the axis of a narrow pencil of light during the passage of the waves through the prism and therefore, since minimum deviation is alone under consideration, $\angle EFI = 90°$. Draw $\overline{IL}$ through I so as to make the angle AIL equal to τ. Also let $\overline{JL}$ be constructed perpendicular to $\overline{IL}$ and meeting $\overline{FE}$ at J.
$\overline{AE} = \overline{AL} + \overline{LE}$ and since $\angle JLE = \angle AIL = \tau$ we have $\overline{AE} = \frac{1}{2}b \tan \tau + \overline{JL} \sec \tau = \frac{1}{2}b \tan \tau + \overline{FI} \sec \tau$. Now, in the $\triangle DIF$, $\angle FDI = 45° + \tau$ and $\angle DFI = 45°$ hence $\overline{FI} : \sin (45° + \tau) = \frac{1}{2}b : \sin 45°$ or $\overline{FI} = \frac{1}{2}b (\sin \tau + \cos \tau)$, therefore

$$\overline{AE} = \tfrac{1}{2}b(1 + 2 \tan \tau). \tag{5}$$

We shall next derive four independent equations involving x_0, y_0, x' and y' from which these coördinates may be obtained separately. The angle of emergence equals the angle of incidence hence $\angle OEH = 90° - i_2$. But $\angle OEH + \angle HEP + \angle PEQ = 180°$ and $\angle PEQ = 90° - \frac{1}{2}(i_1 - i_2)$, consequently $\angle HEP = \frac{1}{2}(i_1 + i_2)$. Furthermore, $\angle IPE = \angle UPQ = 90°$ hence $\angle HPI = \angle HEP = \frac{1}{2}(i_1 + i_2)$. Now, in the $\triangle GPI$, $\overline{GP} = \overline{IG} \cot (\angle HPI)$ or $\overline{GH} + \overline{HP} = \overline{IA} + \overline{HP} = \overline{AH} \cot \frac{1}{2}(i_1 + i_2)$, whence

$$\tfrac{1}{2}b + y' = x' \cot \tfrac{1}{2}(i_1 + i_2). \tag{6}$$

Again, $x' = \overline{AH} = \overline{AE} - \overline{HE}$ so that by referring to equation (5) and to the $\triangle HEP$ we obtain

$$x' = \tfrac{1}{2}b(1 + 2 \tan \tau) - y' \cot \tfrac{1}{2}(i_1 + i_2). \tag{7}$$

In the $\triangle UPI$, $\angle UPI = \frac{1}{2}(i_1 - i_2)$ and $\overline{UP} = x_0$ therefore $x_0 = \overline{IP} \cos \frac{1}{2}(i_1 - i_2)$. Also, in the right $\triangle GPI$, $\overline{IG} = x' = \overline{IP} \sin \frac{1}{2}(i_1 + i_2)$, hence

$$\frac{x_0}{x'}=\frac{\cos\frac{1}{2}(i_1-i_2)}{\sin\frac{1}{2}(i_1+i_2)}. \qquad (8)$$

In the $\triangle$ *QPE*, $\overline{QP}=y_0=\overline{EP}\cos\frac{1}{2}(i_1-i_2)$; but, in the $\triangle$ *HEP*, $\overline{HP}=y'=\overline{EP}\sin\frac{1}{2}(i_1+i_2)$, hence

$$\frac{y_0}{y'}=\frac{\cos\frac{1}{2}(i_1-i_2)}{\sin\frac{1}{2}(i_1+i_2)}. \qquad (9)$$

Solving equations (6) and (7) for x' and y' we find

$$x'=\tfrac{1}{2}b[(1+2\tan\tau)\sin\tfrac{1}{2}(i_1+i_2)+\cos\tfrac{1}{2}(i_1+i_2)]\sin\tfrac{1}{2}(i_1+i_2). \qquad (10)$$

$$y'=\tfrac{1}{2}b[(1+2\tan\tau)\cos\tfrac{1}{2}(i_1+i_2)-\sin\tfrac{1}{2}(i_1+i_2)]\sin\tfrac{1}{2}(i_1+i_2). \qquad (11)$$

The results obtained by substituting these expressions in equations (8) and (9) are

$$x_0=\tfrac{1}{2}b[(1+2\tan\tau)\sin\tfrac{1}{2}(i_1+i_2)+\cos\tfrac{1}{2}(i_1+i_2)]\cos\tfrac{1}{2}(i_1-i_2). \qquad (12)$$

$$y_0=\tfrac{1}{2}b[(1+2\tan\tau)\cos\tfrac{1}{2}(i_1+i_2)-\sin\tfrac{1}{2}(i_1+i_2)]\cos\tfrac{1}{2}(i_1-i_2). \qquad (13)$$

Formulæ (10), (11), (12) and (13) are perfectly general and, in so far as they determine uniquely x_0, y_0, x' and y' when b, τ, i_1 and i_2 are assigned, they are complete. It may be remarked, in passing, that formulæ (10) and (11) show that the position of the axis of rotation with respect to the prism itself is dependent not upon the separate angles i_1 and i_2 but upon the mean angular position $\frac{1}{2}(i_1+i_2)$, whereas x_0 and y_0 involve the interval i_1-i_2 in addition. Moreover, since τ and i_2 are each acute while i_1 can not exceed 90° it follows that x_0 and x' are essentially positive. On the other hand, both y_0 and y' may be simultaneously positive, or zero, or negative. If, for any reason, it should be desirable to have the axis of rotation intersect the optic axis of the telescope this could be accomplished by merely choosing such values for τ, i_1 and i_2 as would satisfy the condition $1+2\tan\tau-\tan\frac{1}{2}(i_1+i_2)=0$. Equations (8) and (9) show that $x_0y'-y_0x'=0$ and this relation affords a simple check on the results of numerical calculations of the four coördinates involved.

By introducing an auxiliary angle ω which satisfies the relation $\tan\frac{1}{2}\omega=1+2\tan\tau$ formulæ (10) to (13) may be transformed into the following expressions which are convenient for logarithmic computation:

$$x'=A\sin\tfrac{1}{2}(i_1+i_2), \qquad (10')$$

$$y' = -B \sin \tfrac{1}{2}(i_1 + i_2), \qquad (11')$$

$$x_0 = \quad A \cos \tfrac{1}{2}(i_1 - i_2), \qquad (12')$$

$$y_0 = -B \cos \tfrac{1}{2}(i_1 - i_2). \qquad (13')$$

In these equations

$$A \equiv \frac{b \cos \tfrac{1}{2}(i_1 + i_2 - \omega)}{2 \cos \tfrac{1}{2}\omega} \quad \text{and} \quad B \equiv \frac{b \sin \tfrac{1}{2}(i_1 + i_2 - \omega)}{2 \cos \tfrac{1}{2}\omega}.$$

The condition $\tan \tfrac{1}{2}\omega = 1 + 2 \tan \tau$ requires $\tfrac{1}{2}\omega$ to lie between 45° and 71° 34′ since τ is comprised within the interval 0° to 45°.

When τ and the angles of incidence, I_1 and I_2, corresponding to the ends of the spectral region to be studied are given, equations (10), (11), (12) and (13) are not sufficient for the unique determination of the axis of rotation, since i_1 and i_2 would not be known. Consequently the next step in the analysis is to find a functional dependence of i_1 and i_2 upon I_1 and I_2. (Generally the indices of refraction, n_1 and n_2, for the ends of the spectral interval would be known, but, when τ is also assigned, this is equivalent to having I_1 and I_2 given since $\sin I_1 = n_1 \sin \tau$ and $\sin I_2 = n_2 \sin \tau$.)

Now, for angles of incidence i which are not equal to either i_1 or i_2 the optic axis of the collimator does not meet the incidence face at the center of the latter and therefore the axis of the emergent beam does not coincide with the optic axis of the telescope. Hence we shall so determine i_1 and i_2 as to cause the axis of the emergent beam to depart as little as possible from $\overline{OX}$ during the rotation of the prism from the angular position I_1 to the final position I_2. During this motion the points of incidence, of total reflection and of emergence oscillate along their respective deviating planes in a manner which apparently makes it difficult to see just what path the point of emergence traces out in the plane of the fixed coördinate axes. It is therefore necessary to find an expression for the ordinate of the point of emergence corresponding to any orientation of the prism as characterized by the variable angle of incidence i. Let $\alpha\beta\gamma\delta$ represent the prism in any such position. For the sake of complete generality and to avoid an excessively complicated figure we shall, from now on, make use of the methods of elementary analytical geometry.

The point of emergence, ϵ, will be found as the intersection of the lines $\overline{\beta\alpha}$ and $\overline{\eta\epsilon}$ where $\overline{\eta\epsilon}$ is perpendicular to the line $\overline{\lambda\eta}$ at the point of internal reflection η. To find the equation of the line $\overline{\beta\alpha}$ in the normal form we may proceed as follows. The perpendicular, p, dropped from the origin O upon $\overline{\beta\alpha}$ is parallel to $\overline{\delta\alpha}$ and hence it makes the angle i with $\overline{OX}$. Accordingly the equation of $\overline{\beta\alpha}$ may be written $x \cos i + y \sin i - p = 0$. Also $\overline{\beta\alpha}$ is at the distance y' from the point (x_0, y_0) hence $y' = x_0 \cos i + y_0 \sin i - p$ and therefore the equation of $\overline{\beta\alpha}$ in final form is

$$x \cos i + y \sin i - (x_0 \cos i + y_0 \sin i - y') = 0. \qquad (14)$$

In order to find the equation of $\overline{\eta\epsilon}$ it will be necessary to know the length of $\overline{O\lambda}$ which is the intercept on $\overline{OY}$ of the line $\overline{\delta\alpha}$, consequently we shall next deduce the equation of $\overline{\delta\alpha}$. The normal, p', drawn from the fixed origin to $\overline{\delta\alpha}$ is parallel to $\overline{\beta\alpha}$ and hence it forms the angle $90° + i$ with $\overline{OX}$. Thus, the equation of $\overline{\delta\alpha}$ may be written $-x \sin i + y \cos i - p' = 0$. Moreover, the distance from the point P to $\overline{\delta\alpha}$ is equal to $-x'$ hence $-x' = -x_0 \sin i + y_0 \cos i - p'$. Consequently the equation of $\overline{\delta\alpha}$ is

$$x \sin i - y \cos i + (x' - x_0 \sin i + y_0 \cos i) = 0. \qquad (15)$$

Putting $x = 0$ in equation (15) we obtain

$$\overline{O\lambda} = x' \sec i - x_0 \tan i + y_0. \qquad (16)$$

We are now prepared to find the equation of $\overline{\eta\epsilon}$. The perpendicular from O to $\overline{\eta\epsilon}$ is $\overline{O\zeta} = \overline{\theta\eta} = \overline{\lambda\eta} - \overline{\lambda\theta}$ and it makes the angle $270° + i - \tau$ with the positive direction of the axis of x.

The equation of $\overline{\eta\epsilon}$ may be written $x \sin (i - \tau) - y \cos (i - \tau) - \overline{\lambda\eta} + \overline{\lambda\theta} = 0$. We shall accordingly evaluate $\overline{\lambda\eta}$ and $\overline{\lambda\theta}$ in terms of $\overline{O\lambda}$ so as to take advantage of relation (16). In the $\triangle\, O\theta\lambda$, $\overline{\lambda\theta} = \overline{O\lambda} \cos (i - \tau)$ and, furthermore, in the $\triangle\, \delta\eta\lambda$, $\overline{\lambda\eta} : \sin (45° + \tau) = \overline{\delta\lambda} : \sin 45° = (b - \overline{\lambda\alpha}) : \sin 45°$ whence $\overline{\lambda\eta} = (b - \overline{\lambda\alpha})(\sin \tau + \cos \tau)$. But $\overline{\lambda\alpha} = \overline{\mu\nu} = \overline{O\nu} - \overline{O\mu}$ and, since $\overline{O\nu}$ is the perpendicular dropped from O upon the line $\overline{\beta\alpha}$, equation (14) gives $\overline{O\nu} = x_0 \cos i + y_0 \sin i - y'$. Finally, in the $\triangle\, O\mu\lambda$, $\overline{O\mu} = \overline{O\lambda} \sin i$ hence, using equation (16), we find $\overline{\lambda\alpha} = x_0 \sec i - x' \tan i - y'$. By making obvious use of this result the relation $\overline{\lambda\eta} = (\sin \tau + \cos \tau)(b - x_0 \sec i + x' \tan i + y')$ is obtained. Once more, by substituting from equation (16) in $\overline{\lambda\theta} = \overline{O\lambda} \cos (i - \tau)$ the equation of $\overline{\eta\varepsilon}$ is found to be

$$x \sin (i - \tau) - y \cos (i - \tau) - (\sin \tau + \cos \tau)(b - x_0 \sec i + x' \tan i + y') + (x' \sec i - x_0 \tan i + y_0) \cos (i - \tau) = 0. \tag{17}$$

Lastly, we must solve equations (14) and (17) for y, which is then the required ordinate of the point of emergence, which we shall denote by y_1. After making simple reductions it comes out that

$$y_1 = x_0 + y_0 - x'(\sin i - \cos i) - y'(\sin i + \cos i) - b(1 + \tan \tau) \cos i. \tag{18}$$

Substituting in equation (18) the expressions for x', y', x_0 and y_0, as given respectively by formulæ (10), (11), (12) and (13) and also carrying out elementary trigonometrical reductions which are far too long for publication the following result is obtained

$$y_1 = 2b\{\cos \tfrac{1}{2}(i_1 + i_2) + [\sin \tfrac{1}{2}(i_1 + i_2) + \cos \tfrac{1}{2}(i_1 + i_2)] \tan \tau\} \sin \tfrac{1}{2}(i - i_1) \sin \tfrac{1}{2}(i - i_2). \tag{19}$$

By equating to zero the first derivative of y_1 with respect to i we find that y_1 has a stationary value when $i = \frac{1}{2}(i_1 + i_2)$. The second derivative shows that y_1 always has an algebraic minimum when i assumes this mean value. The minimum of y_1 may be written

$$y_2 = -2b\{\cos \tfrac{1}{2}(i_1 + i_2) + [\sin \tfrac{1}{2}(i_1 + i_2) + \cos \tfrac{1}{2}(i_1 + i_2)] \tan \tau\} \sin^2 \tfrac{1}{2}(i_1 - i_2). \tag{20}$$

The fact that y_1 attains its minimum when $i = \frac{1}{2}(i_1 + i_2)$ is interesting because this position of the prism is not the one at which the points of incidence, of total reflection, and of emergence experience a reversal of direction of motion along the respective deviating faces. As is easily shown, these turning points are reached when $i = \sin^{-1}[\sin \frac{1}{2}(i_1 + i_2) : \cos \frac{1}{2}(i_1 - i_2)]$, that is when $i > \frac{1}{2}(i_1 + i_2)$.

On the other hand, the simplicity of the geometrical meaning of the condition $i = \frac{1}{2}(i_1 + i_2)$ would naturally lead one to suppose that this result could have been established at once from the geometrical properties of the figure and that therefore the preceding analysis is unnecessary. However, this supposition is not strengthened by the facts that $x_1 - x_0$ does not involve the mean angle, $\frac{1}{2}(i_1 + i_2)$, symmetrically and that the equation in x and y of the point of emergence is, in general, of the fourth degree with very complicated literal coefficients.

Fig. 3 shows the curve traced out in the plane XOY by the point of emergence ε for the special case of Jena glass S. 228 where $b = 44.7$ cm. and $\tau = 30°$. (See also Table II, first row.) Points 6 and 10 are at the ends of the arc which pertains to the axis of rotation through P and hence to the spectral region from A' to G'. The angles of incidence corresponding to the points 1 to 14 are respectively 45°, 50°, 55°, 60°, 65°, 70° 5′ 25″, 74° 26′ 40″, 76°, 77° 31′ 22″, 78° 47′ 54″, 80°, 82°, 83° and 84°.

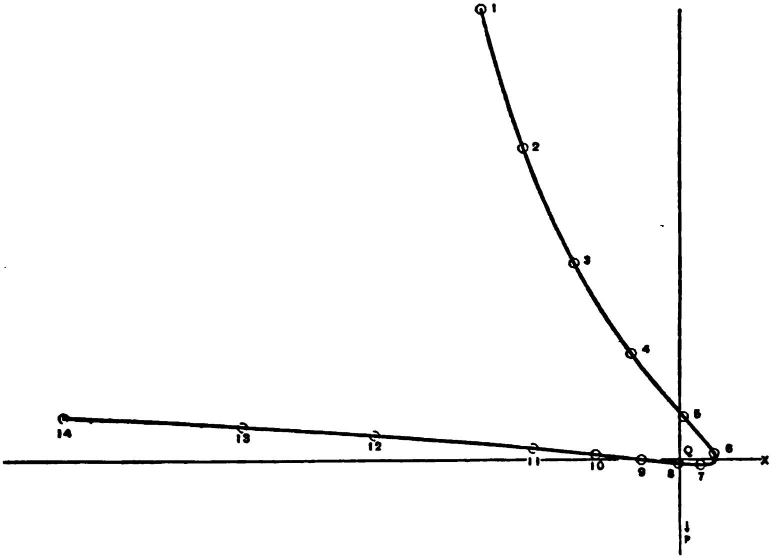

Fig. 3.

It follows at once from equation (19) that y_1 is symmetrical with respect to $\frac{1}{2}(i_1 + i_2)$ in the sense that this ordinate has the same value for any two values of i such that the one exceeds $\frac{1}{2}(i_1 + i_2)$ by the same amount as the other falls short of this mean angle. (This is also illustrated by Table II.) Since such is the case, it is now a simple matter to impose upon i_1 and i_2 the condition that y_1 shall have the least possible arithmetical maximum within, and at the limits of, the angular interval from I_1 to I_2. For this condition to hold it is both necessary and sufficient that the numerical value of y_1 at its minimum shall be equal to the values of y_1 at the two boundaries of the interval. The equality of the values of y_1 at the ends of the interval leads to

$$i_1 + i_2 = I_1 + I_2. \tag{21}$$

Again, as stated above, $|y_2|$ must equal y_1 when $i = I_1$ (or $i = I_2$), hence, referring to equations (19) and (20), we see that

$$\sin \tfrac{1}{2}(I_1 - i_1) \sin \tfrac{1}{2}(I_1 - i_2) = \sin^2 \tfrac{1}{4}(i_1 - i_2)$$

and therefore

$$\cos \tfrac{1}{2}(i_1 - i_2) - \cos [I_1 - \tfrac{1}{2}(i_1 + i_2)] = 2 \sin^2 \tfrac{1}{4}(i_1 - i_2).$$

Substituting for $i_1 + i_2$ the sum given by equation (21) and making simple transformations we find

$$i_1 - i_2 = 4 \sin^{-1} \left[\frac{1}{\sqrt{2}} \sin \tfrac{1}{4}(I_1 - I_2) \right]. \tag{22}$$

Finally, by solving equations (21) and (22) for i_1 and i_2 it is found that

$$\begin{aligned} i_1 &= \tfrac{1}{2}(I_1 + I_2) + 2 \sin^{-1} \left[\frac{1}{\sqrt{2}} \sin \tfrac{1}{4}(I_1 - I_2) \right], \\ i_2 &= \tfrac{1}{2}(I_1 + I_2) - 2 \sin^{-1} \left[\frac{1}{\sqrt{2}} \sin \tfrac{1}{4}(I_1 - I_2) \right]. \end{aligned} \tag{23}$$

As has been tacitly assumed, the preceding equations require that $I_1 > i_1 > i_2 > I_2$.

The solution of the main problem is now complete since the combination of equations (21) and (22) with (12), (13) and (20) leads to the final explicit formulæ,

$$\begin{aligned} x_0 = \tfrac{1}{2}b[&\cos \tfrac{1}{2}(I_1 + I_2) + \sin \tfrac{1}{2}(I_1 + I_2) \\ &+ 2 \sin \tfrac{1}{2}(I_1 + I_2) \tan \tau] \cos^2 \tfrac{1}{4}(I_1 - I_2), \end{aligned} \tag{24}$$

$$\begin{aligned} y_0 = \tfrac{1}{2}b[&\cos \tfrac{1}{2}(I_1 + I_2) - \sin \tfrac{1}{2}(I_1 + I_2) \\ &+ 2 \cos \tfrac{1}{2}(I_1 + I_2) \tan \tau] \cos^2 \tfrac{1}{4}(I_1 - I_2), \end{aligned} \tag{25}$$

$$\begin{aligned} y_2 = -b\{&\cos \tfrac{1}{2}(I_1 + I_2) + [\cos \tfrac{1}{2}(I_1 + I_2) \\ &+ \sin \tfrac{1}{2}(I_1 + I_2)] \tan \tau\} \sin^2 \tfrac{1}{4}(I_1 - I_2). \end{aligned} \tag{26}$$

In virtue of the fact that $i_1 + i_2 = I_1 + I_2$, formulæ (10) and (11) remain unchanged. Since $\tan \tau < 1$ it follows that $x' < 1.58b \sin \frac{1}{2}(I_1 + I_2)$, $x_0 < 1.58b$ and $|y_2| < 2.24b \sin^2 \frac{1}{4}(I_1 - I_2)$. In general, $\sin^2 \frac{1}{4}(I_1 - I_2)$ is a small quantity and hence y_2 is a small per cent. of b. Obviously when $\frac{1}{4}(I_1 - I_2)$ is very small $\sin^2 \frac{1}{4}(I_1 - I_2)$ and $\cos^2 \frac{1}{4}(I_1 - I_2)$ may be replaced by $1.9039 \times 10^{-5}(I_1^\circ - I_2^\circ)^2$ and by 1 respectively.

The auxiliary quantities ω, A and B which occur in equations (10′) to (13′) do not change their values, because of the condition $i_1 + i_2 = I_1 + I_2$, and therefore they may be used to reduce formulæ (24), (25) and (26) to the following expressions which are especially adapted to logarithmic computation.

$$x' = A \sin \tfrac{1}{2}(I_1 + I_2), \tag{10′}$$

$$y' = -B \sin \tfrac{1}{2}(I_1 + I_2), \tag{11′}$$

$$x_0 = A \cos^2 \tfrac{1}{4}(I_1 - I_2), \tag{24′}$$

$$y_0 = -B \cos^2 \tfrac{1}{4}(I_1 - I_2), \tag{25′}$$

$$y_2 = -\frac{b \tan \tau \cos \left[\frac{1}{2}(I_1 + I_2) - \left(\frac{1}{2}\omega - 45^\circ\right)\right] \sin^2 \frac{1}{4}(I_1 - I_2)}{\sin\left(\frac{1}{2}\omega - 45^\circ\right)}. \tag{26′}$$

Some practical illustrations are given in Table II., in which the linear unit is the centimeter. The numbers in the fifth row correspond very closely to the prism of a spectroscope made by Hilger. For this particular piece of apparatus $x' = 4.46$ cm., $y' = 0.60$ cm., $x_0 = 5.08$ cm. and $y_0 = 0.44$ cm.

Commencing with the paragraph in which the methods of analytical geometry were introduced, some of the succeeding results apply strictly only to cases where the incident beam is sufficiently narrow to avoid having a lateral portion of the beam cut off by the vertical edges of the incidence face when $i \underset{<}{\equiv} I_1$. In other words, the final formulæ involve implicitly the condition that the entire beam may be continuously represented by its axis during the rotation of the prism from the angular position I_1 to the position I_2. However, another extreme case is met with in practice. It arises when the aperture of the collimator is large enough to completely fill the incidence face with light when $i \underset{>}{\equiv} I_2$. For example, the

TABLE II.

Jena No.	b	τ	$\frac{1}{2}\omega$	n_1	n_2	I_1	I_2	x'	y'	x_0	y_0	$-y_2$	$-y_4$
S. 228	4	30° 0′ 0″	65° 6′ 14″	1.9619	1.8805	78° 47′ 54″	70° 5′ 25″	4.516	−0.7428	4.681	−0.7700	0.00565	−0.00111
O. 113	4	30 0 0	65 6 14	1.7721	1.7218	62 22 56	59 25 12	4.140	+0.3042	4.737	+0.3481	0.000850	+0.0000582
O. 102	4	30 0 0	65 6 14	1.6744	1.6374	56 50 48	54 57 14	3.883	+0.6292	4.689	+0.7598	0.000372	+0.0000518
O. 198	4	30 29 57.5	65 20 20	1.8181	1.7610	67 19 41	63 20 59	4.356	0.0000	4.792	0.0000	0.00144	0.0000
O. 469	3.73	32 17 10	66 9 58	1.6756	1.6382	63 30 41	61 2 59	4.076	+0.2768	4.604	+0.3127	0.000495	+0.0000361

latest form of Hilger's constant deviation prism spectroscope, having an index of refraction of about 1.65 for mean D-light, is constructed on this principle. When this condition is fulfilled the width of the effective incident beam equals the projection of the incidence face on any plane perpendicular to the optic axis of the collimator. Hence, the point where the axis of the effective incident beam meets the incidence face is the center of this face for all orientations of the prism within the given angular interval. Therefore, the axis of the emergent beam continually passes through the center of the emergence face. Since the last named point is constrained to move on the circumference of a circle having P as center and PE as radius it is a very simple matter to impose upon i_1 and i_2 the condition that, on the whole, the center of the emergence face shall depart from the optic axis of the telescope by as small an amount as possible within and at the boundaries of the region from I_1 to I_2. Accordingly we shall omit the mathematical details and merely state the final results.

If y_3 denotes the ordinate of the center of the emergence face corresponding to the position i then

$$y_3 = \frac{2y_0}{\cos \frac{1}{2}(i_1 - i_2)} \cdot \sin \tfrac{1}{2}(i - i_1) \sin \tfrac{1}{2}(i - i_2). \tag{27}$$

This equation is the precise analogue of formula (19) and, since it involves the variable angle i in identically the same manner as (19), relations (21) and (22) again apply and lead to the final ordinate

$$y_4 = -y_0 \tan^2 \tfrac{1}{4}(I_1 - I_2). \tag{28}$$

This expression is, of course, the analogue of formula (26) or (26′). Equation (28) shows that y_4 vanishes simultaneously with y_0 and, as has already been shown, y_0 equals zero when

$$1 + 2 \tan \tau - \tan \tfrac{1}{2}(I_1 + I_2) = 0.$$

Thus, it is possible, in general, to so choose τ as to cause no displacement of the axis of the emergent beam with respect to the optic axis of the telescope during the rotation of the prism. In this connection, attention may be called to the fact that when the indices of refraction n_1 and n_2, corresponding to the ends of the interval of rotation are given, it is not a simple matter to calculate τ so as to make $y' = y_0 = y_4 = 0$ because I_1 and I_2 involve the unknown quantity τ by virtue of the relations $\sin I_1 = n_1 \sin \tau$ and $\sin I_2 = n_2 \sin \tau$. In fact, the rationalized equation for $\tan \tau \equiv t$ is the following inconvenient polynomial of the sixth degree, namely:

$$\begin{aligned}8(n_1 - n_2)^2 t^6 + 16(n_1 - n_2)^2 t^5 + 2[(n_1 + n_2)^2 + 7(n_1 - n_2)^2 - 4]t^4 \\ + 2[(n_1 + n_2)^2 + 3(n_1 - n_2)^2 - 4]t^3 \\ + [(n_1 + n_2)^2 + (n_1 - n_2)^2 - 10]t^2 - 8t - 2 = 0.\end{aligned}$$

The simpler plan to follow, in any numerical case, is to use the so-called method of successive trial and error. In the latter way the numbers in the fourth row of Table II. were calculated. The last column of this table contains the values of y_4 as obtained from formula (28).

In conclusion it may not be superfluous to state *en résumé* that what we have done was first, to generalize the usual statements relative to Pellin and Broca's type of prism; second, to have demonstrated formulæ (10), (11), (12) and (13) so that i_1 and i_2 may be subjected by the designer to any conditions whatsoever compatible with the nature of the problem; and third, to have subjected i_1 and i_2 and the various formulæ involving these angles to a set of conditions which, in the two most important and extreme cases, cause the axis of the emergent beam to suffer very small displacements from the optic axis of the telescope. The numbers in the columns of Table II. headed $-y_2$ and $-y_4$ show how well conditions (21) and (22) solve the main problem proposed for solution.

Sloane Physical Laboratory,
Yale University, April, 1909.

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THE ELECTROLYTIC DETECTOR, STUDIED WITH THE AID OF AN OSCILLOGRAPH.

BY GEORGE W. PIERCE.

INTRODUCTION.

WHILE engaged in a series of experiments [1] on certain crystal rectifiers for alternating electric currents — these rectifiers also serving as detectors for electric waves — the writer made use of a sensitive form of Braun's cathode-tube oscillograph. The same apparatus is here applied to the study of the electrolytic detector. The purpose of the investigation is, first, to attempt to contribute to the understanding of the mode of operation of the electrolytic detector, and, second, to find out whether the experiment with the electrolytic detector will throw any light on the crystal detectors which, like the electrolytic detector, are rectifiers for small alternating currents.

Method of Employing the Detector in the Reception of Electric Waves. — The electrolytic detector for electric waves, as described by Fessenden [2] and shortly after by Schloemilch,[3] consists of a cell containing an electrolyte and having one electrode of very small area, usually in the form of an extremely fine wire of platinum, and as the other electrode a larger area of platinum or some other metal. When used in wireless telegraphy the two electrodes are connected in a circuit upon which the electric oscillations are impressed, so that the rapidly oscillating electric currents in the circuit are made to traverse the cell of the detector. An example of a simple form of receiving circuit, with the detector connected in the antenna, is shown at *MDG* of Fig. 1. A local circuit *TED*, through the detector, contains a telephone receiver *T* and an adjustable source of E.M.F., which is used to polarize the detector by sending through it and the telephone a small direct current. Under the action of the electric oscillations through the detector the current in the telephone receiver is modified so as to produce a sound in the telephone with a period determined by the train frequency of the incident electric waves. The action is localized at the contact of the fine wire with the electrolyte.

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[1] Pierce, Proc. Am. Acad., 44, pp. 317–349, March, 1909; PHYS. REV., 23, pp. 153–187, March, 1909.

[2] Fessenden, U. S. Patent No. 727,331, May 5, 1903.

[3] Schloemilch, E. T. Z., 24, p. 959, 1903.

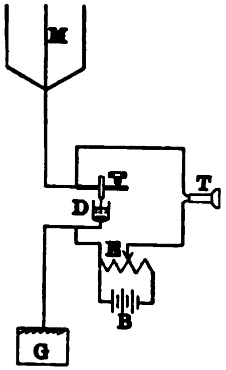

Fig. 1.

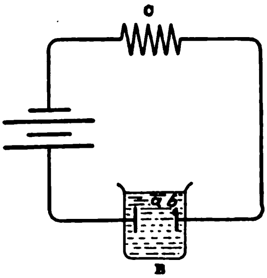

Fig. 2.

The Electrolytic Detector as a Rectifier. — That an electrolytic cell with one of the electrodes small, when suitably polarized with a direct current, is a rectifier for alternating currents was first shown by Pupin[1] before such a cell came into commercial use as a detector for electric waves. The following account of Pupin's rectifier is translated from the "Jahrbuch der Elektrochemie," Vol. 6, p. 35, 1899: "In Fig. 3" (here reproduced as Fig. 2) "*A* is a battery, *B* an electrolytic cell with the platinum electrodes *a* and *b* and acidulated water. If the polarization of the cell *B* is as great as the E.M.F. of *A*, no current flows in the circuit. If one allows an alternating current to act upon the circuit *ABC*, the circuit contains resistance, self inductance, and a capacity localized in the plates *a* and *b*. The cell *B* acts, however, as a condenser only so long as the potential difference of the plates *a* and *b* is smaller than the decomposition voltage. If this value is exceeded, a current goes through the circuit. If the alternating current, for example, has an amplitude that is twice as great as the E.M.F. of *A*, in case the phase has the same direction as *A* a current flows in the circuit, *e. g.*, in the direction *BC*; when the phase is oppositely directed, the condenser *B* sends a current in the opposite direction. This last can be diminished by making the capacity of *B* very small. If, for example, the area of one of the electrodes is only one square millimeter, one may easily rectify alternating currents with a frequency of 1,000 per second; with greater frequency the electrode must naturally be made still smaller. It is best to employ a platinum wire sealed into glass — the wire being cut off immediately at the end of the glass. The author succeeded in rectifying electric oscillations of Hertzian frequency, and producing electrolytic effects with them; the wire for this purpose was .025 mm. in diameter."

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[1] Pupin, Electrical World, 34, p. 743, 1899; Zeitsch. f. Elektrochemie, 6, p. 349, 1899; Jahrbuch. d. Elektrochemie, 6, p. 35, 1899; Bul. Am. Phys. Soc., 1, p. 21, 1900.

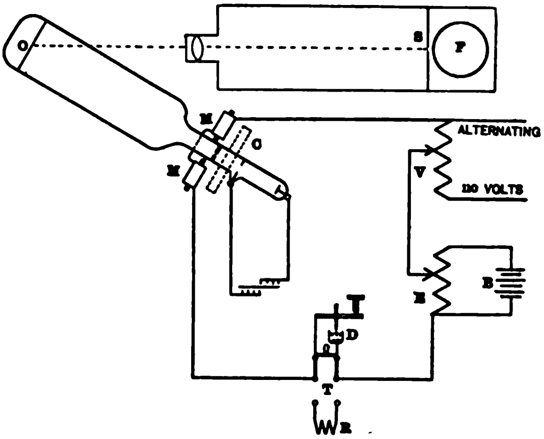

Fig. 3.

This quotation is introduced to show that Pupin had employed the electrolytic detector as a rectifier for electric waves of Hertzian frequency, and that he had a well-defined explanation of the processes occurring in the rectifier. The present experiment falls into close agreement with Pupin's explanation of the phenomenon.

Further study of the electrolytic detector, as a rectifier, has been made by Armagnat,[1] and Austin.[2] Armagnat has taken current-voltage curves of the detector with a galvanometer and a source of steady voltage, and by arguments similar to those employed by Pupin (but without reference to the polarization capacity of the electrodes) has shown more in detail how electrolytic polarization accounts for the rectification of alternating currents by the detector. Austin, also using a galvanometer as the indicating instrument, studied the effect of alternating currents of slow frequency and also the effects of electric waves, and came to the opinion that heat, chemical action, rectification and electrostatic attraction across the gas film might have a part in the explanation of the phenomenon.

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[1] Armagnat, Bul. soc. française, session of April, 1906, p. 205; Journal de Physique, 5, p. 748, 1906.

[2] Austin, Bul. Bureau of Standards, 2, p. 261, 1906.

In the present experiments the current through the detector under the action of an alternating E.M.F., superposed on a polarizing current, is determined by means of an oscillograph. The application of the oscillograph to the problem gives the instantaneous values of the current through the detector, and permits an examination of the wave form of the rectified cycle.

OSCILLOGRAPHIC APPARATUS.

Circuits Employed with the Detector in Taking the Oscillogram. — The electrolytic detector used in these experiments made use of a platinum point, .005 mm. in diameter, dipping into 20 per cent. nitric acid, and was adjusted to high sensitiveness as an electric wave detector immediately before taking the oscillograms. A diagram of the circuits employed in the experiment, together with a sketch of the oscillographic apparatus, is shown in Fig. 3. The detector is at *D*, and is connected in series with the deflecting coils *MM* of the oscillograph and with the variable sources of voltage *V* and *E*. The voltage *V* is taken from a potentiometer connected with the 60-cycle alternating mains of the laboratory. *E* is an adjustable steady voltage taken from a battery. The voltage at *E* could be reversed. By opening the switch at *S* the electrolytic detector could be disconnected from the circuit, and by throwing this switch to the right an ohmic resistance could be substituted for the detector.

The Braun's Tube Oscillograph. — The oscillographic apparatus was essentially the same as that employed in the experiments on crystal rectifiers (Part II.).[1] In the present experiments the high potential through the tube was produced by a Holtz influence machine, driven by an electric motor, whereas in the previous experiments Professor Trowbridge's high potential battery was used. The moving plates of the influence machine were of plain glass and did not carry any sectors or discs such as are used on the most common types of Toeppler-Holtz machines. This is important, since such sectors or discs produce a pulsewise discharge through the cathode tube, which results in a broadening of the cathode spot when it is deflected. The present machine in which the electricity was taken directly from the moving glass plates was very constant and gave cathode rays of great homogeneity, so that the luminescent spot produced at *O* on the screen of the tube remained sharply defined however far it was deflected.

The photograph of the moving luminescent spot was taken on a film carried by a rotating drum *F*, which made 20 revolutions per second about a horizontal axis. The drum was driven by a synchronous motor operating on the 60-cycle alternating mains of the laboratory, from which the alternating current sent through the rectifier was also taken. The synchronism of the drum with the deflections of the spot was so perfect that very long exposures could be made without any failure of perfect superposition, and without any appreciable fogging of the film.

The deflecting electromagnets *MM* had a combined resistance of 436 ohms, and were provided with soft iron cores about 6 mm. in diameter. With the small current employed the iron showed no appreciable hysteresis, and the deflections of the light spot were proportional to the currents. In oscillogram No. 1 (Plate I.) a deflection of .85 cm. on the film was obtained with a current of 1 milliampere. In taking the pictures, except No. 1, a Ryan focusing coil was placed about the tube so as to give a longitudinal field, which could be adjusted to bring the luminescent spot to have a circular area about .5 millimeter in diameter. This focusing device diminished the sensitiveness of the apparatus to about one third so that 1 milliampere gave a deflection of .28 cm. on the film, but the employment of the Ryan coil is a great advantage in producing a narrow line for the curves.

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[1] G. W. P., *l. c.*

In taking the oscillograms the following steps were taken: The drum carrying the film was set rotating. The high potential current was started in the tube. The chosen value of the polarizing current was applied to the circuit and was read on a direct-current milliammeter. The alternating current was superposed on the circuit, and by adjustment of the potentiometer at *V* the voltage of this alternating current was given any desired value.

THE EXPOSURES.

After the preliminary adjustment of the direct and alternating currents through the detector, four exposures were made on each picture, while the film was being carried around continuously by the synchronously driven drum.

Axis of Zero Current. — This is the lower straight line across the pictures, and was obtained by an exposure of 20 seconds taken with the circuit open.

Axis of Polarizing Current. — This is the upper straight line across the picture, and was obtained with the detector in circuit and traversed by the polarizing current. The exposure was 20 seconds. In oscillogram No. 1 this axis is not apparent because on account of the small value of the polarized current employed it falls into coincidence with the axis of zero current.

The Rectified Cycle. — This cycle may be identified in the oscillograms as a positive[1] loop for a half-period, followed by a nearly straight portion lying along the axis of zero current for a part of a half cycle, and going over into the positive loop through an intermediate "building up" segment. This cycle (exposure of 60 sec.) was taken with the detector in circuit, with the alternating E.M.F. applied to the circuit, and with the polarizing current also flowing.

The Voltage-Phase Cycle. — This the sine-curve of the pictures, and was taken in order to obtain the E.M.F. immediately about the detector.[2] A similar curve was made use of in the writer's experiments on crystal rectifiers and is there discussed. In the present experiments, because of the employment of the polarizing current with the rectifier, a question arises as to the appropriate method of taking this cycle. Two different methods were tried, either of which by proper elimination of the constants of the oscillographic apparatus, will give the desired result. The method yielding simplest results for the voltage-phase cycle is the following: After the exposure for the rectified cycle had been made, the alternating voltage was left unchanged, and a resistance was substituted for the rectifier. A double adjustment of the substituted resistance and the direct voltage was made by successive approximations until the result was attained that (1) the direct voltage alone gave through the substituted resistance a current equal to that used in polarizing the rectifier and (2) the alternating voltage superposed on this direct current gave a deflection of the luminescent spot to a point coincident with the maximum point attained with the rectifier in the circuit. This means that the voltage-phase cycle was taken with the axis of polarizing current as axis, and with amplitude equal to the maximum amplitude of the rectified cycle. This method was employed in oscillograms 1, 2 and 5.

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[1] In describing the oscillograms, values *above* the axis of zero current are called positive; values below this axis are called negative.

[2] The ordinary method, which would be to take the leads from the two sides of the detector through a high resistance to the oscillograph, could not be used because the oscillograph was working at the limit of its sensitiveness on the full voltage without the added resistance.

TABLE I.

Tabular Description of the Oscillographic Records.

No.	Polarizing Direct Current in Milliamperes.	Polarizing E.M.F. in Volts.	R.M.S. Volts A.C.	Maximum Positive Current through Detector in Milliamperes.	Equivalent Resistance in Ohms.
1[1]	Less than .1	1.45	2.09	2.37	440 [1]
2	1.0	5.5	4.00	9.6	70
3[2]	1.2	5.5	4.00	9.6	00 [2]
4[2]	1.4	Not measured.	5.00	10.0	00 [2]
5	2.2	"	5.00	11.0	150

The second method of taking the voltage-phase cycle was as follows: The polarizing voltage was reduced to zero, the detector was short-circuited, and an alternating voltage equal to that used with

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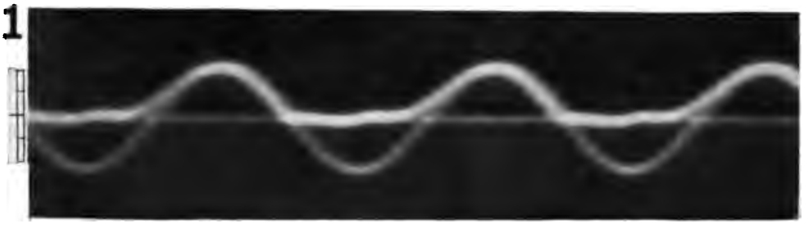
[1] It should be noticed that the sensitiveness of the oscillograph when No. 1 was taken was three times as great as when the other oscillograms of the plate were taken

[2] The voltage-phase cycle of oscillograms 3 and 4 were taken with the polarizing current omitted, so that they have the axis of no current as axis of the cycle.

1



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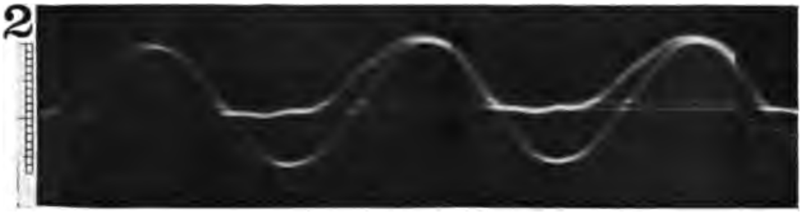

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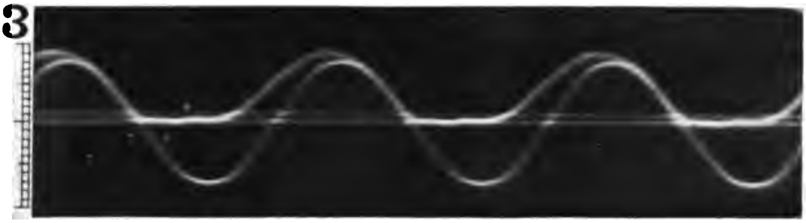

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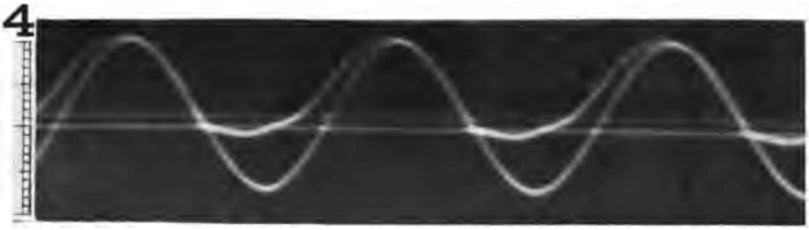

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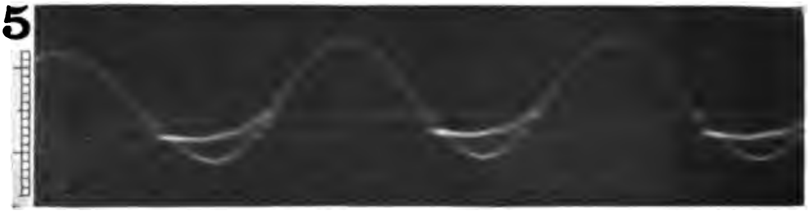

G. W. PIERCE.

The Electrolytic Detector.

the detector was applied to the circuit. This method was employed in oscillograms 3 and 4.

Coördinates of the Oscillographic Curves. — In taking all of the curves of the oscillograms, the motion of the light spot over the film is from left to right; the time coördinate is, therefore, the abscissa of the curves and is drawn as usual from left to right. The current coördinate is given in the scale drawn in ink at the left hand margin of each picture — one division being one milliampere.

DISCUSSION OF THE OSCILLOGRAMS OF PLATE I.

The oscillograms shown in Plate I. are reproductions of positives printed from the films carried by the rotating drum. The reproduction is one third the size of the original. They were taken with a 60 cycle alternating current applied to the circuit containing the electrolytic detector. The several curves shown in the plate were obtained with different polarizing currents superposed on the circuit. Table I. contains a tabulation of the polarizing current and voltage, the applied alternating voltage, the maximum current through the detector and the substituted resistance employed in taking the voltage curve.

Point Anode or Cathode — the Large Loop in the Direction of the Polarizing Current. — Some of the oscillograms were taken with the polarizing current from the point to the electrolyte and some with the polarizing current in the opposite direction. Although the values of the polarizing voltage required to produce a given polarizing current were different in the two cases the general characteristics of the cycle were the same. A reversal of the polarizing current reversed the rectified current, and whether the polarizing current was from the point to electrolyte or in the opposite direction the large loop of the rectified cycle (always oscillographed positively) was obtained when the alternating current was flowing in the same direction as the polarizing current.

The Form of the Rectified Cycle. — The cycle obtained with the rectifier in the circuit has the same general form in all the pictures. When the current, having traversed the positive loop, comes to the axis of zero current, it follows along this axis for a short way, then takes a small negative dip, becomes positive again, follows along just above the axis of zero current for a short time, and then rises along a transition curve to the positive loop.

Calculations Concerning the Form of the Cycle. — The rectified cycle, when examined by comparison with the voltage-phase cycle, makes a misleading impression unless one takes carefully into account the condition under which the curves are obtained. One must bear in mind that the form of the current through any rectifier is not determined by the rectifier alone, but is a function also of the constants of the circuits employed with the rectifier. In the present experiments, the deflecting coils of the oscillographic apparatus possessed appreciable self inductance and resistance, and these factors must be taken into account. It is proposed, therefore, to examine the problem in the light of the elementary theory of alternating currents.

The examination will be confined to oscillogram No. 2. The data for this curve are contained in Table I. The polarizing current had a value of .001 ampere, obtained by impressing a steady voltage of 5.5 volts on the circuit containing the detector and the deflecting coils. The impressed alternating voltage had a R.M.S. value of 4 volts, and therefore a maximum value of $4\sqrt{2} = 5.66$ volts. The equation of this voltage is

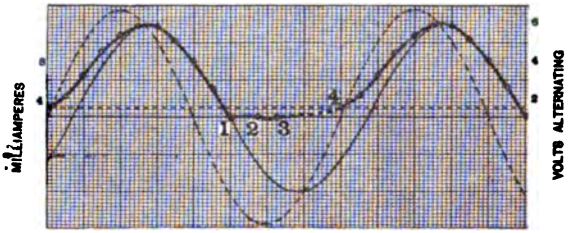

Fig. 4.

$$e = 5.66 \sin \omega t, \tag{1}$$

which is plotted as the dotted sine-curve of Fig. 4. The ordinate scale for this curve, in volts, is at the right of the diagram.

The voltage-phase cycle taken with this alternating E.M.F. and with a direct current of .001 ampere impressed upon the circuit, which consists of the coils of the oscillograph and a resistance of 70 ohms substituted for the rectifier, has the equation

$$(2)\qquad i_1 = \frac{5.66}{\sqrt{R^2 + L^2\omega^2}} \sin\left(\omega t - \tan^{-1}\frac{L\omega}{R}\right) + 1 \times 10^{-3}.$$

R is made up of the resistance of the coil, 436 ohms, plus the added resistance of 70 ohms, making 506 ohms. The maximum value of i_1, taken from the oscillogram, is 9.8×10^{-3} amperes. The values of R and $i_{1(\max)}$ substituted in equation (2) gives

$$9.8 \times 10^{-3} = \frac{5.66}{\sqrt{506^2 + L_2\omega}} + 1 \times 10^{-3};$$

whence

$$(3)\qquad L\omega = 397,$$

and

$$\varphi_1 = \tan^{-1}\frac{397}{506} = 38.1^\circ,$$

where φ_1 is the angle of lag of the voltage-phase cycle behind the voltage e of equation (1). The value of i_1 is now completely known, and is plotted as the *continuous* sine-curve of Fig. 4. This curve is the voltage-phase cycle of the oscillogram No. 2.

Let us next compute some points of the rectified cycle; namely, the points indicated by circles and numbered 1, 2, 3 and 4 on the diagram of Fig. 4. Let us at first assume that the rectified cycle and the voltage phase cycle come to the axis of zero current together. This gives us the point 1 as a starting point. Then the value of the current at the points 2 and 3 is also zero, because the impressed voltage is zero, being 5.5 volts (the polarizing E.M.F. externally applied) and -5.5 volts (the corresponding ordinate of the dotted sine-curve). Thus the externally applied voltage is zero. Also, since the current is practically zero and the rate of change of current is practically zero in the neighborhood of the points 3 and 4, the internal E.M.F. of the circuit due to resistance and inductance is zero. This gives the total voltage as zero and the resultant current zero at these points. The point 4 has the abscissa 2π, therefore the external alternating voltage (dotted curve) is zero; whence the current is equal to the polarizing current, and the point is on the axis of polarizing current. (This assumes that the current between points 3 and 4 is small and its rate of change is small, which is borne out by the fact that the current in this region has to remain between the two axes.)

The points 2, 3 and 4 are thus located by calculation. The march of the current intermediate between the points 3 and 4 is difficult to determine, on account of the unknown way in which the current grows during the polarization of the electrode. I have, therefore, indicated the current in this interval by a dotted line. We know, however, that between the points 2 and 3 there must be a tendency to a small negative maximum, because in this interval the negative value of the alternating voltage, taken from the dotted sine-curve, is larger than the positive polarizing voltage (5.5 volts). This small negative maximum is indicated on the diagram.

Having obtained the location of the four points 1, 2, 3 and 4, and of the negative loop between 3 and 4, let us next consider the large loop of the rectified cycle. In this loop the alternating voltage and the polarizing voltage aid each other, so as to produce large values of the current. With these large values of the current the resistance and the inductance of the coils come to play an important part, and the current tends to traverse an arc of a sine curve. However, since the current has been nearly zero for a large part of a half period, the "building-up" terms of the current equation are of importance. Let us attempt to obtain an approximate equation for this part of the cycle. To do this we need to keep in mind the experimental fact that it takes several volts to overcome the E.M.F. of polarization of the lectrolytic cell with small electrode, but after this polarization has been overcome, the application of additional voltage produces additional current proportional to the additional voltage. This fact is illustrated by the current-voltage curve of Fig. 5, taken from the work was obtained by Armagnat by applying various values of steady of Armagnat.[1] This curve voltage and measuring the current through the detector with a galvanometer. If we assume that a similar relation of current to voltage obtains when the voltage is rapidly changing with the

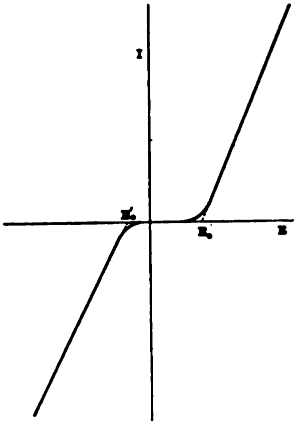

Fig. 5.

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[1] *L. c.*

time, we shall be able to compute approximately the upper loop of the cycle. When the current in the positive direction in our experiments has reached the axis of polarizing current it is on the straight part of the curve of Fig. 5, so that above this axis the resultant current is made up of .001 ampere due to the polarizing voltage plus the current under the action of the alternating voltage — this added current being obtained by assuming that the resistance of the detector is constant. This assumption gives for the current of the upper loop of the rectified cycle, the following equation:

$$(4) \qquad i_2 = \frac{5.66 \sin\left(\omega t_1 - \tan^{-1}\frac{L\omega}{R}\right)}{\sqrt{R^2 + L^2\omega^2}} + .001 + c\epsilon^{-\frac{Rt}{L}},$$

in which t_1 is measured from the instant at which ωt becomes 2π, 4π, 6π, etc. $L\omega$ has the value given in equation (3) above, and because of the equality of the positive amplitude of the rectified cycle and that of the voltage-phase cycle, we know that R has the value it had above; namely, 506 ohms. Putting these values in equation (4) and remembering that when $\omega t = 2\pi$, $i_2 = .001$, and $t_1 = 0$, we may determine the constant c. The value obtained is $c = .0054$ ampere. By a comparison of equations (4) and (2) we have

$$(5) \qquad i_2 = i_1 + .0054\epsilon^{-\frac{Rt}{L}}.$$

By computing the values of the exponential term and adding them to the corresponding values of i_1 we obtain the values of i_2. These are plotted as the positive loop in Fig. 4, which, together with the points 1, 2, 3 and 4, make up the complete computed curve for the current through the rectifier.

In justification of our assumption of the point 1 as a starting point it should be noted that, since the exponential becomes inappreciable in less than a half period, i_2 becomes equal to i_1 and the two currents approach the axis of zero current together.

Comparison of the Computed Cycle with the Cycle of the Oscillogram No. 2. — The scale used in plotting the computed curves is not identical with that of the original oscillogram. To show the agreement of the form of the computed curve of Fig. 4 with oscillogram No. 2, a tracing of the original oscillogram is drawn in Fig.

6. The computed values of the rectified cycle, reduced to the same scale, are also plotted as the circles of Fig. 6.

The computed points agree well with the oscillogram in the location of the negative maximum, the position of which with respect to the positive maximum is seen to be entirely due to the inductance and resistance of the circuit. The computed points also fall well into agreement with the oscillogram on the right-hand slope of the positive loop, but on the left-hand slope there is a considerable departure. On this slope the computed values of the current are all too low. This might have been expected because the "building up" had really begun earlier than the time assumed; that is, before the cycle had reached the point 4; and the computation in this segment is to be regarded only as a rough approximation.

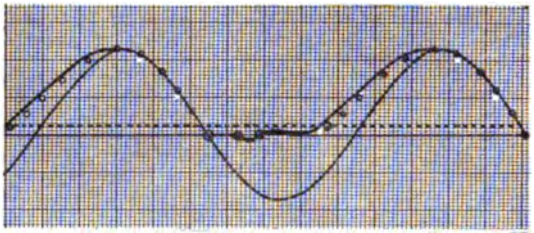

Fig. 6.

Evidence of Polarization Capacity. — On oscillograms 1, 2 and 3 there is a small positive rise of the photographic curves in the region to the immediate right of the negative maximum. This rise is more striking in the original photographs than in the reproductions; and, though small, it deserves attention, because the occurrence of this small positive maximum is evidence of the existence for about 1/1,500 of a second of a positive E.M.F. greater than the E.M.F. immediately following. Now in this part of the cycle the externally applied E.M.F. is greater following the rise than during the rise; therefore the rise indicates the existence of a positive E.M.F. in the circuit itself. This is capable of the following explanation in terms of the theory of polarization. After the prevalent external E.M.F. has been in a negative direction and has returned to zero, the polarization tension which has been opposing the negative current at the electrode continues to exist for a short time and produces a positive current. This action, resembling that of a capacity, is familiarly known as the polarization capacity of the electrode. By the existence of the small positive maximum near the axis of the cycle, the oscillogram shows that the polarization capacity of the electrode is not entirely negligible. Evidence of the existence of this polarization capacity is clearly given by the oscillograms 1, 2 and 3. The oscillograms 4 and 5, while not having a positive maximum near the axis, show also a striking tendency toward a maximum at this point, which is, however, masked by the rapid rise of the building-up curve in this part of the cycle.

Conclusion in Regard to the Electrolytic Detector.

1. The whole phenomenon of the rectification of small alternating currents by the electrolytic detector seems to be explicable in terms of the theory of electrolytic polarization.

2. The polarization capacity of the small platinum electrode is not entirely negligible, even with currents making only 60 cycles per second. The polarization capacity may, however, aid in producing rectified current as well as oppose this effect, and apart from the effect of this capacity on the tuning of the circuit need not detract from the utility of the rectifier as a detecter for electric waves.

3. The present conclusions in regard to the action of the detector is entirely in accord with Pupin's original brief description of the phenomenon as quoted above.

Comparison of the Electrolytic Detector with the Crystal Rectifiers.

The resemblance of the oscillograms with the electrolytic detector to those with the crystal rectifiers[1] is close, in so far as depends on the fact that both classes of rectifiers are nearly perfect[2] rectifiers when employed under their best conditions. The electrolytic rectifier in order to approximate perfection[3] as a rectifier must be polarized by the superposition of a direct current; while the use of the direct current with the crystal rectifier, though slightly lowering its resistance, does not materially improve the rectification. Also the two rectifiers are different, in that the electrolytic rectifier shows evidence of electrolytic polarization capacity, which, so far as may be judged from the oscillograms, is absent with the crystal rectifier. The experiment with the electrolytic detector, since it shows in the matter of polarization capacity the integrative action of this detector, which was sought for and not found with the crystal rectifier, is thus an interesting "control" experiment. Also an examination into the resemblances and differences of the two classes of rectifiers ought to aid us in seeking a rational explanation of the crystal rectifier.

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[1] Pierce, Part II., *l. c.*

[2] A rectifier is called "nearly perfect" when the ratio of the current in one direction to that in the opposite direction is large.

[3] The current through the electrolytic rectifier is slightly asymmetric when no polarizing current is employed.

However, before attempting to give definite expression to any theory as to the nature of the action in the crystal rectifiers, it is proposed to attempt to study in this connection the action of some form of vacuum-tube rectifier, so that the vacuum-tube rectifier may also be compared with the crystal rectifier under similar conditions.

In the meanwhile, the results of some experiments on a crystal rectifier making use of iron pyrites is presented in a separate paper to appear soon.

The expense of this research was in part defrayed from a grant from the Bache Fund of the National Academy, for which the author wishes to make grateful acknowledgment.

Jefferson Physical Laboratory,
Harvard University, Cambridge, Mass.
April 14, 1909.

THE DEPENDENCE OF THE PHOTO-ELECTRIC CURRENT ON LIGHT INTENSITY.

By F. K. Richtmyer.

THERE seems to be a general tendency to assume that the rate of discharge of negative electrons from the surface of a metal under the action of light is directly proportional to the intensity of the incident light, at least within fairly wide limits. However, the available data on this subject seem rather contradictory. Elster and Geitel (1893),[1] working with cells made of potassium, show data that seems to prove that the two quantities are directly proportional for small ranges of light intensities. Lenard (1902)[2] worked with a very much greater range of intensities and reached the same conclusion, but his data when plotted do not give rigorously a straight line. On the contrary Griffith (1907)[3] working with ultra-violet light and a zinc plate concludes that the ratio of the current to the light intensity is not constant but increases with increasing illumination, and he gives reasons why this increase in the ratio ought to be expected.

Considering the present theory of the photo-electric discharge it is not quite clear why this proportionality should exist. The experiments of Lenard (1900, 1902, 1903),[4] Millikan and Winchester (1907)[5], and others seem to show that the emission of negative electrons is due to sort of an atomic disintegration. The complex atoms, becoming unstable under the action of light vibrations, eject electrons whose velocities do not differ greatly from the velocities which they possessed in the atom. Ladenburg and Markau (1908)[6] have shown that only those electrons are emitted whose natural periods correspond to the period of vibration of the incident light.

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[1] Ann. der Ph., 48, 625, 1893.
[2] Ann. der Ph., 8, 149, 1902.
[3] Phil. Mag., p. 297, 1907.
[4] Ann. d. Ph.
[5] Phil. Mag., p. 188, 1907.
[6] Phys. Zs., p. 821, 1908.

But why should the number of such electrons emitted be strictly proportional to the light intensity? The following experiments were undertaken in order to get some data in addition to that given in the references cited above.

The apparatus is shown in Fig. 1. A 6-volt storage battery is connected in series with two resistance boxes R_1 and R_2, whose sum is kept constant, while either may be varied at pleasure, thus keeping a constant current in the circuit. The positive terminal of R_2 is connected to earth and the negative terminal to the Na cell, *C*.[1] The receiving wire *r*, of platinum, is connected to one pair of quadrants of the Dolezalec electrometer *Q*, the other pair being earthed. The electrometer was charged to + 55 volts by contact with the laboratory circuit and did not leak noticeably for several hours. The cell and the electrometer with all their connecting wires were encased in a tin box *abcd*, painted dull black on the inside. A window at *W* admitted light to the electrometer mirror, the cell being protected from stray light from this source by a partition *pp*. At *G* provision was made for making or breaking ground connection.

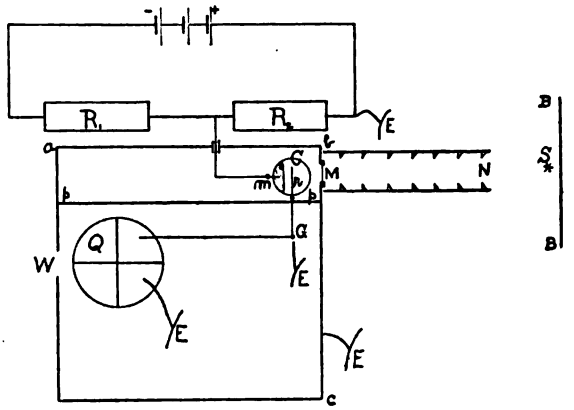

Fig. 1.

The cell was illuminated through the window *M* by a 16 c. p. lamp at *S*, a wooden box *MN* (square cross-section) protecting the cell from the stray light of the room. This box was at first simply blackened on the inside, but serious error was caused by reflection from its sides. This difficulty was avoided by gluing laterally to the inside of the box small right-angled strips of wood as shown. Some annoyance was also caused by the irregularities in the transmission of light through the walls of the glass, due to varying angles of incidence as the position of *S* was varied. Also the distance between the plane of the filament of the lamp and the rather irregular surface of the metal was not easy to measure. The window *M* was therefore covered by a piece of medium weight writing paper. The intensity of illumination on this paper could be measured very accurately and the light transmitted to the cell was strictly proportional to the light intensity on the paper. Finally the source *S* was backed up by a black screen *BB*. Under these conditions a very great range of intensities was available by varying the distance *MS* and the number of layers of paper covering the window.

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[1] See Elster and Geitel, Ann. d. Ph., 43, 225, 1891.

It may be of value to record some of the numerous difficulties that were encountered during the process of this investigation. It was noted that if the cell *C* was kept in the dark, the Na insulated (except for the high resistance ground through the walls of the

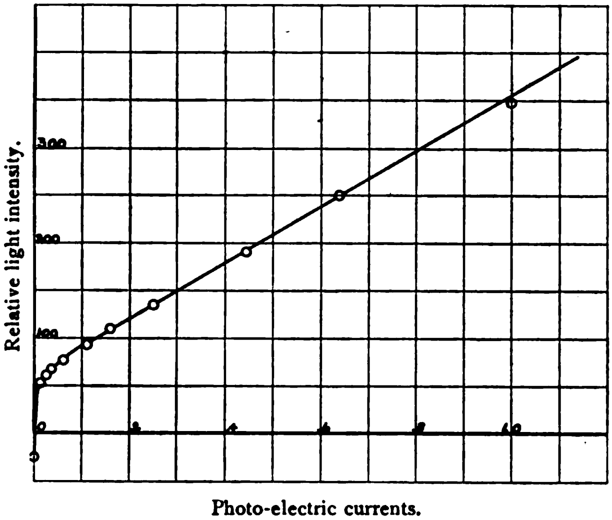

Fig. 2.

cell) and the ground connection at *G* broken, the electrometer showed a small rate of deflection in a direction to indicate that the needle was receiving a positive charge. If, however, the cell be illuminated by a gas flame 15 or 20 feet distant a small rate of deflection in the opposite direction was observed. And if the intensity of illumination was then gradually increased a curve of the form shown in Fig. 2, data Table I, resulted. The rate of charge was observed by a stop watch. (For these data the paper window at *M* was removed to get greater intensity.)

Table I.

Flame to Na Surface.	Light Intensity.	Time to Charge 100 Divisions.	Relative Currents.
1.00 ft.	1	28.8 sec.	.0347
1.25	.64	40.0	250
1.50	.445	52.6	190
2.00	.25	74.0	135
2.50	.16	91.0	110
3.00	.111	107	93
4.00	.0625	129	77
5.00	.040	149	67
6.00	.028	161	62
8.00	.0156	197	51
Inf.	.000	400[1]	−25

The reason for the non-linear character of this curve is apparently connected with the reason for the positive charge being given to the electrometer when the cell is in the dark. This latter is probably due to the fact that the Na surface is highly electro-positive with respect to the platinum wire *r*. When *G* is grounded, *r* is at zero potential and the Na is at a positive potential equal to the contact potential difference between Na and Pt, the resistance between the two being the glass walls of the cell which are probably made conducting by a very thin film of Na, deposited by sublimation at the time of filling the cell.

If this explanation is correct, by charging *m* to a negative[2] potential we ought to decrease this current or possibly reverse it. In fact we may consider the connections as represented diagrammatically in Fig. 3. The cell *C* tends to send positive electricity in a counter-clockwise direction, while the "battery" at *e* (the pd. around R_2, Fig. 1) tends to send current in the opposite direction, through the high resistance of the cell. The direction of the current will depend on which E.M.F. is the greater. We ought to expect a linear relation between this current and *e*.

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[1] Direction of drift opposite to that of the preceding.

[2] It is to be remembered that when we charge *m* to a negative potential, we really charge the platinum wire sealed in through the glass to that negative potential, whereas the Na itself is at a potential higher than this by an amount representing the contact potential difference between Na and Pt.

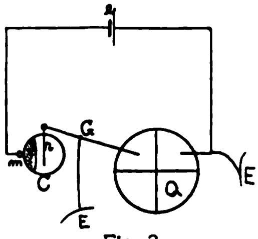

Fig. 3.

To test this the value of *e* was varied and the corresponding rate of charge of the electrometer measured by a chronograph. In taking these observations it was noted that the rate of deflection decreased as the electrometer became more highly charged, due to the fact that as the wire *r* became positive the effective E.M.F. of *C* was decreased. To avoid this disturbance due to the electrometer itself becoming charged, the following method of taking data was employed: Chronograph record was made of the transit of the spot of light on the electrometer scale past the 10, 20, 30, 40, etc., mm. mark. From this data a curve was plotted with rate of charge as ordinates and the position of the spot of light on the scale as abscissæ. This curve was then projected backward until it cut the ordinate representing the electrometer zero. See Fig. 4 and Table II.

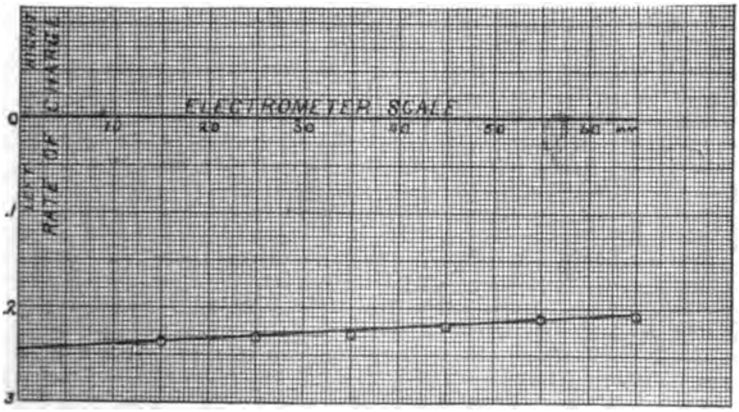

Fig. 4.

TABLE II.

For $e = 0$. Between Divisions	Requires	Rate of Charge.
10–20 mm.	42.1 sec.	.237 mm./sec.
20–30	43.2	.232
30–40	43.6	.229
40–50	45.4	.220
50–60	47.1	.212
60–70	48.1	.208

Rate of charge at electrometer zero = .248 mm./sec. as read from intersection on curve.

A series of such curves was obtained, one for each value of "e" used and their intersections with the ordinate for the electrometer zero were taken as comparative values of the current. A summary of this data is shown in Table III. and graphically in Fig. 5, 1.

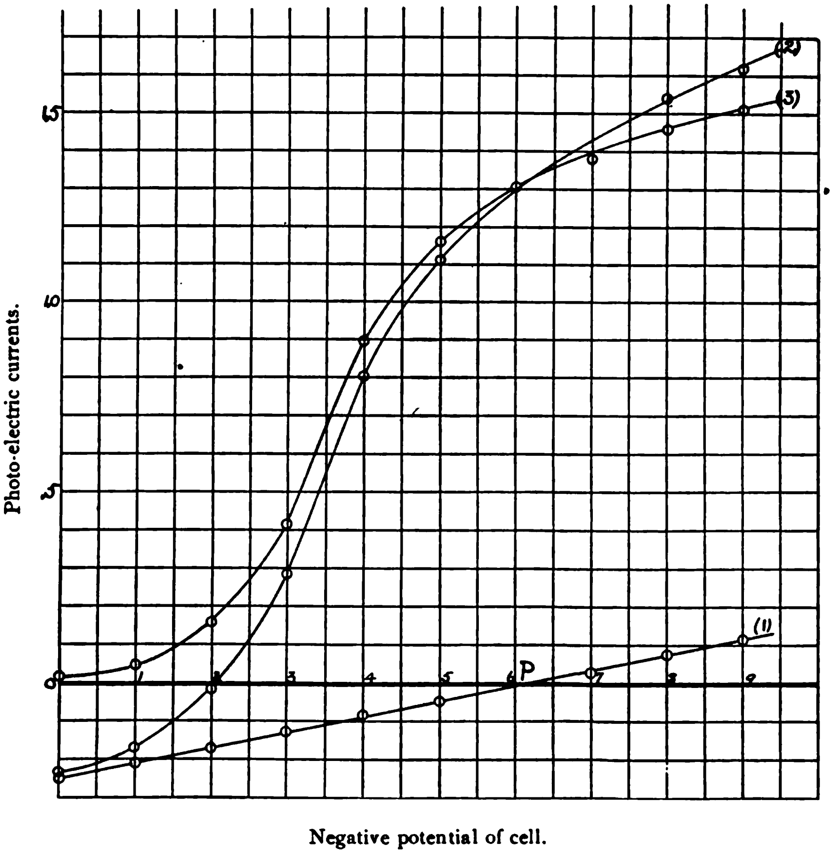

Fig. 5.

TABLE III.

Values of e (Arbitrary).	Direction of Charge.	Rate of Charge at Elec. Zero.
0	left	.248 mm./sec.
1	left	.217
2	left	.172
3	left	.126
4	left	.080
5	left	.044
6	too small to measure	
7	right	.039
8	right	.080
9	right	.117

(These arbitrary units of potential are nearly equivalent to .5 volt.) Since Fig. 5, 1 is a straight line the above explanation as to the origin of the current in the cell when not illuminated seems to be verified.

To compare the magnitude of this current due in part to contact pd. with that produced under the action of light a second curve was obtained in exactly the same manner as 5, 1 except that the cell was illuminated by a 16 c. p. lamp several feet away shining on the paper window *M*. This curve is shown in Fig. 5, 2. The effect of the light alone is clearly the difference between the ordinates of these two curves and is shown in Fig. 5, 3.

Now as is explained above, the potential *P* (Fig. 5, 1) represents the contact pd. between Na and Pt (about 3 volts). And when the platinum wire at *m* leading into the Na is charged to this negative potential the Na itself is really at zero potential. To represent properly the relation between the potential of the Na and the photo-electric current from it, Fig. 5, 3 should be replotted with the origin shifted to point *P*. This is done in Fig. 6. It may be noted that this curve agrees in the essential details with those obtained by Ladenburg and Markau[1] for the photo-electric current from platinum under the action of ultra-violet light.

Referring to Fig. 5 it is evident that the currents indicated by curve I. are large enough to cause serious error unless suitable corrections are made. This would not only be undesirable but would tend to lessen the accuracy of observations, since the correction is a fairly large percentage of the whole effect. These corrections would however be reduced to zero if during the investigation the cell were kept charged to a potential (about 3 volts) represented by point *P*. Whatever current is now obtained must be due to light alone. Under these conditions the variation of the photo-electric current with variation in light intensity could be quite accurately studied.

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[1] *Loc. cit.*

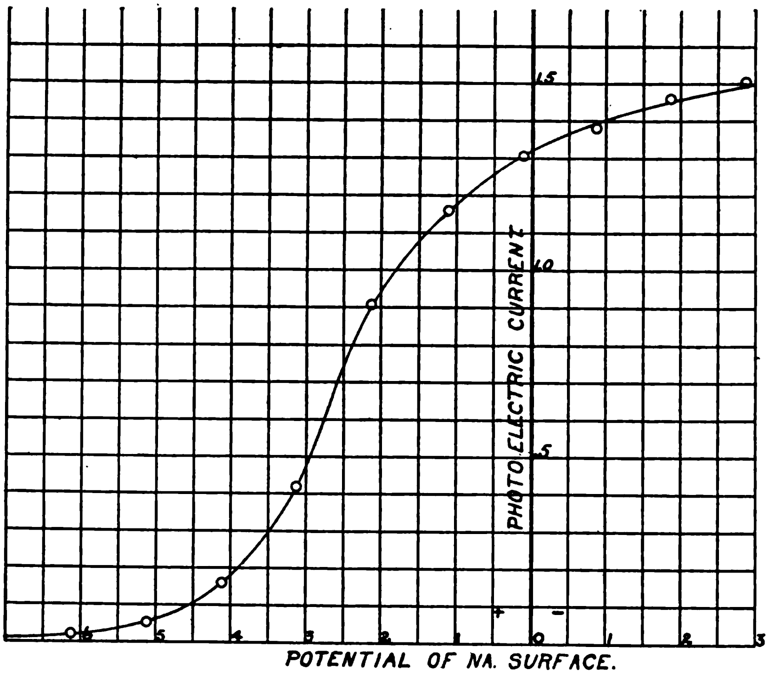

Fig. 6.

The measurement of current by use of the chronograph had several disadvantages. It took a long time to "read" the chronograph records. Besides, as the currents to be measured became larger the rate of drift of the electrometer increased proportionally. This caused certain irregularities due apparently to an increasing lag of the needle. This method was consequently abandoned and the current was measured by observing the permanent deflection given to the needle after being illuminated for a measured time interval, say 30 or 40 seconds. This time interval could be quite accurately obtained by switching on the lamp *S* simultaneously with a beat of a 2-second sounder, counting the desired number of seconds, and breaking the circuit at another beat, after which the electrometer would come to rest and the deflection could be read. As the light intensity increased the duration of illumination could be correspondingly lessened so as to give approximately the same deflection in each case. From this data the deflection of the electrometer, had the duration of the illumination been only one second, could be readily computed, and thus the currents uniformly compared. The currents so obtained are the average of from 6 to 10 observations for each light intensity.

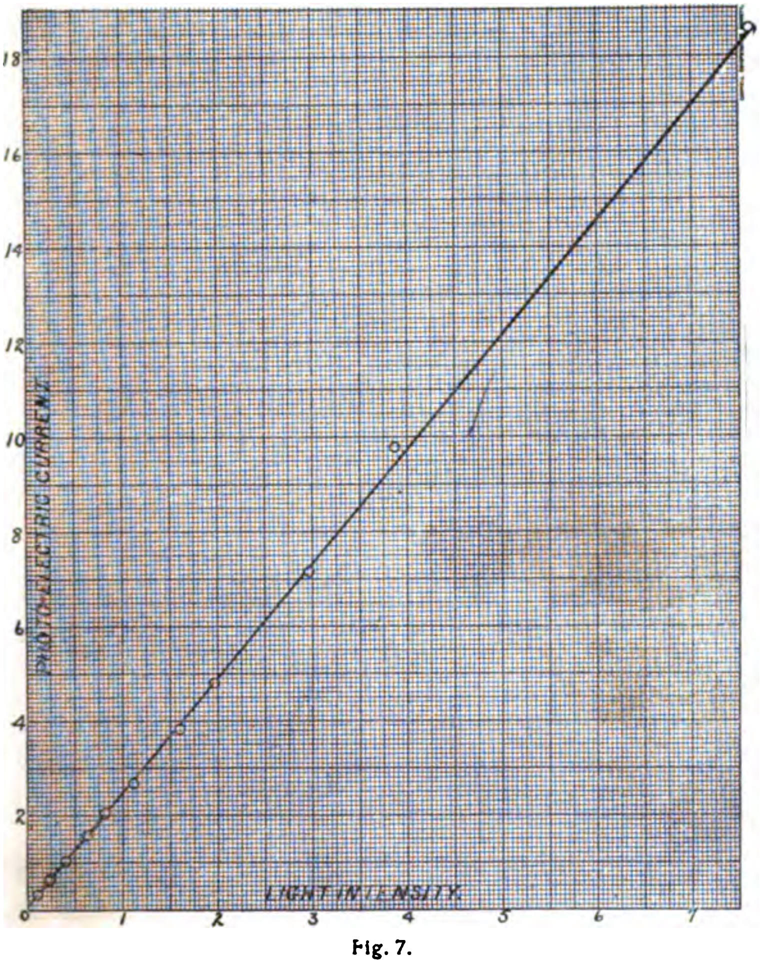

Fig. 7.

The results are shown in Fig. 7, data Table III. The first nine readings were made by allowing the light from the 16 c.p. lamp to fall on *two* thicknesses of paper at the window *M*, the intensity of illumination on the cell being estimated at .007 candle-feet for the first reading. Intensities and currents are given in relative values only. For the last three readings one sheet of paper was removed, its absorption coefficient being determined by observing the deflections before and after removing.

TABLE III.

Lamp to Window.	Relative Light Intensity.	Deflection per Second.
10.00 ft.	.0100	.318 mm.
6.50	.0236	.646
5.00	.0400	1.033
4.00	.0625	1.573
3.50	.0816	2.055
3.00	.111	2.682
2.75	.131	3.20
2.50	.160	3.93
2.25	.197	4.80
One paper removed, increasing illumination 4.75 times.		
4.00	.297	7.51
3.50	.387	9.79
2.50	.760	18.53

CONCLUSION.

The photo-electric current from a Na surface at zero potential, under the action of light from an incandescent lamp is strictly proportional to the light intensity for ranges of intensities from (approximately) .007 c.f. to .5 c.f.

It is hoped to extend this investigation to much higher intensities by use of improved apparatus. It is with pleasure that the writer acknowledges his obligations to Professor Merritt for his many suggestions.

CORNELL UNIVERSITY,
April, 1909.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

THE CRITICAL DENSITY OF WATER.[1]

BY HARVEY N. DAVIS.

IN 1897, Thiesen suggested that the empirical formula

$$L = L_0(t_e - t)^n$$

for the heat of evaporation of a substance would give, at the critical temperature t_e, the right value not only of L itself but also of dL/dt, if n is less than 1. His value, $n = \frac{1}{3}$, was based on the observations of Regnault, Dieterici and Griffiths. In 1906, Henning deduced from his own newly published observations, with those of Dieterici, Smith, Griffiths and Joly, the values $L_0 = 94.21$ calories and $n = 0.31249$, using $t_e = 365°$ C. A still better set of constants is $L_0 = 92.93$ mean calories and $n = 0.3150$. With these constants, Thiesen's formula represents not only the seventeen experimental values below 100° reported by the five investigators just mentioned, but also the eighteen means between 100° and 200° presented to this society by the writer a year ago, all with an average deviation of only one sixteenth of one per cent. The formula can, therefore, be used with some confidence for qualitative extrapolations above 300°.

The relation between the pressure and the temperature of saturated steam has been determined up to the critical point by several observers. The values of Cailletet and Colardeau will be used in this paper. They enable one to compute the dp/dt of Clapyron's equation and thus to get from the values of L contained above a set of values for the change of volume during vaporization at temperatures above 200°. But the 1905 edition of Landolt and Bornstein's tables gives values of the density of water up to 320°. It is therefore easy to compute a set of values for the density of saturated steam up to that temperature, and to verify, as in the accompanying figure, Cailletet and Mathias's "law of the straight diameter." The diameter turns out to be, as usual, nearly but not quite straight. Its points above 200°[2] are represented well within their limits of error by the equation

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 26 and 27, 1909.

[2] An attempt was made to include also the points below 200° but even a third degree formula fails to represent satisfactorily the values near 0°.

$$s = 0.4552 - 0.0004757\,(t - 160) - 0.000000685\,(t - 160)^2 \text{ gr./cm.}^3$$

The substitution of $t_c = 365^\circ$ in this formula gives as the critical density of water

$$s_c = 1/v_c = 0.329 \text{ grams per cu. cm.},$$

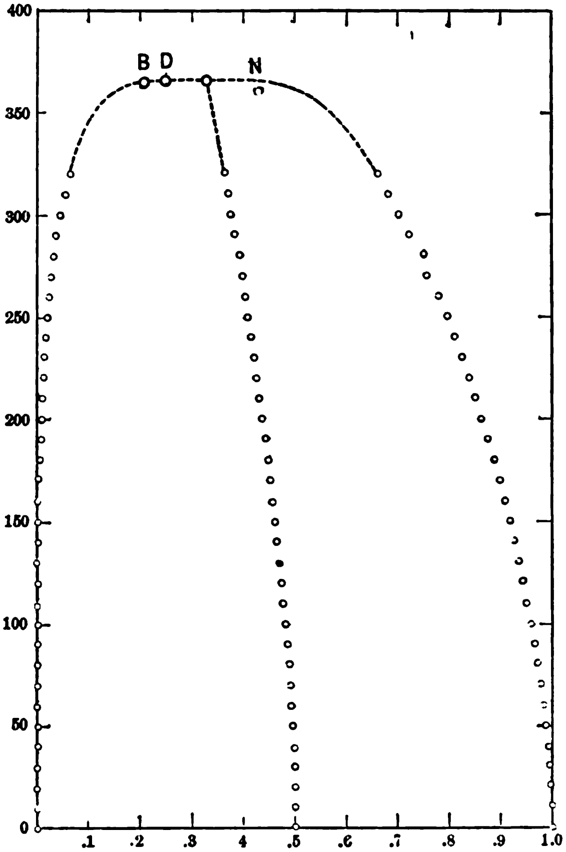

Fig. 1.

from which

$$v_c = 3.04 \text{ cu. cm. per gram.}$$

The accuracy of this determination is difficult to estimate because of its dependence on the extrapolation formula for L. It should, however, be noticed that comparatively large percentage errors in the computed densities of saturated steam make very small percentage errors in the mean densities and therefore in the critical volume. It should also be noticed that a considerable error in the assumed critical temperature makes only a small error in the critical volume.

In 1885, Nadejdime reported $v_c = 2.33$ cm.^{3}/gr., and in 1890, Battelli found $v_c = 4.812$ cm.^{3}/gr. Both used methods which were primarily designed for the determination of critical temperatures, and could be expected to give critical volumes only with considerable uncertainty. In 1904, Dieterici computed $v_c = 4.025$ cm.^{3}/gr. from an empirical formula of Young, assuming that at the critical point water is "normal." The disagreement between the present value and his indicates a polymerization factor of 1.3, which agrees very well with the first set of numbers given by Ramsay and Shields for water at ordinary temperatures, but cannot easily be reconciled with Ramsay's later corrected values.

From the equation of the diameter and the previously computed change of volume during vaporization one can compute the volumes and densities of water and of steam at temperatures above 320°. The results are indicated by the dotted line in the figure. The flatness of the top of the steam dome is due to the crowding of a large pressure range into a small temperature range.

Jefferson Physical Laboratory,
Cambridge, Mass.

Distribution of Gases in the Atmosphere.[1]

By W. J. Humphreys.

THE distribution of the principal gases of the atmosphere is calculated according to Ferrel's formula for places above the plane of the upper inversion. Below this plane, because of convection, it is assumed to be constant in volume per cent. except as modified by the presence of water vapor.

It is a striking fact that water vapor, which is so important in the economy of nature, often forms a smaller percentage of the atmosphere, even at the surface of the earth, than does argon; and that for the entire atmosphere it is seldom more than one fourth as great as is argon.

The Exponent of Gas Radiation.[1]

By W. J. Humphreys.

BY taking into account the fact that, for earth radiation, water vapor is practically a black body, it is possible with a knowledge of its effective temperature, that is, the temperature of an equally radiating black body, and with a knowledge of the temperature of the isothermal region, to determine approximately the total-radiation exponent of the upper air.

In this way it can be shown that the total radiation of the dry air is nearly proportional to the fourth power of its absolute temperature, at least when its temperature is not far from that of the earth.

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 26 and 27, 1909.

METHODS AND ERRORS IN MAGNETIC MEASUREMENTS.[1]

BY CHARLES W. BURROWS.

AFTER a brief introduction in which the merits of the common types of apparatus are discussed, the compensated double yoke method used at the Bureau of Standards is described. This is followed by a discussion of experimental data, showing the nature and magnitude of the various disturbing influences which enter into the measurement of normal magnetic induction and hysteresis data. These disturbing influences include galvanometer corrections; non-uniformities in specimens, magnetizing coils, and magnetic flux distribution; effect of ends of solenoids, of compensating coils, and of yokes; influence of imperfect demagnetization and of repeated reversals of the magnetizing current; effect of mechanical vibrations and temperature.

A comparison of data obtained by different methods is given, and the desirability of having magnetic data presented in round numbers and with some uniformity is shown. It is suggested that the magnetizing forces corresponding to the even thousands of gausses of induction would give satisfactory and easily comparable data.

In conclusion, specifications of the conditions and manner of measuring induction and hysteresis data, and an estimate of the limits of accuracy attainable, are given.

ACHROMATISM OF INTERFERENCE.[1]

BY C. W. CHAMBERLAIN.

THE production of a large number of interference bands from originally white light, in channelled spectra for example, is not a proof of the regularity of white light but of the resolving power of the spectroscope. The phenomena of white light may be produced by a succession of entirely irregular impulses. The production of a spectrum by means of a grating or prism involves the spreading out of an impulse over a length of time which increases with the resolving power. Interference in a spectrum involves a retardation of one portion of the impulse over the remainder. If an interfering system is illuminated with white light and the interfering beams fall upon a grating, the two portions of each white light impulse are spread out into a series of periodic impulses the number of which depends upon the number of lines in the grating. The relative retardation of the two portions of each impulse depends upon the angle between the lines of the grating and the interference fringes and the distance between the interfering system and the grating.

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 26 and 27, 1909.

Heretofore in the use of the interferometer the central fringe alone has been made achromatic in white light, and only a very narrow region on either side of this fringe shows any periodicity whatever. The number of fringes seen in white light can be enormously increased by viewing the region on either side of the central fringe through a grating, the lines of which are perpendicular to the fringes, rotating the grating through ninety degrees, then increasing its distance from the dividing surface of the instrument as the distance from the central fringe increases. In this manner black fringes in white light can be observed with large difference of path.

THE ABSENCE OF PHOTOELECTRIC FATIGUE IN A VERY HIGH VACUUM.[1]

BY R. A. MILLIKAN AND G. WINCHESTER.

IN view of conflicting testimony as to the existence of photoelectric fatigue in a very high vacuum an extended series of observations was made upon silver, zinc, iron, nickel, and copper. In no case was any positive evidence of photoelectric fatigue obtained. On the contrary, continuous illumination by a very powerful arc largely increased the photoelectric current from all of the metals experimented upon. In each case, however, a maximum was reached which remained constant, within the limits of experimental error, even under the most prolonged illumination. Also after this maximum had once been attained it remained altogether invariable with time during the period during which tests were carried out — a period amounting, in the cases of silver and copper to more than three weeks.

The initial increase in the photoelectric current produced by powerful illumination can be accounted for by assuming either that the light produces a mechanical roughening of the surface and hence an increase in the effective illuminated area, or that the light cleans the surface of oxides, adherent or occluded gases, or other surface films. The absence of fatigue under powerful and prolonged illumination lends support to the view that the emission of electrons by metals under the influence of ultraviolet light is a specific property of the atom of each metal rather than a property imposed upon the metal by the occlusion of a gas. It further shows that the phenomenon of photoelectric fatigue as ordinarily observed is one which has its seat in the gaseous layer surrounding the metal rather than in the metal itself.

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 26 and 27, 1909.

THE KINETIC ENERGY OF THE POSITIVE IONS EMITTED FROM HOT BODIES.[1]

BY F. C. BROWN.

THESE determinations of the kinetic energy of the positive ions were carried out under essentially the same conditions as were those on the kinetic energy of the positive ions from platinum.[2]

The ultimate object of the experiment was not so much to measure the kinetic energy of the ions as it was to test the theory proposed in a paper by Richardson and Brown.[3] According to this theory the kinetic energy of the ions arises from thermal agitation. This requires that the current carried from a portion of an infinite plane to a neighboring parallel plane should be expressed by the formula,

$$i = i_0 e^{-\frac{nev}{R\theta}}$$

where R is the ordinary gas constant in the equation $R\theta = \frac{1}{3} nm\overline{v}^2$.

The summary of the result is given in the accompanying table. Within the limits of experimental error these results are in agreement with the theory.

	Form of Radiator.	Temperature, Absolute.	Current at Zero Potential, $i_0 \times 10^{12}$.	Pressure, mm.	Gas Constant, $R/10^3$.
Gold (1)	Disc	1030 973	1	.007	4.2
Gold (2)	Disc	1190 1163	60	.01	3.9
Silver (1)	Disc	1020	.8	.002	3.0
Silver (2)	Disc	1150	35	.008	2.9
Palladium	Disc	1170	25	.04	3.4
Aluminium phosphate	Disc	1230	100	—	3.9
Aluminium phosphate (2)	Disc	1170	120	.006	3.4
Nickel	Strip	1120	2.5	.003	3.6
Iron (1)	Disc	1100	1	—	4.6
Iron (2)	Disc	1100	.8	.005	5.2
Iron (3)	Disc	1240	—	.01	4.4
Platinum	Wire	1695	—	—	5.1
Tungsten	Filament	1150	1	.0003	5.1
Tantalum	Filament	1050	.7	.0005	9.6
Tantalum	Strip	1050	1.0	.002	3.0
Osmium	Filament	1120	3.0	—	2.5

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 26 and 27, 1909.

[2] Paper by F. C. Brown, Phil. Mag., 36, vol. 17, p. 355 ; PHYS. REV., XXVII., 528.

[3] Phil. Mag., S. 6, vol. 16, p. 353, 1908.

A SPECTROSCOPE OF HIGH RESOLVING POWER.[1]

BY C. W. CHAMBERLAIN.

LIGHT falls upon a concave or plane grating having previously been separated by successive reflections into a succession of impulses separated from one another a distance in wave-lengths approximately equal numerically to the number of lines in the grating. A preliminary report. The theory has been developed and will be published later. The experimental work is now in progress.

ACCOUNT OF THE TREASURER OF THE AMERICAN PHYSICAL SOCIETY FOR THE YEAR ENDING DECEMBER 31, 1908.

DISBURSEMENTS.

Expenses of Secretary, 1907	$106.70
Expenses of Treasurer, 1907	30.45
Allowance to Secretary, 1908	100.00
Mileage, etc., of Secretary, 1908	73.88
Allowance to Treasurer, 1908	50.00
Stationery and Printing, orders of Secretary	147.64
Stationery and Printing, orders of Treasurer	92.95
Stationery and Printing, account of A.A.A.S.	5.68
Extra expenses in connection with transfer of office of Treasurer	35.95
Repayments to members owing to errors in 1907	18 00
PHYSICAL REVIEW for 1908 and in part for 1907	1,744.00
("Science Abstracts") for 1908 and in part for 1907	1,359.25
Deposit in Bank, December 31, 1908	114.44
	$3,878.94

RECEIPTS.

William Hallock, Treasurer	$1,020.00
Dues for 1906 and before	30.00
Dues for 1907.	457.50
Dues for 1908	2,224.78
Dues for 1909 and after	20.00
Life Membership	50.00
Entrance Fees	45.00
Payments for Section "B," Science Abstracts	13 00
Interest on Deposit	18.66
	$3,878.94

J. S. AMES, *Treasurer.*

I certify that I have audited these accounts and that I find them correct.

J. B. WHITEHEAD.
February 11, 1909.

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[1] An abstract of a paper presented at the Washington meeting of the Physical Society, April 26, 1906.

NEW BOOKS.

An Elementary Manual of Radiotelegraphy and Radiotelephony for Students and Operators. By J. A. Fleming. 8vo, pp. xiv + 340. London, Longmans, Green & Co., 1908.

This manual is more than an abridgement of the author's "Principles of Electric Wave Telegraphy," still the standard reference book on the subject. The larger work has been intelligently and carefully condensed into the smaller volume, and many of the important results of the rapid development of the art since the publication of the earlier book in 1906 have been included. The manual includes, for instance, the most complete discussion of the Poulsen arc to be found anywhere in the literature up to the present, and a short account of the crystal rectifiers is added to the list of oscillation detectors treated in the older work. The chapter on Radiotelephony includes the important developments in this subject down to August, 1908.

Space has been gained by omitting much of the historical matter and the derivation of most of the formulas. The rapid progress of the last two years has also enabled the author to sift out many of the tentative devices and arrangements of apparatus which were included in the earlier work, and to concentrate his attention on those features which have proven most valuable in practice. But the absence of all references to more complete treatments of even the most important subjects seems unfortunate.

Some errors of the larger work have been corrected in the manual. In the chapter on Electromagnetic Waves, which is written with all the author's usual clearness and force despite its highly condensed form, the Maxwell relations between the time and displacement rates of change of the magnetic and electric forces are now given correctly for a plane wave, as

$$\frac{d^2H}{dt^2}=\frac{1}{\mu K}\frac{d^2H}{dz^2} \quad \text{and} \quad \frac{d^2E}{dt^2}=\frac{1}{\mu K}\frac{d^2E}{dz^2}$$

(page 125). Other errors have crept in, however, and Fig. 7, on page 126, which represents the electric and magnetic forces as sine and cosine waves, displaced 90°, is incorrect and misleading. The correct diagram for this case, showing the electric and magnetic waves in the same phase, is given in Maxwell's Electricity and Magnetism, Vol. II., p. 439, Fig. 67.

Besides its uses as a hand-book for operators and an introduction to the more advanced treatises for students, the manual will serve as a valuable supplement to the author's "Principles of Electric Wave Telegraphy" until a new edition of the larger work shall become necessary.

G. W. Nasmyth.

Volume XXIX. *August, 1909.* *Number 2.*

THE

PHYSICAL REVIEW.

A SYSTEMATIC STUDY OF VIBRATORS AND RECEIVERS FOR SHORT ELECTRIC WAVES.

By Harold W. Webb and L. E. Woodman.

1. Introduction.

THE work described in this paper was undertaken to find, if possible, such definite relations between the dimensions and properties of vibrators and receivers used in measurements with short electric waves, as would make possible a more systematic procedure in future investigations. In the past it has been customary to construct vibrators and receivers of any arbitrary form, and to separately determine the properties and the wave-length corresponding to each. This somewhat haphazard method has proved unnecessarily tedious and has led to many inconsistent results and in many cases it has not been possible for later workers adequately to check the wave-length determinations made by their predecessors. It has also been very difficult to select in advance apparatus of suitable dimensions for new experiments of a definite character. It seemed, then, of great importance to eliminate this chance procedure by determining how the wave-lengths depended upon the dimensions and other details of vibrators and receivers, and to find the absolute value for certain definite standard forms, which could be reproduced at any time from given data. In addition, the solution of several minor problems was attempted. The methods of measurement, such as the resonance and interference curves which have been extensively used in the past, also formed the object of considerable study, as it is probable that they have caused much of the inconsistency which disfigures this branch of physical investigation. In this manner it was hoped to bring the investigation of the short electric waves to such a condition, that future results could be interpreted without incorrect assumptions, based largely on the very misleading analogies with other types of radiation.

2. Method of Measurement.

The greater part of this investigation was based on wave-length determinations made by the Boltzmann mirror interference method, previously used by many investigators.[1] The essential feature of this method is the use of two plane parallel mirrors, which split the beam from the exciter, or "vibrator" as we shall call it, into two equal parts with a difference in path depending upon their relative displacements. The two reflected beams then fall on a "collecting" mirror, which brings them to a focus on the receiver. The method used in this work differed fundamentally from that employed by most of the earlier workers, in that a non-selective or untuned receiver was used in place of the selective or tuned receiver.

The "tuned receiver" method of measuring the principal wave-length emitted by a vibrator consists of two steps. First, the "tuning" of a receiver to the vibrator, that is, the determination of the form and dimensions of a receiver which gives the maximum energy; and second, the using of this tuned receiver to measure the wave-length by an interference method. Now it has always been found[2] that the wave-length measured in this way depends mainly on the receiver and only slightly on the emission of the vibrator, which indicates that the result depends mainly upon the accuracy of the tuning, and that any error here will introduce an equal error into the wave-length measurement. In the course of the present study, and as a result of over one hundred resonance curves taken under a variety of conditions, it was found that the tuning process, unchecked, oftener than not, gave untrustworthy results and was seldom free from errors that made accurate measurements impracticable. This is one probable cause of many of the discordant results found in earlier work. A more complete discussion of this point will be taken up in the latter half of this paper.

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[1] Klemencic and Czermak, Wied. Ann., 50, 177, 1893.

[2] G. F. Hull, Phys. Rev., V., 241, 1897; Willard and Woodman, Phys. Rev., XVIII., 18 1904; A. D. Cole, Phys. Rev., XX., 271, 1905.

This difficulty is avoided with the non-selective receiver and the wave-length determined is a function of the vibrator only. The disturbing factors and the difficulties of measurement are greatly increased by its use, but with proper care satisfactory and consistent results were obtained, which was not the case when tuned receivers were used. The non-selective receiver in wave-length measurements is not wholly new as the coherer may come under this head.[1] G. F. Hull employed a nail coherer with his interferometer method, although the question of its non-selectiveness was somewhat in doubt. For this study, however, the coherer was not suitable, as its usefulness in quantitative determinations is doubtful.

In seeking for a suitable non-selective receiver the effect of a resistance on the selective qualities of a resonator was investigated. If to the thermo-junction at the center of a Klemencic receiver[2] a large resistance be added, or if we employ a bolometer wire[3] of high resistance to measure the energy received, the resulting damping might be expected to prevent resonance.[4] This was tested by introducing near the thermo-couple short resistance wires of platinum, .045 mm. to .000 7 mm. in diameter, and "tuning," that is, measuring the energy for different receiver lengths. Although this resistance was varied from 0 to 500 ohms, this method did not give a non-selective receiver, since all the curves indicated a marked resonance of some kind. With a resistance of 500 ohms the energy received increased from 65 per cent. to 100 per cent. as the length of the receiver was shortened from 17 to 7 centimeters, decreased to 75 per cent. at 5.5 centimeters, and rose again to 100 per cent. at 3.5 centimeters. Again, with 150 ohms the energy dropped from 100 per cent. at 7 centimeters to 45 per cent. at 45 centimeters, and similar results were obtained with resistances of 25 and 70 ohms. Since the change for a receiver with no resistance added to the thermo-junction is not more than 75 per cent. between the lengths 14 centimeters and 5 centimeters, we see that the introduction of large resistances does not greatly increase the non-selectiveness.

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[1] J. G. Bose, Phil. Mag., 43, 60, 1897; G. F. Hull, *l. c.*, p. 238.

[2] I. Klemencic, Wied. Ann., 45, 78, 1892.

[3] Rubens and Ritter, Wied. Ann., 40, 58, 1890.

[4] W. P. White, PHYS. REV., XXV., 138, 1907.

To explain this result we need only consider the independent resonance of the two antennæ or wings, between which lay the constriction consisting of the thermo-couple and the added resistance. When these wings were of proper length for individual resonance, the current passing between their adjacent ends through the constriction was a maximum, the action being very similar to that of the thermo-electric receiver employed by Lebedew in his work with short waves.[1] Thus the effect of increasing the resistance at the center was simply to split the single resonator into two, each of which possessed all the qualities of resonance.

The form which was finally adopted consisted of a Klemencic receiver, the length of which was ten or more times that of the average "tuning," or resonance length corresponding to the wave-lengths measured. The non-selective action of this arrangement is due to two causes. First, the absence of standing waves because of the rapid damping of the exciting wave, which has entirely ceased to affect the receiver by the time the first impulse has returned to the thermo-junction after reflection from the free ends. And, second, the overlapping of the broad resonance maxima of neighboring wave-lengths, which differ but little. For example, a receiver 70 centimeters long corresponds approximately to multiple tones of the wave-lengths, . . . , 6.5, 7.0, 7.6, 8.5, 9.4, 10.8, etc., centimeters, the lack of sharpness in the resonance, which is also due in part to the large damping in the incident wave, preventing any selective action. For the same reason a receiver whose length is very short compared with the resonance length should be non-selective, but in this case we should lose the advantage gained by the absence of standing waves, which is probably the chief factor in the success of the long receiver.

To prove the non-selectiveness of this new form by showing the absence of resonance maxima, a "tuning curve" was taken, varying the length of receiver from 110.0 centimeters to 3.0 centimeters. The result is shown in Fig. 1. No appreciable resonance could be observed until it was shortened to 27 cm., five times the fundamental resonance length, when a small maximum appeared (C). At three times the resonance length a sharper maximum was found (B), and at the resonance length (A) the resonance was very marked, the energy being six times that received with the greater lengths. Thus beyond the first three maxima we have practically no resonance and still retain about 20 per cent. of the energy.

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[1] P. Lebedew, Wied. Ann., 56, 3, 1895.

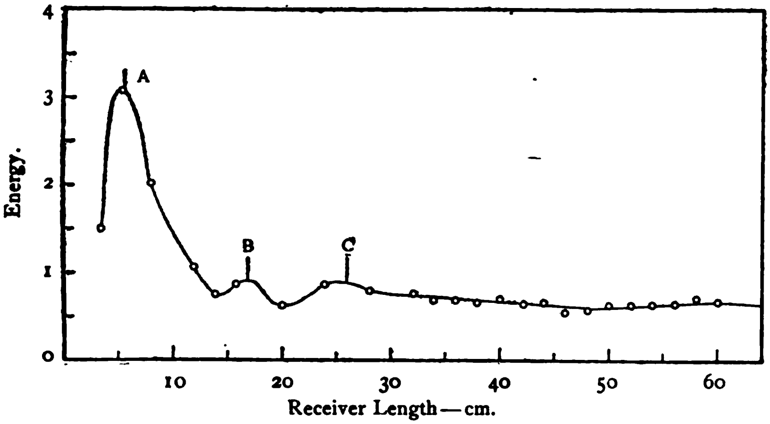

Fig. 1.

A further test was made by attaching capacities to the ends of a receiver, 58 centimeters long. As these capacities were diminished the energy received steadily increased without maxima or minima.

3. Details of Apparatus.

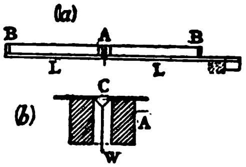

Fig. 2.

Receiver. — The arrangement of the non-selective receiver is shown in Fig. 2. The two collinear antennæ, or wings AB, were strips of copper foil, 0.15 mm. thick, 3 mm. wide, and 35 cm. long, making the total length of the receiver 70 cm. A wooden support L held the wings in position, their inner ends, which were 1 mm. apart, being attached to a wooden block A, glued to the support, while their outer ends were drawn tightly over similar blocks B. Care was taken to avoid bringing any dielectric into contact with the copper strips between the center and the ends, and to make them uniform. The effect at any point of a discontinuity of the dielectric or of the metal would be a reflection of the current, producing there a loop of potential and giving the system a resonance period. For this reason the block *A* was made as small as possible, 1 cm.

The thermo-couple joining the inner ends of the wings was made of commercial iron and constantan wires, .04 mm. and .025 mm. in diameter, respectively, or, when a greater sensitiveness was required, of platinum and constantan wires of smaller diameter. The arrangement of the lead wires originally used by Klemencic[1] was found more satisfactory than that employed by recent writers.[2] Instead of attaching the leads from the galvanometers to the free ends of the thermo-couple wires, they were connected directly to the lower side of the wings, *W* (Fig. 2, *b*). The thermo-couple wires *C* were then simply short pieces of wire soldered to the wings, and bent around one another, so that they remained in contact. The junction between them was made permanent by soldering, or by an electric weld.[3] This method of connecting the leads makes the construction much simpler, and renders the junction less liable to breakage when jarred, than if the relatively heavy lead-wires were attached directly to the fine wires. Furthermore, it makes possible the use of very thin wire in the thermo-element. For very sensitive receivers a platinum-constantan couple of wire about .0045 mm. in diameter was used. The platinum was obtained from platinum-cored silver wire. For the constantan a piece of the commercial wire, .025 mm. in diameter, was dissolved by repeatedly dipping into warm concentrated nitric acid, until a finely pointed end was obtained. With the aid of a binocular microscope, this was soldered to one of the copper strips with the pointed end extending about 0.5 mm. The platinum wire, soldered to the second strip with about 1 mm. free, was then bent so as to make a spring contact with the end of the constantan, and the two metals were welded together by passing a very small electric spark to the copper wings. The sensitiveness of such a junction was about fifteen times greater than one made with the larger iron and constantan wires with a welded junction.

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[1] I. Klemencic, Wied. Ann., 42, 416, 1891.

[2] I. Klemencic, Wied. Ann., 45, 78, 1892.

[3] Fountain and Blake, Phys. Rev., XXV., 257, 1907.

The two methods of arranging the lead wires were compared, but no difference was found either in the energy received or in the resonance length of the receiver.

The use of the non-selective receiver greatly increased the difficulty of measurement. In all this work a control or "check" receiver,[1] placed near the vibrator, was used to eliminate the errors due to the variations in the emitted energy. The result of any reading was then the ratio of the galvanometer throw, due to the non-selective or "main" receiver, to that obtained simultaneously from the check receiver. However, due to the differences in the character of the energy sent out by the vibrator, this ratio was not constant for any given arrangement of the apparatus,[2] the variation being very much larger with a non-selective main and a tuned check receiver, than with two similar receivers in resonance with the emitted wave-length. A non-selective check similar to the main receiver was tried, but it proved more convenient and satisfactory to have the auxiliary receiver tuned. To eliminate the errors due to the "change of ratio," the method of alternation was used, in which every second reading was taken with the apparatus in a standard condition. For example, when the position of a maximum or minimum in an interference curve was to be determined, alternate readings were taken on each point and some point of reference, chosen preferably near the maximum or minimum in question. The height of the different parts of the curve was thus given in terms of the height of the standard point, free from the error due to the gradual change of ratio. Two or three alternations were usually sufficient to accurately determine any point.

Interference Apparatus. — The arrangement of the interference apparatus is shown diagrammatically in Fig. 3 (*a*, plan; *b*, elevation). The "collecting" mirrors M and N, at the foci of which were, respectively, the vibrator V and the receiver R, were cylindrical-parabolic with horizontal axes, apertures 70 by 72 cm., and focal-length 36.0 cm. Mirrors of other types and dimensions described later were also employed. The two movable plane interference mirrors B were of plate glass, 76.0 cm. wide and 38 cm.

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[1] Klemencic and Czermak, *l. c.*

[2] Blake and Fountain, PHYS. REV., XXIII., 261, 1906.

high, covered with tin-foil. Each was fastened with three adjusting screws to a vertical carriage *P*, with horizontal arms *A*, sliding on a rigid frame *F*. Millimeter scales on the arms and indexes on the frame measured the position of the mirrors. The adjustment of the planes of the mirrors was carefully tested with a plumb-bob and a steel straight edge, as it was found that accurate alignment was very essential for correct results. The angle between the incident and reflected beams was 15° 30′ = $2 \cos^{-1} .992$, so that the actual difference of path was about one per cent. greater than that indicated by the scale.

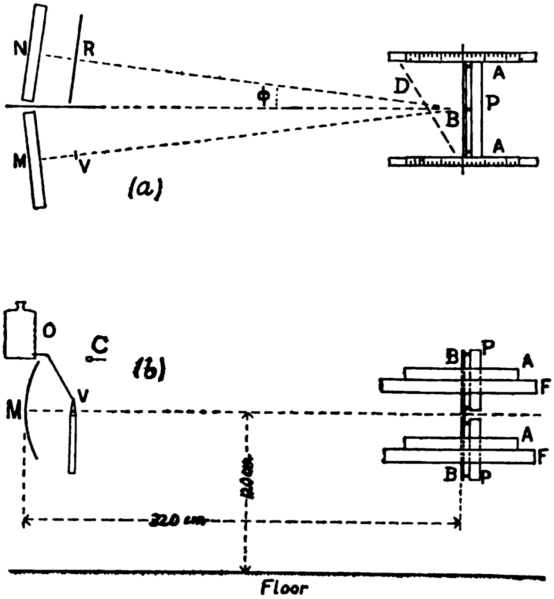

Fig. 3.

The main receiver (non-selective) *R* was mounted on a stand so as to permit readily altering its height by a measured amount. The check receiver *C* was fixed above the vibrator on a level with the top of the collecting mirror, as it was found that 9 per cent. of the energy in the direct beam was absorbed, when it was placed in front of the source. To facilitate changes in its length, it was made with detachable wings.

The galvanometers and the method of using them were similar to those described by Blake and Fountain.[1] The energy received by the check was measured on a du Bois-Rubens instrument, with a sensitiveness 4×10^{-9} volts, and for the main receiver a galvanometer was used of the type designed by Nichols and Williams,[2] with a sensitiveness 2×10^{-9} volts. The half-period of each galvanometer was 2.8 seconds.

Vibrators. — Two different types of vibrator were used in this investigation, the cylindrical or "rod" vibrator,[3] consisting of two metal cylinders placed end to end and separated by a very thin film of oil, and a modified form of Righi vibrator.[4] The former was selected as more suitable for an extended study, as it lent itself more readily to exact measurement, the conditions affecting its action being easily determined and controlled, as is not the case with the Righi or other similar forms. A convenient shape was arbitrarily chosen, which we shall call the "standard rod vibrator," and to which all the measurements of wave-length will be referred. Eight different sizes were used in which all the essential *relative* dimensions, including those of the supporting dielectric, were maintained constant. (See Table I., columns 1–5. The numbers 1–8 in column 1 will be used to refer to the individual vibrators.) The proportions between the dimensions are given in the following scheme (cf. Fig. 4):

Total length of vibrator (l) = 8 × diameter (d).

Length of dielectric (s), parallel to the axis of the cylinder, and consisting of 1 part oil, 2 parts wood-fibre = $\frac{3}{2}$ × diameter.

Width of dielectric (w), perpendicular to the axis = 3 × diameter.

The details of the construction of the rod vibrator are given in Fig. 4, *a*, *b*, *c*.

The circular brass rods B (Fig. 4, *a* and *c*) were cut off plane at their free ends, while into the inner ends of each a steel sphere of the same diameter was set in such a way that only one hemisphere was exposed. The spheres were soldered into their sockets, being removed and again soldered into place, whenever fresh sparking surfaces were required. The rods were supported by fitting them into holes drilled in wood-fibre plates *F*, which were attached to the upper ends of two flat wooden strips *W*, 35 cm. long, the lower ends of which were fastened rigidly to a wooden block *M*. The wood-fibre screw *S* served for the rough adjustment of the length of the "oil-gap," as we shall call the distance between the hemispherical ends of the rods. For the final adjustment the brass screw and nut *K* were employed. The effect of these was to spring together the two strips of wood and force apart the rods, the screw *S* acting as the fulcrum of a long lever. As the length of the oil-gap used was very small (.03–.07 mm.), such a delicate adjustment was necessary. To facilitate changing the size of the rods used in the vibrator, the wood-fibre plates *F* were made to accommodate six sizes, ranging from 0.24 to 0.79 cm. in diameter, being carefully tapered to keep the dielectric in its proper proportions (*b*, Fig. 4). To bring any desired set of holes into the focus of the collecting mirror, the block *M* was made to slide against a vertical stop *N* on the fixed supporting frame, a set screw *L* clamping it in the proper position, indicated by a scale.

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[1] Blake and Fountain, *l. c.*, p. 259.

[2] Nichols and Williams, PHYS. REV., XXVII., 250, 1908.

[3] Cf. A. D. Cole, PHYS. REV., IV., 55, 1896; K. F. Lindman, Ann. d. Phys., 7, 826, 1902.

[4] Cf. Willard and Woodman, *l. c.*

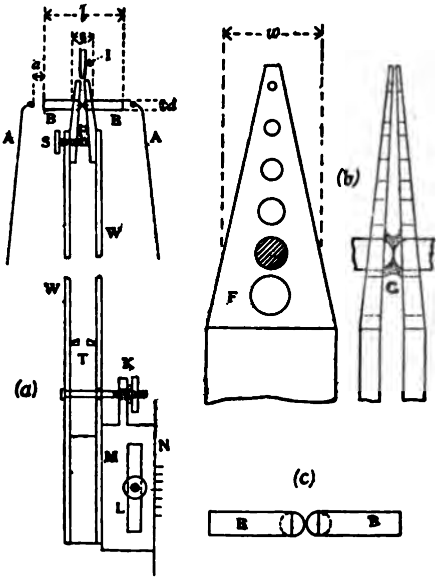

Fig. 4.

The copper wires *A*, terminating in small knobs, were connected to the terminals of an induction coil which supplied the current to the vibrator. Except for a few millimeters, these wires were nearly vertical, so that they were approximately perpendicular to the electric force. The length of the "air gaps", *a*, was 7 mm. for all the rod vibrators, this dimension, as will be shown later, having no effect on the wave-length emitted. The axial position was selected for them, to avoid the fouling due to the decomposition of the oil straying from the center. When they were placed near the dielectric, they were soon choked by a carbon deposit, and the action of the vibrator became very irregular. The induction coil was operated on alternating current, 110 volts, 5–7 amperes, and was capable of a 3 cm. spark.[1]

The oil portion *G* of the dielectric was unenclosed, except on two sides by the wood-fibre plates. This was made possible by using a continuous flow from the nozzle *I*, connected by a glass tube to a reservoir *O* (Fig. 3, *b*). The escaping oil was caught by wooden troughs *T* and collected in a receiving jar. This arrangement was of great advantage, since it enabled the observer to examine or clean the sparking surfaces without dismounting the vibrator. Furthermore, it was necessary that the gap be in view at all times, so that its length could be regulated and kept nearly constant. During the earlier part of the work the oil-gap was measured by a microscope with a micrometer eye-piece, but it was later found possible to adjust it with sufficient accuracy from the appearance of the spark in the oil.

It should be emphasized that all the measurements described in this paper refer only to vibrators with *oil*-gaps *less than 0.1 mm.* in length. This is important as an air-gap at the center in place of the oil-gap, or an oil-gap several millimeters in length[2] introduces new conditions, and such vibrators should therefore be considered in another class.

Sources of Error in Wave-length Measurements. — The measurement of wave-lengths by the system of mirrors described above is attended by several difficulties and sources of error. Among the most important of these is the disturbing action of the collecting mirrors, especially when a non-selective receiver is used. As this point is of considerable importance in all measurements of this nature, the results of an extended study of "mirror action" will be briefly described. The different collecting mirrors tested in the interference apparatus will for convenience be designated by the letters *A–E*, in the following way:

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[1] Blake and Fountain, *l. c.*, p. 258.

[2] Cf. W. R. Blair, PHYS. REV., XXVI., 57, 1908.

	Type of mirror.	Aperture.	Focal length (cm.).
A	Spherical.	50 cm. — circular.	19.1
B	"	50 cm. — "	18.8
C	Cylindrical-parabolic.	70 × 72 cm. — rectangular.	36.0
D	" "	70 × 72 cm. — "	50.0
E	" "	50 × 72 cm. — "	50.0

In the first stage of the investigation the spherical mirrors *A* and *B* were used with very unsatisfactory results. In nearly every

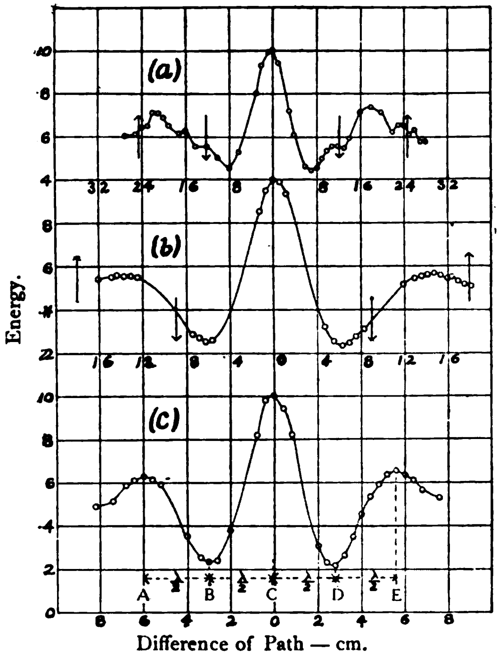

Fig. 5.

measurement of the wave-length the maxima and minima in the interference curves were displaced and distorted. This peculiar action of the mirrors can be illustrated best by a typical curve obtained under these conditions (see Fig. 5, *b*). Here the most noticeable features are the displaced maxima and minima, which indicate values of the wave-length, 14.4 cm. and 12.8 cm., respectively. The correct value, as determined by later experiment, was about 18.0 cm., the true position of the maxima and minima being indicated on the curve by arrows. It will also be observed that the maxima are considerably flattened, and that on the left hand side a double crest appears. With the cylindrical-parabolic mirror *C*, of double the focal length, and the spherical mirror *B* for vibrator and receiver, respectively, correct results were obtained for wave-length values less than 13.0 cm. For larger values the curves were extremely irregular with multiple maxima and minima. This is shown by the curve *a*, in Fig. 5. The true wave-length was about 24 cm., the arrows indicating as before the proper position for the maxima and minima. The distortion is due in part to other causes, such as the large difference in path between the two portions of the reflected beam, which tends to flatten the maxima. For purposes of comparison a normal wave-length curve free from mirror action is shown in Fig. 5, *c*.

With both mirrors of the cylindrical-parabolic type *C*, the wave-length curves were much more regular, and this combination was used for the greater part of the measurements on which this study is based. While it was evident that the disturbances were not wholly avoided, they were small and their effect could be largely eliminated by averaging the results over a great range of wave-lengths. As we shall see later the wave-lengths greater than 14 cm. were measured with errors of three or four per cent. For wave-lengths greater than 18 cm., the large difference in path made the maxima indeterminate, and in such cases only the minima could be used.

That the results for the shorter wave-lengths were not affected appreciably by the mirrors of the type *C*, was shown by comparing the results for a given vibrator with several different combinations of mirrors. With vibrator 5 the mirrors *A* and *B* gave a wave-length 11.6 cm., about five per cent. lower than the correct value, 12.2 cm. But with the combinations *C* and *C*, *D* and *D*, and *E* and *E* (see Table I., VII.–XI.) the values of the wave-length were approximately the same, showing that neither the change of focal length from 36 to 50 cm. (cf. *C* and *D*), nor the change in the height of the aperture from 70 to 50 cm. (cf. *D* and *E*) affected the results. With the mirrors *C* and *B* behind the vibrator and the receiver, respectively, the same value for the wave-length was obtained (VI.), proving that the use of the spherical mirror behind the receiver was permissible for wave-lengths less than 13 cm. Combinations *C* and *B*, and *C* and *C* were again compared with a vibrator emitting a wave-length of 10 cm., with the same result.

No complete explanation can be given for this peculiar action of the mirrors. It seems to depend principally upon their focal length, but it is also closely related to their form. The investigation of the phenomenon proved very complex, and in this study no attempt was made to push it further than was necessary to find mirrors which would give results free from such disturbances.

As will be seen later, the measurements made to determine the resonance lengths of receivers are also subject to mirror action, and in general this must be true of all measurements made with short waves. In a number of earlier investigations[1] the focal length of the mirrors used has been less than a single wave-length, and there the mirrors undoubtedly influenced the beam sent out from the combination of vibrator and mirror. However, the error produced was probably not of as great importance as would at first appear, since the effect of short focal lengths would be to give the emitted beam a definite "wave-length," which was determined by the focal length rather than by the nature of the vibrator, especially if the focal length differed but little from an odd multiple of the quarter wave-length of the vibrator. Nevertheless, the influence of the mirrors must be considered, whenever they are used, if satisfactory results are to be obtained.

The symmetry of the apparatus was found to be of very great importance in obtaining correct results. The two collecting and the two plane interference mirrors should be adjusted and aligned geometrically with great care. But it proved to be of much greater importance to so adjust the apparatus, that an equal amount of energy was reflected from each of the plane mirrors in its zero position. Failure to observe this precaution resulted in errors in measuring the wave-length, with tuned as well as with non-selective receivers, of five or more per cent. when this difference of energy was thirty to forty per cent. The test of this symmetry could be made by alternately removing one or the other of the mirrors and thus measuring the reflected energy from each, but it was more convenient and satisfactory to use an oblique deflecting screen *D* (Fig. 3) to intercept the upper or lower half of the beam. Usually, any dissymmetry could be eliminated by a slight vertical movement of the main receiver. In some cases the height of the vibrator was altered by a few millimeters, and in one or two measurements a metal screen was used to cut off some of the energy near the edge of the beam.

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[1] Cf. Willard and Woodman, *l. c.*; Blake and Fountain, *l. c.*

The "shadowing" of one mirror by the other[1] was avoided by having both the vibrator and receiver in the same horizontal plane with the dividing line between the interference mirrors. The sidewise shift of the reflected beams, when these mirrors were displaced from the zero position, resulted in some error, but this was small because of the small angle between the incident and reflected beams, and was still further reduced by the use of cylindrical mirrors focusing on the long non-selective receiver. When measuring large wave-lengths the center of gravity of the mirrors was kept nearly constant, to minimize the dissymmetry of the reflected beam, but even then wave-lengths greater than 20 cm. could be determined from the first minima only, as the maxima became indefinite.

In judging whether or not a wave-length curve was correct, the following criteria were employed: first, the maxima and minima should be well defined and single — in general the curve should be smooth and symmetrical; secondly, the wave-length determined from the maxima (*A* and *E*, Fig. 5, *c*) should agree with that found from the minima (*B* and *D*), within one or two per cent.; and thirdly, the central or principal maximum (*C*), corresponding to no difference of path, should not be displaced more than one or two millimeters. These criteria were not in themselves sufficient for correct results, but they were usually necessary, and hence served as guides.

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[1] Willard and Woodman, *l. c.*, p. 12.

Definition of the Wave-length. — The quantity which we shall call the wave-length in these measurements was determined by averaging the two values found from the distances between the central maximum on the interference curve and the first maxima on either side, and the single value found from the distance between the first minima on either side. Thus, referring again to Fig. 5, *c*, we have $\lambda = \frac{1}{3}(CA + CE + BD)$. Greater weight is given the determinations from the maxima, as they are somewhat more accurrate than those based on the minima, since in the latter the errors are doubled. The distances AB and DE were not used, for averaging these values with the others would result in losing entirely the advantage gained by the use of the minima. Maxima and minima for differences in path greater than a single wave-length were not determined, as with a non-selective receiver they were very indefinite. Furthermore, they would depend upon the nature of the wave-train within the second wave-length, while with these highly damped radiations only the first wave-length is of importance. Hence, the wave-length is practically the distance between the first two crests in the wave-train emitted by the vibrator.

4. WAVE-LENGTHS EMITTED BY VIBRATORS.

Rod Vibrator. — The results of the measurements of the wave-lengths emitted by the "standard" cylindrical or rod vibrators are tabulated in Table I. In the sixth column the Roman numerals are used to designate the individual determinations. In the seventh and eighth columns are given, respectively, the wave-lengths measured, and the ratios of the wave-lengths to the lengths of the vibrator ($\lambda \div l$). In the last two columns the collecting mirrors used in the determination are indicated by the letters *A–E*, the meaning of which has already been given.

Fig. 6, *a*, gives these results in graphical form, the abscissas representing vibrator lengths and the ordinates wave-lengths. The curve is a straight line passing through the origin, showing that the wave-length is in exact proportion to the dimensions of the vibrator.

The mean value for the ratio $\lambda \div l$ for the eight vibrators is 2.40. In computing this, the determinations I. and II. were omitted, since the conditions under which they were taken were unfavorable. It will be noticed that the values for the determinations XII. and XIII. lie on either side of the mean value, the variation amounting to about three per cent. This was due mainly to the disturbing action of the mirrors, which as we have already seen becomes more effective as the wave-length increases. However, the two errors very nearly balance one another, so that their mean value seems practically independent of the mirrors.

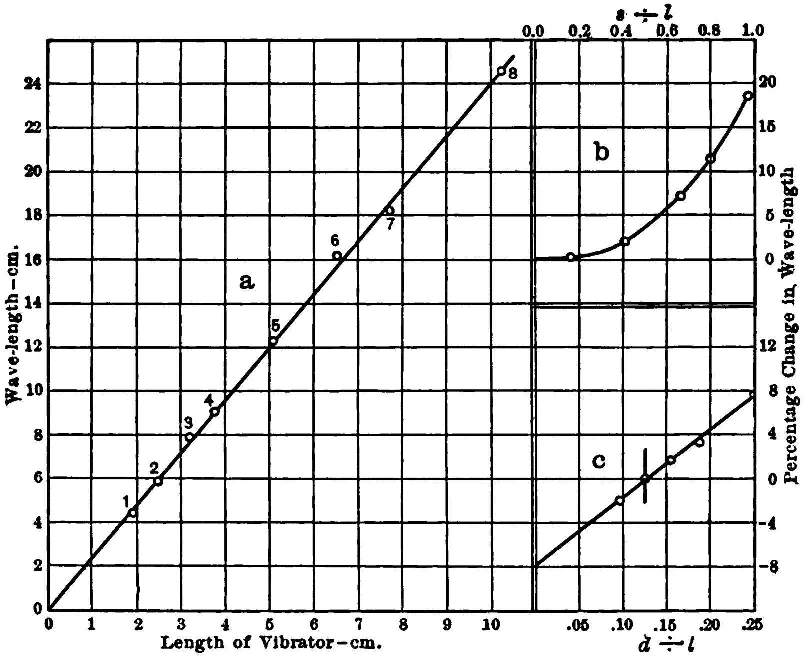

Fig. 6.

As mentioned before, the wave-length of vibrator 5 was determined with four different combinations of collecting mirrors, other than *A* and *B*, all of which give approximately the same values, the average of the six determinations VI.–XI. being 2.42. The wave-length of vibrator 8 (XIV.) was determined from the minima of the interference curve, since the maxima became indefinite for such large differences in path.

Conditions Affecting the Wave-length of the Rod Vibrator.—The results just described are of two-fold importance; first, because they prove a linear relation between the dimensions and the wave-length of a vibrator with constant *relative* dimensions; and second, because of their statistical value. They enable us to obtain any desired wave-length by constructing a vibrator whose form and dimensions are given by the four relations (cf. Fig. 4);

TABLE I.

Vibrator.	Dimensions of Vibrators.				Wave-length Determination.	Wave-length $= \lambda$ (cm.)	$\frac{\lambda}{l}$	Mirrors.	
	Diameter of Rods $= d$		Length of Rods $= l$ cm.,	Length of Dielectric $= s$ (cm.)				Vibrator.	Receiver.
	Inches.	cm.							
	2	3	4	5	6	7	8	9	10
1	[illegible]	0.24	1.90	0.4	I.	4.38	(2.31)	*A*	*B*
2	[illegible]	0.32	2.50	0.5	II.	5.78	(2.31)	*C*	*B*
3	[illegible]	0.40	3.20	0.6	III.	7.60	2.38	*C*	*B*
"	"	"	3.22	0.6	IV.	7.91	2.45	*C*	*C*
4	[illegible]	0.48	3.80	0.7	V.	8.96	2.36	*C*	*B*
5	[illegible]	0.64	5.08	1.0	VI.	12.15	2.40	*C*	*B*
"	"	"	5.10	"	VII.	12.30	2.41	*C*	*C*
"	"	"	5.10	"	VIII.	12.25	2.40	*C*	*C*
"	"	"	5.08	"	IX.	12.15	2.39	*C*	*C*
"	"	"	5.05	"	X.	12.27	2.43	*D*	*D*
"	"	"	5.05	"	XI.	12.40	2.46	*E*	*E*
6	[illegible]	0.80	6.55	1.2	XII.	16.15	2.46	*C*	*C*
7	[illegible]	0.96	7.78	1.5	XIII.	18.18	2.33	*C*	*C*
8	[illegible]	1.27	10.25	1.9	XIV.	24.60	2.40	*C*	*C*
						Mean	2.40		

$$l = 8d = \tfrac{16}{3}s = \tfrac{8}{3}w,$$

$$\lambda = 2.40l,$$

[$a = 7$ mm. Oil-gap < 0.1 mm.].

The relative dimensions of our standard vibrator are, however, entirely arbitrary, having been chosen for simplicity and convenience. For this reason a study of the rod vibrator was undertaken to determine approximately how the wave-length varies with varying relative dimensions and other conditions, thus enabling us to compute the wave-lengths for vibrators of other than the standard form. Vibrator 5 was used for this purpose, with the apparatus and non-selective receiver as before. To facilitate the measurements only the minima of the interference curves were taken to determine the wave-length, as it was found that this brought in no considerable error. The work was divided into three parts; determining the effect of varying,

1. The position and size of the air-gaps.
2. The length of the dielectric.
3. The ratio of the diameter to the length of the vibrator.

No change of wave-length was observed as the air-gaps, lying in the axis of the cylinders produced, were varied from 3 to 7 mm. With 3.5 mm. air-gaps, perpendicular to the axis of the cylinders, and 10 mm. from the oil-gap, unvarying results were obtained. The wave-length is therefore independent of the length and position of the air-gaps. Earlier work with tuning curves, however, indicated a more complex emission[1] when the air-gaps were very small. This will be shown to better advantage later, in the discussion of the Righi vibrator (see Fig. 8*b*).

As was to be expected, the length of the supporting dielectric at the center of the vibrator, measured along the axis, has a very appreciable effect, especially as it approaches the free ends. Without altering the metal rods of the vibrator, the dielectric was built up symmetrically from the center outwards, by slipping hard rubber collars, 2 cm. square, over their free ends. Fig. 6, *b*, shows the relation found between the length of the dielectric and the wave-length. For the sake of computation, the abscissas have been made to represent the fraction of the total length of vibrator covered by the dielectric, and the ordinates the corresponding percentage change in wave-length. The curves show that the dielectric is practically ineffective, if its length is less than one fourth the total length of the metal, but that the effect becomes more and more marked for larger values.

The relation between the wave-length and the diameter of the rods, for a given length of vibrator, is shown in Fig 6, *c*, in which the abscissas give the ratio of the diameter to the length ($d \div l$), and the ordinates the percentage change in wave-length, 100 per cent. corresponding to the standard condition $d \div l = 0.125$. The curve is a straight line and may be represented by the equation,

$$100\,\Delta\lambda \div \lambda = 60(d \div l - 0.125),$$

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[1] Cf. K. F. Lindman, *l. c.*

where $\Delta\lambda$ is the actual change of wave-length. In this determination five sets of rods were used, with diameters ranging from 0.48 to 1.27 cm. The total lengths of the vibrator and of the dielectric were kept constant at 5.08 cm. and 0.8 cm., respectively.

If the straight line in Fig. 6, *c* be prolonged, it cuts the axis of zero diameter at $\Delta\lambda = -7.5$ per cent., or in the unreduced coördinates at $\lambda = 11.4$ cm. The length of the vibrator being 5.08 cm., we find the limiting value of the ratio of the wave-length to the vibrator length, as the diameter approaches zero, to be $11.4 \div 5.08 = 2.24$.

Within reasonable limits, we should now be able to calculate the wave-length of any rod vibrator from its dimensions alone, using the relation $\lambda = 2.4l$, found for the standard form, and the corrections for the dielectric and the diameter, just described. For example, the calculated wave-length for a vibrator with the dimensions $l = 4.2$ cm., $d = 0.40$ cm., $s = 0.55$ cm. is 9.9 cm., the correction for $d \div l$ being -2 per cent., and for $s \div l$, 0 per cent. The experimental value for this vibrator was 9.6 cm., but the interference curve was irregular, the indications being that this was somewhat low, so that the calculated value is probably more nearly correct.

The effect of varying the length of the oil-gap was not investigated with this type of vibrator, but the question will be discussed in the following section of this paper, when the results of a similar study with the Righi vibrator are described.

Wave-length of a Righi Vibrator.— For the purpose of obtaining further statistics of the relation of the wave-length to the dimensions of vibrators, a Righi vibrator[1] of special form and construction was next studied. The arrangement and dimensions of the eight sizes of vibrator used are given by Fig. 7, and Table II. The steel spheres *S* rested in shallow ground sockets in the ends of glass tubes *T*. The weight of the larger spheres was sufficient to keep them steady, but to hold the smaller sizes in place, it was necessary to use air suction, obtained by connecting an aspirator pump to the lower ends of the glass tubes. No oil vessel was used, the layer of oil *O* separating the spheres being maintained by a constant flow from a glass tube *I*, connected to a reservoir, while the wooden troughs *L* collected the escaping oil into a suitable receptacle. The adjustment of the length of the oil-gap was made by springing the glass tubes by means of the link *P* and the turn-screw *K*. A wooden clamp *R*, 3 cm. below the spheres, held the tubes rigidly apart at a fixed distance, and rendered the system stiff against vibration. Acting as a fulcrum, it further served to reduce the motion of the spheres due to the screw *K*, thus making possible a very small change in the oil-gap. The air-gaps were placed in the axis of the vibrator, and were 6 mm. long for all sizes.

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[1] Cf. Willard and Woodman, *l. c.*, p. 2.

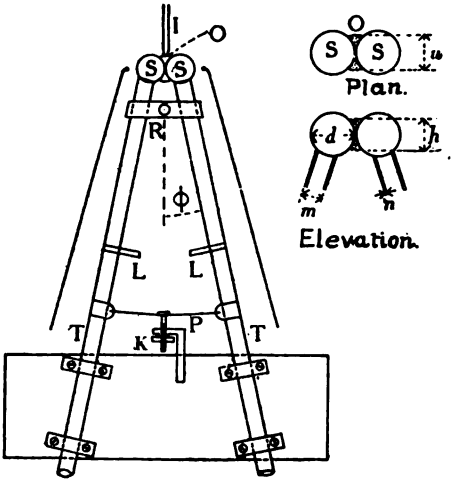

Fig. 7.

This peculiar form of the Righi vibrator was found to have very great advantage over that usually employed, in that it gave a very close approximation to a point source, and made the adjustment and renewal of the oil-gap very simple. Further details of its use will be given in a subsequent paper.

The dimensions for the eight sizes of the "standard Righi vibrator," as it will be called, are tabulated in Table II., columns 1–7, with the corresponding wave-lengths determined with the non-selective receiver in column 8, the letters used referring to Fig. 7. The constancy of the relative dimensions was not as carefully maintained as in the case of the rod vibrators. This was due partly to the necessity of choosing convenient sizes of glass tubes, and partly to the fact that the dimensions of the oil layer could not be kept proportional to the diameter of the spheres. With large spheres its size is limited by the surface tension, while with the smaller vibrators the vigorous flow, necessary to wash out the carbon deposit from the oil-gap, gave a relatively large layer. Since the variations in the relative dimensions of the glass mountings were small, and did not greatly alter the amount of dielectric in contact with the metal, they may be neglected.

TABLE II.

Spheres.			**Oil layer (cm.).**		**Supporting Rods (cm.).**		**λ (cm.).**	**Remarks.**
Diameter $=d$		**Metal.** 3	**Height** $=h$ 4	**Width** $=w$ 5	**Diam.** $=m$ 6	**Thick. of walls** $=n$ 7	8	
Inches. 1	**cm.** 2							
$\frac{3}{8}$	0.95	Steel	0.6	0.6	0.60	0.10	6.7	From 1 max.
$\frac{1}{2}$	1.27	"	0.9	0.75	0.60	0.10	8.9	
$\frac{5}{8}$	1.59	"	1.0	0.7	0.60	0.10	10.4	
$\frac{3}{4}$	1.91	"	1.1	0.8	0.88	0.12	11.4	
$\frac{7}{8}$	2.22	Brass	1.3	0.85	0.88	0.12	12.8	
1	2.54	Steel	(1.35)	(1.0)	1.21	0.22	14.7	
$1\frac{1}{4}$	3.18	"	1.6	1.2	1.35	0.20	17.8	From 1 max. and 1 min.
$1\frac{1}{2}$	3.81	"	1.7	1.3	1.35	0.20	20.7	From min. alone.

As shown in Fig. 8, *a*, the relation between the wave-length and the diameter of the spheres, between the limits $\lambda = 7$ cm. and $\lambda = 20$ cm., can be represented very well by a straight line with the equation

$$\lambda = 4.95d + 2.2.$$

The intercept 2.2 cm. indicates a large change of curvature between $\lambda = 6.0$ cm. and $\lambda = 0$, for we should expect the curve to pass through the origin. This discontinuity in the relation between λ and d is probably due to the increasing size of the oil layer, which, as we have just seen, becomes larger and larger relatively to the decreasing diameter of the spheres. If the diminution in size were continued in the same way, the sphere would eventually become entirely immersed in the dielectric, and the relation between λ and d would then be given by a straight line with a much greater slope. We should therefore regard this equation simply as a convenient and close approximation, from which we can obtain, within the given limits, the wave-length for any vibrator having the standard form.

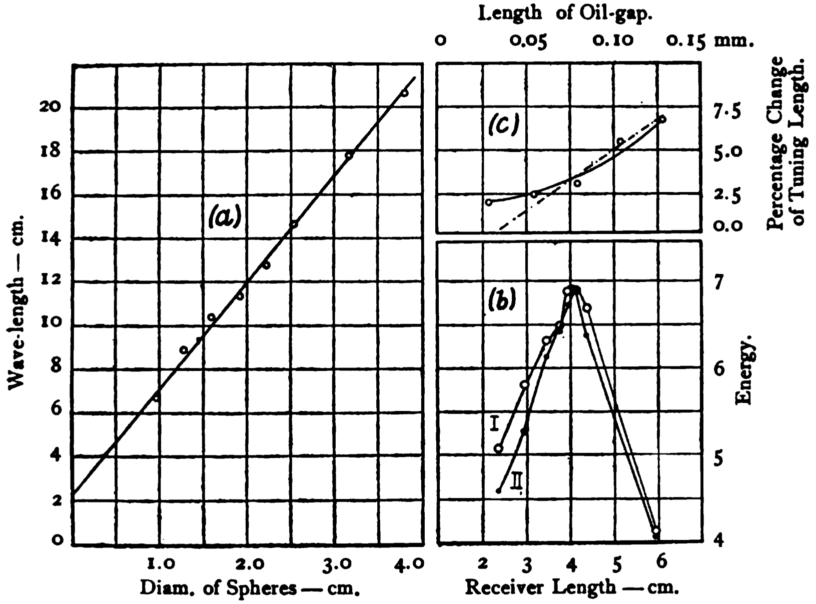

Fig. 8.

For convenience, the dimensions and wave-lengths of the Righi vibrators are given in the following compact form:

$$m = 0.5d$$
$$n = .07d$$
$$\varphi = \tan^{-1} \tfrac{1}{5}$$
$$h = .35d + 0.4$$
$$w = .25d + 0.4$$
$$\lambda = 4.95d + 2.2$$

The effect upon the wave-length of varying the conditions for this vibrator was investigated by means of tuning curves, the resonance length being taken as the measure of the wave-length. As already emphasized this method is open to objection, but in this case it was found sufficiently accurate. The following conditions were studied:

1. The length and position of the air-gap.

2. The dimensions and nature of the dielectric between the spheres.
3. The size of the oil-gap.
4. The nature of the metal.

No change in the wave-length was observed as the length and position of the air-gaps were altered. During this test the gaps were varied from 6 to 1.5 mm., and moved to positions both in front and above the vibrator. There was, however, as in the case of the rod vibrator, an indication of a greater complexity in the emission for the shorter air-gaps of 1.5 and 3 mm., with which the tuning curves were less smooth than those taken with 6 mm. gaps and showed a greater tendency to develop secondary maxima. This is shown very well by the Curves *I* and *II*, Fig. 8, *b*, which were taken simultaneously with air-gaps, 3 and 6 mm. long, respectively. The latter (*II*, *b*) is much sharper and has a well defined maximum, while the former (*I*, *b*) tends to give additional crests. Quantitative conclusions, however, cannot be drawn from these curves, since their form depends upon the collecting mirrors used, as well as upon the character of the emission of the vibrator.

The observations taken with these vibrators show that small variations in the dimensions of the dielectric are not sufficient to appreciably affect the wave-length. Complete data bearing on this point could not be obtained, as the nature of the vibrator prevented large variations in the size of the oil layer. Paraffin oil was used in the greater part of the work and proved very satisfactory. In the earlier measurements olive oil was tried, but because of its greater viscosity, the gap quickly clogged with the particles of carbon from the decomposed oil, causing the vibrator to break down after a very few discharges. The breaking potential of the olive oil was, however, much less than that of paraffin oil, with which it was impossible to use gaps larger than .08 mm., as then the spark between the spheres passed through the air instead of through the oil. Kerosene was also tried with this vibrator, but its action proved irregular and unsatisfactory. No difference in the wave-length emitted with the different oils was observed.

In a similar way the effect of the length of the oil-gap was studied by tuning curves. For measuring this length, a microscope with a micrometer eye piece was mounted on a long arm, so that it could be swung into the observing position in front of the vibrator, or out of the beam, at will. The measurement of the gap was always made at the beginning of a series of readings, before the oil was flowed on the spheres. The results of these observations are given by the curve *c* in Fig. 8, the ordinates representing percentage change of tuning length (*i. e.*, also of wave-length), and the abscissas the corresponding length of the oil-gap in millimeters. It is probable that the dotted line more nearly represents the relation than the heavy curve drawn through the observed points, since the pitting increases the length of the small gaps very rapidly, so that in determining the first point, the average length was about .04 mm., instead of .03 mm. as measured. We see, however, that within the limits, .03 to .08 mm., the effect of the gap length is small, and need not therefore be accurately determined, which conclusion applies also to the rod vibrators, since the effect of the gap is essentially the same in both types. Since the limit .03 to .08 mm. includes all sizes of gaps which can be used to advantage with either type of vibrator, this factor was neglected in all subsequent measurements. In practice it was found best to regulate the gap by the appearance of the spark, as the rapid pitting made micrometer measurements difficult and unreliable for long series of readings. This regulation was repeated at short intervals, as a large, bright, sputtering action at the oil-gap was usually accompanied by large variations in the ratios of the energy measured by the two receivers.

The tuning curves, taken with steel, brass, and aluminium spheres in the vibrator, coincided with one another almost exactly, showing that the emission was independent of the nature of the metal.

5. ENERGY AND DAMPING.

Energy Emitted by the Vibrators. — A study of the energy emitted by different vibrators leads to some interesting conclusions. Much of the data available is not sufficiently accurate for the formulation of definite relations, because the measurement of the energy was complicated by the condition of the sparking surface, the presence of carbon particles in the oil, and the varying length of the spark gap, of which only the initial value could be determined.

We can, however, show approximately how the energy depends upon the various conditions and dimensions of the vibrator; viz., its size, the length of the oil-gap and of the air-gaps, and the nature of the metal and of the oil. These determinations were made on a non-selective receiver, which was taken as measuring the absolute energy, or when the wave-length was not changed by the variation, a tuned receiver was sometimes substituted. Unless otherwise stated the length of the oil-gap will be assumed constant in the following relations.

The energy emitted by a vibrator with constant relative dimensions was approximately proportional to the square of the wave-length. This was true for both types of vibrator, as shown by the following tables:

Rod Vibrator.

Wave-length, λ.	Energy.	Energy ÷ λ^2.
15.8	207	.83
12.2	114	.77
9.1	62	.75
7.7	51	.86

Righi Vibrator.

Wave-length, λ.	Energy.	Energy ÷ λ^2.
17.8	440	1.4
12.8	210	1.3
10.4	180	1.7
8.9	110	1.4

The energy corresponding to a given wave-length was very nearly the same for both types of vibrator.

The relation between the length of the oil-gap and the emitted energy was carefully studied with the standard Righi vibrator. As the wave-length and damping were practically constant, the energy was measured on a tuned receiver. The results varied considerably, but the mean of thirty measurements showed the energy to be inversely proportional to the length of the gap.

In both types of vibrator the energy emitted was found to be practically independent of the length of the air-gaps, within the limits 3–7 mm.

No difference in the energy was observed when spheres of brass or aluminium were substituted for the steel. With zinc spheres the energy was much smaller for the same initial length of oil-gap, but this was doubtless due to the extraordinarily rapid pitting of the soft metal, which caused a decrease of 75 per cent. in the emission during the first five discharges. The action of the steel, brass, and aluminium spheres was very similar. However, the life of the surfaces of the commercial steel spheres appeared to be somewhat longer, and furthermore their high polish and exact form and size made them much the best for continued work.

The energy varied with the oil used in the vibrator. With paraffin oil it was nearly 30 per cent. greater than with olive oil. This was due to the greater potential required to break through the layer of paraffin oil separating the sparking surfaces, for as we have seen the olive oil has a much lower breaking potential.

With a constant length of cylindrical vibrator, the energy was approximately proportional to the diameter of the rods.

Damping. — If we assume the wave-train emitted by a vibrator to be a simple damped sine wave with the equation $e^{-\gamma nt} \cos 2\pi nt$, where γ is the logarithmic decrement, n the frequency, and t the time, we find two expressions for γ,

$$\gamma = - \text{nat. log.} \left(2 \frac{E_3}{E_1} - 1 \right) \text{ for the first maximum,}$$

and

$$\gamma = - 2 \text{ nat. log.} \left(1 - 2 \frac{E_2}{E_1} \right) \text{ for the first minimum,}$$

E_1, E_2, and E_3 being the energy measured by the receiver in the interference apparatus for differences of path, zero, $\frac{1}{2}\lambda$, and λ, respectively. In this way Klemencic and Czermak[1] found $\gamma = .50$. In their determination the receiver used was tuned to the vibrator, and therefore this value of γ was much too low, as it depended chiefly upon the smaller decrement of the receiver. The non-selective receiver is not open to this objection since it measures the absolute value of the incident energy without regard to its phase.[2] But there is another source of error in determining γ, due to the wrong assumption of the form of the wave-train. For if we compare the values of γ found from the two expressions given above, we find that the minima give a value twenty to thirty per cent. lower than that computed from the maxima, showing that a damped sine wave does not nearly represent the emission. This arises from the fact that the damping is due almost entirely to radiation, and we cannot therefore express the differential equation for the vibrator in the form

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[1] Klemencic and Czermak, *l. c.*, p. 181.

[2] Cf. G. F. Hull, *l. c.*

$$\frac{d^2\varphi}{dt^2} + 2n\gamma\frac{d\varphi}{dt} + (\gamma^2 + 4\pi^2)n^2\varphi = 0,$$

as in the case of damping due entirely to the Joule effect. But γ has no real meaning unless it can be defined in this way. We must therefore arbitrarily define it as the average value of the two expressions given above and regard the sine wave $e^{-\gamma nt} \cos 2\pi nt$, which it determines in connection with the wave-length measurements, as a rough approximation to the true wave-form. This is useful as it gives an idea of the rate of disappearance of the train.

Most of the measurements with the non-selective receiver in the present investigation are not available for computing γ, since those in which the cylindrical mirrors were employed cannot be used because of the divergent beam, and many of those taken with the spherical mirrors are unreliable. However, among the latter were a few curves which appeared to be very nearly correct, and from these the following results were obtained:

From four curves taken with the rod vibrators the average value of γ was 1.28. With the Righi vibrator, steel spheres 1.9 cm. in diameter, the average of seven determinations gave $\gamma = 1.30$. With spheres 0.95 cm., 2.54 cm., and 3.18 cm. in diameter, γ was 1.48, 1.41, and 1.75, respectively, but these higher values were no doubt due to mirror disturbances, since the curves were all very much distorted. Moreover, we should expect the damping to be the same for all sizes of vibrator, especially in the case of the cylindrical type, in which all the relative dimensions were constant. With 1.9 cm. brass spheres γ was 1.33, with zinc 1.34, and with aluminium 1.28. The average for these three metals was 1.32, approximately equal to the values for steel, showing that the nature of the metal did not affect the damping appreciably.[1] No difference in γ was found for different lengths of oil-gap, up to .135 mm.[1]

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[1] Cf. O. Bartenstein, Ann. d. Phys., 29, 245, 1909. These measurements were, however, made with a tuned receiver.

From the wave-length curves taken with a tuned main receiver, the computed values of γ were approximately equal to 0.50, agreeing with the results of Klemencic and Czermak. As already pointed out this value for γ has no real significance, since it depends on the damping of the receiver as well as on that of the vibrator.

6. Standard Receivers.

Description. — We have thus far considered the properties of what has been called a standard vibrator. We shall now take up the study of the receivers in resonance with given wave-lengths. For this purpose a "standard receiver" was constructed, with form and dimensions arbitrarily chosen, but based on previous experience. As in the standard rod vibrator, the essential *relative* dimensions of the standard receiver were maintained constant, as its size was altered.

The details of construction are given by Fig. 9. The copper wings A were flat strips, fastened with wax to a hard rubber base D. In the "constriction gap" C between their inner ends was the thermo-junction. The contact between the wires was electrically welded; or soldered, if the receiver was subject to jarring. The lead wires W were attached to the wings, as already described in connection with the non-selective receiver. The dimensions of the standard receiver are given by the following relations: cf. Fig. 9.

$\left.\begin{array}{l}\text{Length of dielectric, } s\\ \text{Height of dielectric}\end{array}\right\} = \frac{1}{4}$ length of corresponding standard rod vibrator.

$= 0.104\,\lambda$ (approx. $\frac{1}{10}\,\lambda$) = approx. $l/4$.

Depth of dielectric, $h = 3.0$ cm. for all sizes.

Thickness of wings, $t = 0.015$ cm. for all sizes.

Width of wings, $w = \frac{1}{4}s$.

Length of constriction, $c = \frac{1}{10}s$.

Diameter of thermo-couple wires in constriction: iron, 0.004 cm.; constantan, 0.0025 cm.

As an example, the standard receiver, corresponding to the standard rod vibrator 5 ($\lambda = 12.2$ cm.), had the dimensions $l = 5.3$ cm. (from experiment), $s = 1.27$ cm., $w = 0.32$ cm., and $c = 0.13$ cm., h and t having their usual values. This will be called "standard receiver 5," and this method of designating each receiver by the number connected with the corresponding rod vibrator will be observed in the rest of this paper.

Tuning Method. — The first step in determining the properties of a standard receiver was to find l, the resonance length corresponding to a given wave-length λ. This was first attempted by the usual method of tuning curves,[1] with unsatisfactory results. As mentioned earlier, the process of tuning by determining the length of receiver which takes up the most energy, is subject to numerous errors and accidental disturbances, which render the resulting curves uninterpretable. The question is one of very great importance in work with electric waves and therefore merits a somewhat detailed discussion.

The first great difficulty met with in these measurements was the distortion of the tuning curves by the vibrator and receiver mirrors, which were placed facing one another at a distance of 3 meters, and served to concentrate the energy from the source upon the receiver. This disturbance was especially marked in the earlier stages of the work, when spherical mirrors of focal length 19.0 cm. were used. In the majority of the curves two maxima appeared, one of which depended upon the focal length, and the true resonance maximum was displaced, as shown by subsequent tests. A typical instance of this is curve *a*, Fig. 11, in which the arrow indicates the correct position of the resonance maximum. With this mirror system not even approximate results were obtained for wave-lengths greater than 13 cm., as shown by the tuning curve reproduced in *b*, Fig. 11, taken with a wave-length of 18 cm. It will be seen that the resonance maximum, the position of which is marked by the arrow, is almost entirely covered up by the large maximum produced by the mirrors. Similar results were obtained with two paraboloidal mirrors, of focal length 12.0 cm., using a wave-length of 6.5 cm. The curves with this system show great variation, having sometimes two or three prominent maxima, as in Fig. 11, *c*. It should be remarked here that these variations may be due, to a small extent, to the form of the receiver used, as well as to the mirrors. This point will be subsequently discussed more fully.

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[1] Willard and Woodman, *l. c.*

Smooth curves with single maxima were obtained with two sets of cylindrical-parabolic mirrors, of focal lengths 36 cm. and 50 cm., respectively, for wave-lengths less than 18 cm. However, the results obtained from them were inconsistent, the ratio of the wave-length λ (determined by the non-selective receiver) and the resonance length l varying from 2.14 to 2.38. With $\lambda = 18.7$ cm. two prominent maxima appeared at 7.0 and 8.5 cm. (Curve *I*, Fig. 11, *d*). When the direct radiation and the portion of the reflected beam which passed close to the vibrator were eliminated by an oblique screen directly in front of the vibrator, a single maximum at 8.3 cm. was obtained (Curve *II*), indicating that the effect of the mirrors was to cause interference between the direct and the reflected beams. This would account for the simple displacement of the resonance maxima, as well as for the double crests.

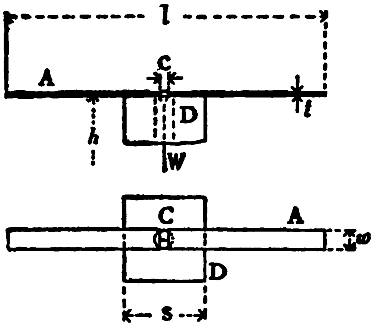

Fig. 9.

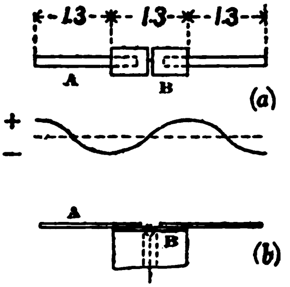

Fig. 10.

To avoid this difficulty a series of tuning curves were taken without mirrors, with the vibrator and receiver 150 cm. apart and the check receiver above the vibrator as before. The results were still inconsistent, but agreed within 5 per cent. with the correct values subsequently found by another method. One possible cause of this variation is seen in the form of the resonance curves (Fig. 11, *e*, *f*.), which are not symmetrical, but very much flattened on the side of the longer receiver lengths. This was due to the greater energy taken up by the longer receivers, because of their greater length and capacity, which is superimposed upon the effect due to resonance. While it cannot be said that this flattening actually displaced the maxima, it can be readily seen that, since the variation of the energy measured by the receiver as its length is varied by several centimeters is only a few per cent., a slight disturbance would be sufficient to displace the maximum 5 or 10 per cent. Experience shows that such disturbances are present in these determinations and cannot be readily controlled. For example, in these measurements the method of gradually altering the length of a receiver, while determining its resonance length, was to start with the flat wings very long and cut small sections from the ends. This gave no opportunity of alternating on the two sides of the resonance length, to eliminate the error due to the change in the ratio between the energies received by the main and check receivers.[1] Now this is progressive with the age of the vibrator, and in the case which we are considering it was a continuous decrease. Hence the maxima of the tuning curves were displaced toward longer receiver lengths, the error being very large because of their flatness.

To avoid this difficulty by taking alternate readings on the two sides of the maximum, adjustable receivers have been used,[2] the length of which could be altered at will. This method was tried as a substitute for the "clipping" method just described, but a new source of error was found, which caused it to be abandoned. In the construction of most forms of replaceable wing receivers there occurs a discontinuity in the shape or cross-section of the metal wings. Now the effect of an abrupt change in the contour of a resonator is to produce at that point a loop of potential, introducing a new mode into the natural vibrations. Now if the total length is such that this superposed period bears a simple relation to the fundamental period, that is, if the loop of potential due to the discontinuity coincides with a loop in the normal vibration, the activity of the receiver will be a maximum, since the vibrations depend only slightly on the highly damped wave exciting them. Experiment showed that this action almost entirely covered up the effect of resonance with the vibrator, the maxima of these curves depending chiefly upon the position of the discontinuity in the resonator. The two adjustable receivers tested in this investigation are shown in Fig. 10, *a*, *b*. The first consisted of two brass blocks *B*, 3 mm. in cross-section, with holes in their ends, into which the variable wings *A* (rods 1 mm. in diameter) were inserted, perfect contact being secured by amalgamation. The total length of the two blocks plus the thermo-junction between them was 1.3 cm. With a vibrator correspondiug to a resonance length of about 2.5 cm., the maximum of the tuning curve taken with this receiver came at 3.9 cm., three times the length of the enlarged part. With Fig. 10, *a*, is given a diagram of the distribution of potential in the vibrating system, showing the loop at the point of change of cross-section. Loops at the free ends and a node at the center, where the current is measured, are of course the normal conditions for maximum energy. In the second arrangement, shown in Fig. 10, *b*, the adjustable wings *A* were flat, and contact with the permanent part of the receiver was made by pressing them against similar strips *B*, fastened to a hard rubber base. The total distance between the extremities of the double thickness

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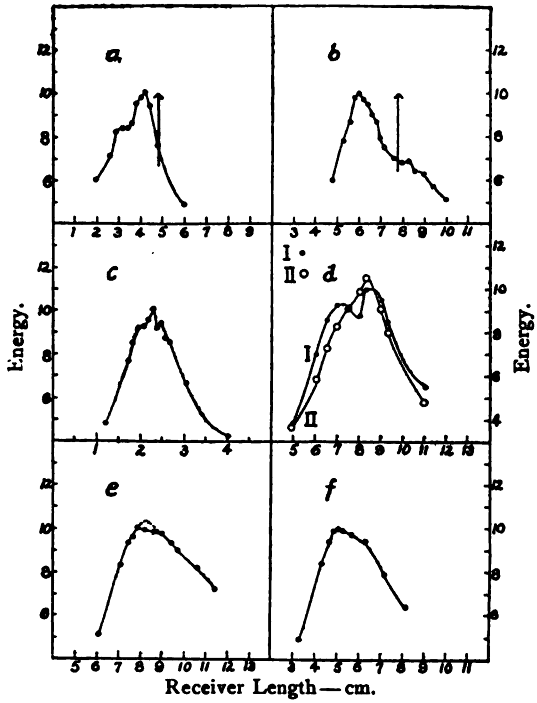
[1] Blake and Fountain, *l. c.*

[2] Cf. A. D. Cole, Phys. Rev., XX., 268, 1905.



Receiver Length — cm.

Fig. 11.

of metal was 1.6 cm. The tuning length found for a wave-length, corresponding to a 5.3 cm. resonance length, was 4.8 cm., again three times the length determined by the discontinuity. Similar results should be expected for any receiver with discontinuities in the dielectric in contact with the metal. These effects are, however, very much smaller, and hence could not be observed, but they were suspected in the case of the very flat resonance curves which have just been described.

TABLE III.

Standard.		λ	Position of Maxima.	Resonance Length.		$\lambda \div l$
Rod Vibrator. 1	Receiver. 2	3	4	By Tuning l 5	By Wave-length Method. 6	7
7	7	18.7	7.8	8.2		
			8.4 and 7.6	8.2		
			7.8	8.0		
				8.1 (mean)	(7.85)	2.32
6	6	15.8	7.2 and 6.6	7.1		
			7.35 " 6.7	(7.3)		
			6.5 " 7.5	(6.95)		
			7.2 " 6.8	7.0		
			7.2 " 6.6	7.0		
				7.0 (mean)	6.9	2.29
5	5	12.2	5.7 and 5.2	5.5		
			5.05	5.1		
				5.3 (mean)	5.3	2.30
					Mean	2.30

Returning to the tuning curves taken without mirrors by the clipping method, it was found that we could obtain the correct resonance lengths, if we used for their determination the maxima of the smooth curves drawn through the points lying on either side and at a little distance from the experimental maxima. (See dotted line in Fig. 11, *e*.) The results found in this way for three wave-lengths are given in Table III., column 5. The first three columns give the standard vibrators and the standard receivers used, with the corresponding wave-lengths. In the fourth column the individual maxima of the tuning curves are recorded, in the order of their importance, if there are more than one. In the sixth column are given the values of the resonance lengths found by the wave-length method, which we are next to consider. The last column gives the ratio of the wave-length λ to the resonance length l, the average for the three receivers being 2.30. The ratio corresponding to the wave-length 18.7 cm. was found from the value of l determined by the tuning method, as in this instance the result by the wave-length method was somewhat doubtful.

"*Tuned Receiver Wave-length*" *Method.* — As we have just seen, the method of tuning could not be used for the investigation of receivers, when great accuracy was required. It was therefore very necessary that a new method be sought, which would give consistent and definite results. This was finally found in the "tuned receiver wave-length" method, in which the special form of receiver under investigation was put in place of the non-selective receiver in the interference apparatus (cf. Fig. 3), and the "tuned receiver wave-length" L determined from the resulting wave-length curve. Without altering the vibrator, the receiver was then varied, and the effects of the change found in terms of L. In this way it was possible to study accurately and rapidly the relations of the receiver to the incident wave-length and the conditions affecting its action.

To apply this method to the determination of the resonance lengths, the tuned receiver wave-length (L) was measured for varying lengths (l) of the resonator, keeping constant the vibrator and all the other dimensions of the corresponding standard receiver. The results are plotted in Fig. 12, *I* and *II*. Curve *I* gives the relation between L and l for receiver 5, with vibrator 5 ($\lambda = 12.2$ cm.). It is a straight line with the equation $L = 1.54l + 4.0$, showing that l is not proportional to L. On the other hand, Curve *II*, taken with receiver 6, and vibrator 6 ($\lambda = 15.8$ cm.), deviates very much from a straight line, largely as a result of mirror disturbances. A curve similar to *II* was obtained for receiver 7, with vibrator 7 ($\lambda = 18.7$ cm.). From each of these curves the resonance length was then found by taking the value of l corresponding to $L = \lambda$, where λ is the true wave-length, found with a non-selective receiver. The results have already been given in column 6, Table III., and agree well with the averages obtained from a number of the modified tuning curves described above. It will be observed that this new definition of resonance length is exactly equivalent to the old, since there it was assumed that the true wave-length was measured by a receiver in resonance with the vibrator.

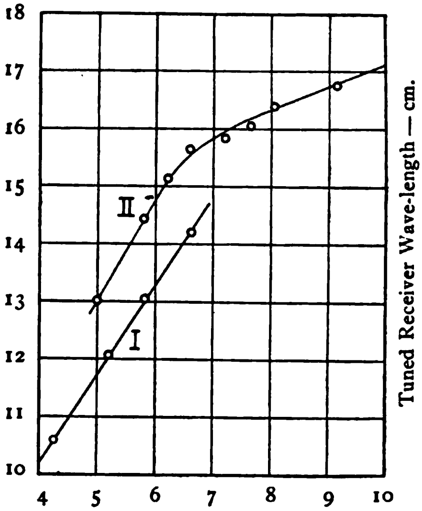


Fig. 12.

These three curves showing the relations between L and l appear to agree with the results of Willard and Woodman,[1] who found L to depend upon both the receiver and the vibrator. On the other hand A. D. Cole,[2] in his study of electric wave receivers found a direct proportionality between L and l and concluded that L was independent of the incident wave-length λ. If we consider the fact that the latter measurements were entirely free from mirror disturbances, it would seem that they were the most reliable, for in the present determinations, and probably in those of Willard and Woodman, the mirrors affected the results. But in none of these determinations are the data complete, or free from errors. We must therefore leave the quantitative determination of the extent to which the vibrator and receiver influence these measurements to a more exact analytical study.

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[1] Willard and Woodman, *l. c.*

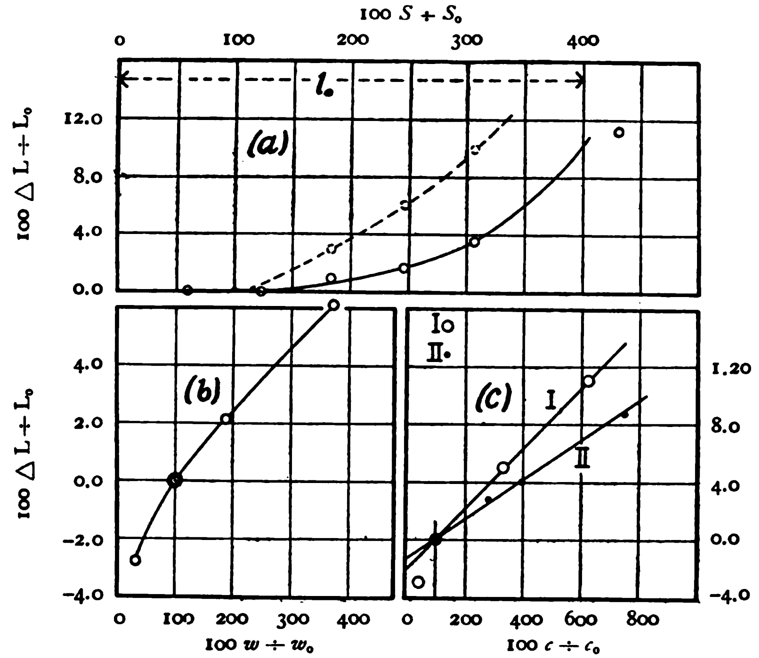
[2] A. D. Cole, *l. c.*, p. 271.

Effect of Varying the Relative Dimensions. — The curves giving the relation between L and l are of value in the further investigation of the properties of receivers. In using the method of tuned receiver wave-length curves the results are given in terms of L, and not of λ. As L depends upon the vibrator, the receiver, and the interference apparatus, it has no quantitative significance. However, from the data just obtained, we can express our results in terms of the equivalent length of a receiver, the other dimensions of which are those of one of the standard receivers. This somewhat extends the application of the method, but quantitatively its scope is still very limited, since L must not differ much from λ, if accurate results are to be obtained.

One of its most important applications was in the study of the effect of varying the relative dimensions of the standard receiver. This was undertaken partly to complete the statistical data which was the original object of this paper, but it was even more useful because of the light thrown on the question of receiver dimensions. The work was divided into four parts, the determination of the effect of —

1. The length of the dielectric along the receiver, s.
2. The width of the metal wings, w.
3. The thickness of the metal wings, t.
4. The length of the constriction, c.

The variation of the length of the dielectric was studied with the standard rod vibrator 6 ($\lambda = 15.8$), with the corresponding receiver 6 ($l = 6.6$, $w = 0.4$, $c = 0.16$ cm., s variable). The hard rubber receiver base was made in 0.5 cm. sections held together with wax, so that its length could be varied from 6.9 to 0.9 cm., without disturbing the wings or thermo-junction. A thin layer of soft wax insured good contact between the metal and the hard rubber. The height of the base (1.6 cm.) was not altered, since this dimension is not of importance. The results are given in Fig. 13, *a*, the abscissas giving in percentages the ratio of the length of the dielectric to the standard length of dielectric, and the ordinates the percentage change in the tuned receiver wave-length. As in the case of the standard rod vibrators (Fig. 6, *b*) no appreciable effect due to the dielectric appears, until its length is more than one fourth of the receiver length. In the dotted curve the corrections given by curve



Standard dimensions: $l_0 = 4\, s_0 = 16\, w_0 = 40\, c_0 \quad L_0 = \lambda$.

Fig. 13.

II, Fig. 12, have been made, and the ordinates now represent the wave-length λ, on the approximate assumption that $\lambda = 2.3l$.

The width of the flat wings also proved to be an important factor. The effect of varying this dimension keeping the other conditions constant is shown in Fig. 13, *b*. Rod vibrator 5 ($\lambda = 12.2$) was used with the corresponding standard receiver 5 ($l = 5.22$, $s = 1.27$, $c = 0.13$ cm., w variable). The curve is not a straight line, the slope increasing as the width is diminished. But it may be represented within small limits of variation by the linear equation, $L = .055w + 94.5$, where L and w are both expressed in percentages of their values for $w = 0.32$, the normal width for this standard receiver. The same test was made with vibrator 6, showing the same variation of L with the width, but qualitatively only, because of the disturbing effect of the mirrors.

The width of the wings was previously investigated by Willard and Woodman,[1] who could detect no difference between flat strips 4.6 mm. and 3 mm. wide. Their failure to observe this was due to the small variation of width, which as the present results show gives only 1.5 per cent. change in L, too small to be detected by the tuning method used.

The effect of varying the thickness (t) of the wings was found to be small. The thicknesses, .06 cm. and .015 cm., were tried, the change being too small to necessitate intermediate determinations. With $\lambda = 12.2$ cm. and $l = 5.24$ cm., the corresponding values of L were 12.42 and 12.18, a variation of but two per cent. for three hundred per cent. variation in thickness. This may be represented by the equation $150 \Delta L = \Delta t$, L and t being expressed in percentages in the usual way.

Similarly the variation in the length (c) of the constriction formed by the thermojunction was tested ($\lambda = 12.2$, $s = 1.27$, $w = 0.32$, $l = 5.23$ cm., c variable). The results are plotted in Curve *I*, Fig. 13, *c*. The curve gives the relation between c and L, expressed in percentage of the normal length of constriction and percentage change, respectively. This is represented by the linear equation, $L = .025c + 96$, or $\Delta L = .025 \Delta c$. A similar linear relation was also obtained with rod vibrator 6 and the corresponding receiver, but the slope of the line was very much less than in the first case, as shown in Curve *II*. This difference may be due largely to error in determining the latter curve, since mirror disturbances were again operative.

The linear relation between ΔL and Δc can be explained by regarding the change in L as due to the increase of self-induction, which is approximately proportional to the change in length of the constriction. For if A is the self-induction of the standard form of receiver corresponding to the tuned receiver wave-length $L = \lambda$, x the increase of self-induction due to a change in c, and ΔL the resulting change in L, we have

$$\Delta L = K\sqrt{A + x} - K\sqrt{A} = \frac{K}{\sqrt{A}}\left(\frac{x}{2} - \frac{x^2}{4A} + \cdots\right),$$

where K is a constant depending upon the units and the capacity, which remains constant since the changes are all made near the center of the receiver. Now for values of c not greater than $l \div 5$, the error introduced by neglecting the term containing x^2 and subsequent terms is within the limits of the experimental error, so that we have practically a linear relation, $\Delta L = Kx \div 2\sqrt{A} = \text{const.} \times c$.

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[1] Willard and Woodman, *l. c.*, p. 7.

Summing up the results obtained with the standard receivers, we have,

$$L = \lambda = 2.30l,$$

for changes in

length of dielectric, $\Delta L = 0$, when $s < l \div 4$,
width of wings, $\Delta L = .055\Delta w$,
thickness of wings, $\Delta L = .007\Delta t$,
length of constriction, $\Delta L = .025\Delta c$,

the variations Δ being expressed in percentages of the corresponding dimensions of the standard receiver. In applying these data to receivers of other than the standard form, it must be remembered that the corrections which we have determined are only applicable in a very limited range. They should be regarded as a means of finding the safe limits of variation from the standard form, rather than as quantitative relations from which the properties of any receiver could be computed. This limitation is due partly to the fact that it is impossible to alter one condition without affecting the others to some extent. For example, the effect of the width of the resonator is closely related to the constriction of the stream lines at the center,[1] and on the other hand the lengthening of the constriction, by removing the point of convergence of the stream lines farther from the center, must affect the influence of the width. We cannot therefore make any great departure from our standard conditions and still use the data determined in this work.

Ratio of Length to Wave-length. — By both the tuning and wave-length method the value found for the ratio $\lambda \div l$ at the resonance length was 2.30, which differs from that found by other observers. The theoretical values deduced by Poincaré[2] and by Abraham[3] were approximately equal to 2.00. Macdonald's[4] theory gave 2.53, and the work of a number of subsequent experimenters appeared to confirm this value. Willard and Woodman[4] found that for three of the four cases tested this ratio was approximately correct. A. D. Cole[1] obtained as an average of five determinations the value 2.52, and Blake and Fountain[4] the value 2.47. In all of these determinations the wave-length was measured by a tuned receiver, so that they were subject to the errors of tuning already considered. However, even if we assume that the results were free from such objection, they have no real bearing on the theoretical conclusions, since, as has been shown, the relative dimensions of the receivers used affected the resulting ratios, which should have a different value for every different form of receiver. The theory can only be verified with a receiver approximating the conditions assumed, that is, with a non-constricted receiver with a diameter or cross-section very small in comparison with the length.

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[1] Blake and Fountain, *l. c.*, p. 277.

[2] Poincaré, "Les Oscillations Electriques."

[3] M. Abraham, Wied. Ann., 66, 435, 1898.

[4] Macdonald, "Electric Waves," p. 111.

The value of the ratio $\lambda \div l$ for such a receiver might be obtained by extrapolation on the curve (Fig. 13, *b*) giving the relation between L and w, but the indeterminate slope between 0 and 0.5 mm. makes this difficult. An infinitely thin receiver was, however, approximated by using wings .025 to .037 mm. in diameter, with no constricted portion. This was done by extending the iron and the constantan thermo-couple wires the full length of the receiver, a fine thread attached to the ends holding them in position. With $l = 5.22$ cm., $s = 1.27$ cm., and $\lambda = 12.2$ cm., the tuned receiver wave-length L was 11.45 cm., giving the ratio $L \div l = 2.20$. To avoid the error due to the difference between λ and L and to eliminate still further the effect of the dielectric, this measurement was repeated with $l = 5.50$ cm., $s = 0.47$ cm., and $\lambda = 12.2$ cm. L was then equal to 12.18 cm., giving for the limiting ratio of wave-length to receiver length, $L \div l = \lambda \div l = 2.22$. This agrees well with the limiting value, 2.24, already found for the ratio of the wave-length to the length of the rod vibrator, as the diameter approached zero. These results fail to confirm either of the theoretical values.[2]

For free linear resonators, consisting of strips of tin-foil 0.2 cm.

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[1] *l. c.*

[2] Cf. O. Bartenstein, *l. c.*, p. 242.

wide and 4.4 cm. long, Blake and Fountain[1] found the ratio $\lambda \div l = 2.25$. This is in good agreement with the present results, even though the width may make a slight correction necessary, which, however, should be very small since the resonators of these observers had no constriction.

7. SUMMARY.

Wave-length determinations for short electric waves should be made with a non-selective receiver. Measurements depending upon tuned receivers are liable to errors arising from the process of tuning, which in general is very unreliable.

A non-selective receiver may be obtained by using a Klemencic receiver of ten or more times the resonance length.

If the relative dimensions of a vibrator or of a receiver are maintained constant, the corresponding wave-length is directly proportional to the dimensions. Hence standard forms of vibrators and receivers may be selected, for which we may easily compute the dimensions for any desired wave-length, if we know from experiment the wave-length corresponding to any one size. Such a vibrator and a receiver were investigated, and their properties determined:

$$\text{Vibrator};\ l = 8d = \tfrac{16}{8} s = \tfrac{8}{8} w;\ \lambda = 2.4l.$$
$$\text{Receiver};\ s = l \div 4 = 4w = 10c, \text{ etc.};\ \lambda = 2.3l.$$

The wave-length emitted by a cylindrical vibrator of given length depends upon the extent and properties of the dielectric at the center, and the ratio of the diameter to the length, but not upon the nature of the metal, nor upon the length of the air-gaps. The length of the oil-gap, if small, was found to have little influence on the emitted wave-length.

The wave-length corresponding to a Klemencic receiver of given length depends upon the extent and nature of the dielectric at the center, the width and thickness of the antennæ, or wings, and the length of the constriction at the center.

A linear relation was obtained between the wave-length and the dimensions of a modified Righi vibrator.

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[1] Blake and Fountain, *l. c.*

The limiting value of the ratio of the length of a rod vibrator or resonator to the corresponding wave-length, as its diameter approaches zero, was found to be 2.23.

The disturbing action of the "collecting" mirrors is very marked in wave-length and tuning curves, affecting both the form and position of the maxima and minima. This depends both upon the focal length and the type of the mirror. By the proper choice of mirrors the resulting error may be avoided in wave-length measurements, but it is a constant source of difficulty in tuning. Tuning without mirrors was tried, but in general the process was found unreliable, and interference curves taken with the receivers under investigation were substituted.

The damping for both types of vibrator studied was approximately 1.3.

In closing, we wish to thank Messrs. H. W. Farwell, J. L. Waldron, and C. R. Webb for their assistance in making measurements, and especially to express our indebtedness to Professor E. F. Nichols, who suggested the problem and gave many valuable suggestions for its solution.

Phœnix Physical Laboratories,
Columbia University,
December, 1908

THE BALLISTIC ELECTRODYNAMOMETER AS AN INSTRUMENT FOR TESTING IRON.

By Martin E. Rice and Burton McCollum.

THE great commercial importance of the magnetic properties of iron and steel, together with their frequent and often wide variation even in samples that have apparently had similar histories, makes imperative the frequent and accurate determination of these properties wherever electrical apparatus is to be built. The necessity for making these determinations has led to the development of a number of methods for measuring the hysteresis effect in iron. These methods may be grouped into four general classes, namely: magnetometer, ballistic galvanometer, wattmeter, and special hysteresis meter methods. The wattmeter and the hysteresis meter methods are the more rapid ones, but they are subject to numerous sources of error that seriously limit their usefulness. In the wattmeter method, an eddy current loss that is very appreciable in amount and hard to determine even approximately is measured along with the hysteresis loss; and the results are rendered still more uncertain by uncertainties in the frequency and wave shape of the electromotive force applied to the magnetizing coil of the specimen under test. In the special hysteresis meter methods, the value of the maximum magnetic flux density and the distribution of the magnetic flux in the specimen under test are more or less uncertain. Moreover, in both of these methods, another set of apparatus and a separate experiment are required for determining the permeability of the specimen. The magnetometer and the ballistic galvanometer methods, if properly carried out, are far more accurate than the first two, but they are extremely tedious as they require the taking of a long series of readings for each test, the plotting to scale of an hysteresis loop, and the measuring of the area of this curve by a planimeter before the final result can be calculated.

The defects inherent in the foregoing methods have led to the development of a new method free from the defects mentioned above, the principal instrument used being an electrodynamometer specially designed for ballistic work and having a rather long period of swing. In developing this method, the objects sought and attained have been to devise a method that is at once simple and rapid in operation and at the same time accurate to a high degree. The method herein described will be found to be fully as rapid in operation as any wattmeter method, while in accuracy it is on a par with the ballistic galvanometer method when carried out in its best form.[1] The ballistic dynamometer may be used for determining both the hysteresis loss per cycle and the permeability of a sample of iron, thereby requiring but one set of apparatus for both purposes. Or, the regular ballistic galvanometer method may be used for determining the permeability curve.

The general arrangement of the apparatus and the method of procedure will be given first. Next will be given the theory of the method and a numerical illustration, followed by comparative results showing the degree of accuracy reasonably attainable in practice.

The sample of iron to be tested should be laminated, laminations of ordinary commercial thickness being entirely satisfactory. The laminæ should be separated by sheets of paper to insure that any slight eddy current loss will not be unduly increased by conduction from one sheet of iron to the next. The laminæ may be cut into the form of closed rings and a number of them assembled and wound by hand, the magnetic circuit in this case being a closed one; or they may be cut into the form of half rings and a number of them inserted into ready wound magnetizing and test coils, the magnetizing circuit in this case having short air gaps in it. The latter arrangement, while not satisfactory for determining the true permeability curve, is, however, commercially as good for determining the hysteresis loss per cycle and the corresponding value of the maximum magnetic flux density. It has, in addition, the advantage of enabling many different samples of iron to be tested in a short time when the apparatus has once been set up.

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[1] "Limitations of the Ballistic Method for Magnetic Induction," by A. Hoyt Taylor PHYS. REV., Vol. XXIII., p. 95.

To measure the hysteresis loss per cycle, the circuits are arranged as shown in Fig. 1. The iron ring *Fe* is magnetized by any desired current which can be adjusted by the battery *B* or by the rheostat *Rh* or by both. This current flows in series through the ammeter *A*, the rheostat *Rh*, the reversing switch *S*, the magnetizing coil C_1, the choke coil *D*, and the fixed or field coils *Fc* of the electrodynamometer. The secondary or test coil C_2 is connected in series with a resistance *Re* and the movable coil *Mc* of the electrodynamometer. Reversing the switch *S* will cause a ballistic deflection in the dynamometer which deflection may be read by a telescope and scale in the usual way. It will be shown later that *W* the energy loss per cycle in the iron, corresponding to a magnetizing current *I*, will be proportional to this ballistic deflection of the dynamometer, or

$$W = K_1\theta, \tag{1}$$

where θ is the observed deflection of the dynamometer and K_1 is a reduction factor depending upon the windings of the iron ring and upon the dynamometer. The value of K_1 is to be determined by discharging a condenser or a mutual inductance through the dynamometer as described later under calibration. Thus when the reduction factor K_1 has once been determined, any number of measurements of hysteresis loss per cycle may be taken for various values of magnetizing current, the loss in each case being calculated simply by multiplying the deflection θ by the reduction factor K_1.

For determining the maximum magnetic flux density by the dynamometer method, the circuits are arranged as shown in Fig. 2. This description is given for the sake of completeness, on the assumption that a ballistic galvanometer is not available or that it is desired to make the entire test with one instrument, as might often be the case in practical work. Maintain in the field coils *Fc* of the dynamometer a separate steady current I'', adjusted to a suitable value by the rheostat *Rh'*. Reversing the magnetizing current by means of the switch *S* gives a deflection in the dynamometer as in the case of an ordinary ballistic galvanometer. The value *B* of the maximum magnetic flux density, corresponding to any magnetizing current *I*, is proportional to this deflection, or

$$B = K_2\theta'', \tag{2}$$

where θ'' is the observed deflection and K_2 is again a reduction factor depending on the dimensions and windings of the iron ring, on the dynamometer and on the current I'' maintained in the field coils of the dynamometer. It is to be determined, as explained later, by the same calibration as for K_1 above.

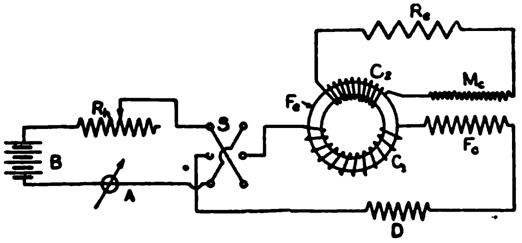

Fig. 1.

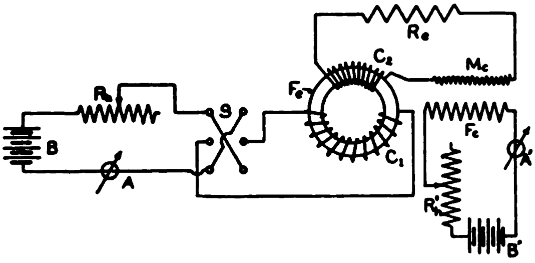

Fig. 2.

For calibrating the dynamometer, the circuits are arranged as shown in Fig. 2, except that the movable coil of the dynamometer is connected to a standard condenser instead of to the test coil C_2. Maintain a steady current I' in the field coils *Fc* of the dynamometer and discharge the condenser through the movable coil of the dynamometer and note the deflection θ'. If C is the capacity in farads of the condenser used and V is the potential difference in volts to which it was charged, the complete expressions for the iron loss per cycle and the maximum magnetic flux density are, respectively,

$$W = \frac{2n_1 r_2 C V I'}{n_2 \theta'} \theta = K_1 \theta, \tag{3}$$

$$B = \frac{10^8 r_2'' C V I'}{2 a n_2'' \theta' I''} \theta'' = K_2 \theta'', \tag{4}$$

where W is given in joules per cycle and B in lines of force per square centimeter, K_1 and K_2 are the two reduction factors mentioned in (1) and (2) respectively, and θ and θ'' are the observed deflections in scale divisions. The currents I' and I'' are measured in amperes and θ' in scale divisions. The other letters not already defined have the meanings: n_1 is the number of turns in the magnetizing coil C_1; n_2 and n_2'' are the numbers of turns in the test coil C_2 for the arrangements of Fig. 1 and Fig. 2, respectively; r_2 and r_2'' are the resistances in the test coil circuit for the arrangements of Fig. 1 and Fig. 2, respectively; and a is the cross sectional area of the iron ring in cm^2.

In this way, corresponding values of the hysteresis loss per cycle and the maximum magnetic flux density may be quickly and accurately obtained after a single calibration of the dynamometer, using as many different values of magnetizing current I as desired.

As will be explained later, the dynamometer deflection measures in reality the total energy loss in the iron ring and in the secondary or test coil circuit. Since the eddy current losses per cycle in the iron and in the test coil circuit vary inversely as the time required for the cycle, while the hysteresis loss per cycle is independent of the time required for a cycle, these eddy current losses which are small in any case can be reduced to a relatively very small and commercially negligible amount, less than one per cent. of the hysteresis loss, by simply increasing the time required for the magnetism in the iron ring to reverse. To accomplish this increase of time is the purpose of the choke coil D, which by its choking action causes the magnetizing current in C_1 to reverse very slowly when the switch S is thrown over. The loss in the test coil circuit is entirely negligible if even a moderately high value of r_2 is used. If a very high degree of accuracy is desired, the slight eddy current loss in joules per cycle in the iron may be calculated approximately from the equation[1]

$$W_e = 1.645 t^2 f B^2 v 10^{-11}, \tag{5}$$

where t is the thickness of a lamina in cm., v is the volume of the iron in cm^3., B is the maximum magnetic flux density in lines per cm^2., and f is the reciprocal of the period of the cycle. The half period of the cycle may be determined by noting the time required for the pointer of the ammeter A to fall and rise again when the magnetizing current is being reversed. Subtracting this value of eddy current loss from the total iron loss as given by equation (3) gives the true hysteresis loss W_h in joules per cycle,

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[1] Steinmetz, Alternating Current Phenomena, p. 199.

$$W_h = W - W_e. \tag{6}$$

The mathematical proof of equation (3) is as follows:

In addition to the letters already defined above, let

φ = the total instantaneous magnetic flux in the iron ring.

e_1 = instantaneous counter E.M.F. in the primary or magnetizing coil.

e_2 = instantaneous generated E.M.F. in the secondary or test coil.

i_1 = instantaneous current in the magnetizing coil.

i_2 = instantaneous current in the test coil.

K = moment of inertia of movable coil of the dynamometer.

g = a constant of the dynamometer depending on its windings only.

b = the constant of torsion of the suspending fiber of the movable coil of the dynamometer.

θ = instantaneous deflection of the moving coil of the dynamometer.

ω = instantaneous angular velocity of the moving coil.

T = instantaneous torque acting on the moving coil due to the currents i_1 and i_2.

P = instantaneous power expended in the iron and the test coil.

$W/2$ = total energy expended per half cycle in the iron and in the test coil circuit.

We have then in the primary circuit,

$$P = e_1 i_1 = i_1 n_1 d\varphi/dt, \quad \text{whence } i_1 = \frac{P}{n_1 d\varphi/dt} \tag{7}$$

and in the secondary circuit,

$$i_2 = e_2/r_2 = \frac{n_2 d\varphi/dt}{r_2}. \tag{8}$$

In equation (8) it is not necessary to consider the circuit as inductive, since its self-inductance is very small compared with its resistance and the rate of change of current di_2/dt is also very small because of the long time used in reversing the magnetism in the iron, and hence $L_2 di_2/dt$ is negligible compared with e_2. The torque acting on the movable coil of the dynamometer when the switch S is thrown over is proportional to the primary current i_1 and to the induced secondary current i_2, or

$$T = g i_1 i_2. \tag{9}$$

Substituting the values of i_1 and i_2 from equations (7) and (8), we have

$$T = \frac{gPn_2 d\varphi/dt}{r_2 n_1 d\varphi dt} = \frac{gPn_2}{r_2 n_1}. \tag{10}$$

Multiplying both sides of this equation by dt, we get

$$Tdt = \frac{gn_2}{r_2 n_1} \times Pdt. \tag{11}$$

The angular acceleration $d\omega/dt$ imparted by the torque T to the moving coil of the dynamometer, on the usual assumption that the entire discharge takes place before the coil has moved appreciably, is given by the equation

$$d\omega/dt = T/K, \text{ whence } Tdt = Kd\omega. \tag{12}$$

Substituting this value of Tdt in equation (11) and integrating between limits, we get,

$$\int_{\omega=0}^{\omega=\omega\max} Kd\omega = \frac{gn_2}{r_2 n_1}\int_{t=0}^{t=\infty} Pdt. \tag{13}$$

The first integral in this equation represents the total angular momentum of the moving coil of the dynamometer at the instant when the secondary discharge has been completed, and the second integral in the equation represents the total energy $W/2$ expended in the iron ring and the secondary coil during a half cycle. Substituting these values of the integrals in the above equation, and solving for $\omega_{\max}$, we obtain,

$$\omega_{\max} = \frac{gn_2 W}{2Kr_2 n_1}. \tag{14}$$

The total kinetic energy of the moving system at any instant is

$$w_1 = \tfrac{1}{2}K\omega^2,$$

and the rate of change of this kinetic energy is,

$$dw_1/dt = K\omega d\omega/dt. \tag{15}$$

The total potential energy stored in the suspension fiber at any instant is

$$w_2 = \tfrac{1}{2}b\theta^2,$$

and the rate of change of this potential energy is

$$dw_2/dt = b\theta d\theta/dt. \tag{16}$$

If the instrument is undamped, the rate of change of potential energy is equal to the rate of change of kinetic energy and opposite in sign, and equations (15) and (16) give,

$$K\omega d\omega/dt = -b\theta d\theta/dt. \tag{17}$$

Multiplying by dt and integrating between limits gives

$$K\int_{\omega=\omega_{\max}}^{\omega=0} \omega d\omega = -b\int_{\theta=0}^{\theta=\theta} \theta d\theta \tag{18}$$

which reduces to

$$\tfrac{1}{2}K\omega^2_{\max} = \tfrac{1}{2}b\theta^2. \tag{19}$$

If the usual correction for damping be made when reading the maximum deflection θ, this integrated equation is true for damped instruments as well as for undamped ones. Solving this equation for the value of $\omega_{\max}$ gives

$$\omega_{\max} = \theta\sqrt{b/K}. \tag{20}$$

Eliminating $\omega_{\max}$ from equations (14) and (20) and solving for W gives

$$W = \frac{2\sqrt{bK}r_2 n_1}{gn_2}\theta. \tag{21}$$

In this equation between W and θ the unknown factor is $\sqrt{bK}/g$. To determine this factor, the dynamometer may be calibrated in either of two ways, as follows:

First method of calibration, using a condenser. With a steady current I' maintained in the field coils of the dynamometer, arranged as in Fig. 2, a condenser is discharged through the moving coil of the instrument. As before, the instantaneous torque acting on the moving coil of the dynamometer during discharge is given by

$$T = gI'i_2 = -gI'dq/dt, \tag{22}$$

where q is the quantity of electricity in the condenser at any instant. Multiplying by dt and replacing Tdt by its value from equation (12), and integrating, we get

$$K\int_{\omega=0}^{\omega=\omega_{\max}} d\omega = -gI'\int_{q=Q}^{q=0} \tag{23}$$

Puttting in the value for the limits and solving for $\omega_{\max}$ gives

$$\omega_{\max} = gI'Q/K = gI'CV/K. \tag{24}$$

By reasoning exactly the same as in deriving equation (20), we get

$$\omega_{\max} = \theta'\sqrt{b/K}, \tag{25}$$

where θ' is the ballistic deflection of the dynamometer in calibration. Eliminating $\omega_{\max}$ from equations (24) and (25) and solving for the unknown factor that was to be determined, we get

$$\frac{\sqrt{bK}}{g} = \frac{CVI'}{\theta'}. \tag{26}$$

Substituting the value of this factor in equation (21) gives the final working equation for W,

$$W = \frac{2n_1 r_2 CVI'}{n_2\theta'}\theta = K_1\theta. \tag{3}$$

Second method of calibration, using a mutual inductance. With a steady current I' maintained in the field coils of the dynamometer, arranged as in Fig. 2, the secondary of a mutual inductance is discharged through the moving coil of the instrument. If I_m is the current which flows in the primary coil of the mutual inductance at the instant before the primary circuit is opened, then the quantity of electricity discharged through the moving coil of the dynamometer when the primary circuit is suddenly opened is

$$Q = \frac{MI_m}{r_3}, \tag{27}$$

where M is the coefficient of mutual induction of the mutual inductance and r_3 is the total resistance in the circuit of the moving coil of the dynamometer during calibration. Substituting this value of Q for CV in equation (26) gives

$$\frac{\sqrt{bK}}{g} = \frac{MI_m I'}{r_3\theta'}, \tag{28}$$

and the corresponding working equation for W is

$$W = \frac{2n_1 r_2 M I_m I'}{n_2 \theta' r_3}\,\theta = K_1\theta. \tag{3}$$

This method of calibration requires the measurement of an additional resistance r_3 and hence the condenser method of calibration is slightly more rapid. When the value of K_1 has been determined by either of the above methods, a large number of hysteresis measurements may be made in rapid succession with varying values of magnetic flux density in the iron under test by merely throwing the switch S and noting the ballistic deflection of the dynamometer.

In commercial work it may frequently be desirable to use the dynamometer also for measuring the magnetic flux density in the iron ring. The necessary working equation (4) is proved as follows: With a steady current I'' maintained in the field coils of the dynamometer, arranged as in Fig. 2, and the switch S being thrown over, the torque tending to deflect the moving coil of the dynamometer is given by

$$T = gI''i_2 = gI''e_2/r_2 = \frac{gI''n_2}{r_2}\frac{d\varphi}{dt}. \tag{29}$$

Multiplying by dt, replacing Tdt by its value $Kd\omega$ as in equation (12), and integrating, we get

$$K\int_{\omega=0}^{\omega=\omega_{\max}} d\omega = \frac{gI''n_2}{r_2}\int_{-\phi_{\max}}^{+\phi_{\max}} d\varphi, \tag{30}$$

whence

$$\omega_{\max} = \frac{2gI''n_2}{Kr_2}\varphi_{\max}. \tag{31}$$

The equation corresponding to (20) is in this case

$$\omega_{\max} = \theta''\sqrt{b/K}. \tag{32}$$

Eliminating $\omega_{\max}$ from equations (31) and (32) and solving for $\varphi_{\max}$, we obtain

$$\varphi_{\max} = \frac{r_2''\sqrt{bK}}{2I''n_2 g}\theta''. \tag{33}$$

Substituting the value of the factor $\dfrac{\sqrt{bK}}{g}$ from equation (26), replacing $\varphi_{\max}$ by its value $aB_{\max}$, and solving for B, we obtain

$$B = \frac{10^8 r_2'' CVI'}{2an_2''I''\theta'}\theta'' = K_2\theta'', \tag{4}$$

where the factor 10^8 is introduced to bring the units to lines of force per cm^2.

It is assumed that in all cases the observed dynamometer deflections have been corrected for damping in the usual way.

A complete numerical illustration of the method follows, the data being taken from a test made by the writers on a sample of transformer iron. The iron, of 15 mils thickness, was cut into closed rings of inside diameter 7 inches and outside diameter 9 inches and 56 pieces were piled up, separated by paper and wound with the necessary coils. The exact data of the iron were: mean length of magnetic path $l = 63.71$ cm., mean cross section $a = 5.042$ cm^2., volume of iron $v = 321.2$ cm^3., mean thickness of one lamina $t = .036$ cm. The data for calibration of the dynamometer were

$V = 4.077$ volts, $C = 2.0135$ microfarads,

$I' = .892$ amperes, $\theta' = 5.905$ scale divisions,

which gives for the calibration factor of the dymamometer,

$$\frac{CVI'}{\theta'} = \frac{2.0135 \times 10^{-6} \times 4.077 \times .892}{5.905} = 1.240 \times 10^{-6}.$$

The data for determining the iron loss W were

$n_1 = 99$ turns,

$n_2 = 100$ turns,

$r_2 = 1122$ ohms,

$\theta = 20.93$ scale divisions for magnetizing current of $I = .947$ amperes.

Substitution of these data in equation (3) gives the total iron loss per cycle in joules for a magnetizing current of $I = .947$ amperes,

$$W = \frac{2 \times 99 \times 1122 \times 1.240 \times 10^{-6}}{100} \times 20.93 = .002755 \times 20.93 = .05766$$

where the reduction factor .002755 can be used for all other values of θ observed, thus giving a series of results with very little calculation. The data for determining the magnetic flux density B were

$r_2'' = 1122$ ohms,

$n_2'' = 20$ turns,

$I'' = 1.468$ amperes,

$\theta'' = 16.06$ scale divisions for magnetizing current of $I = .947$ amperes.

Substitution of these data in equation (4) gives the maximum magnetic flux density in lines per square centimeter for a magnetizing current of $I = .947$ amperes,

$$B = \frac{10^8 \times 1122 \times 1.240 \times 10^{-6}}{2 \times 5.042 \times 20 \times 1.468} \times 16.06 = 470. \times 16.06 = 7548,$$

where the reduction factor 470 can be used for all other values of θ'' observed and thus a series of results obtained if desired. The slight eddy current loss can be calculated from equation (5), the period of a half cycle having been observed to be approximately one second, making the value of $f = .5$. Substitution gives the eddy current loss per cycle in joules,

$$W_e = 1.645 \times (.036)^2 \times .5 \times (7548)^2 \times 321.2 \times 10^{-11} = .00019.$$

Deducting this value from the value of the total iron loss found above gives the true hysteresis loss of the iron sample

$$W_h = W - W_e = .05766 - .00019 = .05747 \text{ joules per cycle}$$

for a maximum magnetic flux density of $B = 7548$ lines of force per square centimeter.

This same sample of iron was also tested by the ballistic galvanometer method, using the same value of magnetizing current. The results were

$$W = .05732 \quad \text{and} \quad B = 7552,$$

showing a discrepancy of about one-fourth of one per cent. in the values of the hysteresis loss by the two entirely different methods. As a further comparison of results obtained by the two methods, the curve in Fig. 3 is given. A sample of ordinary sheet iron of

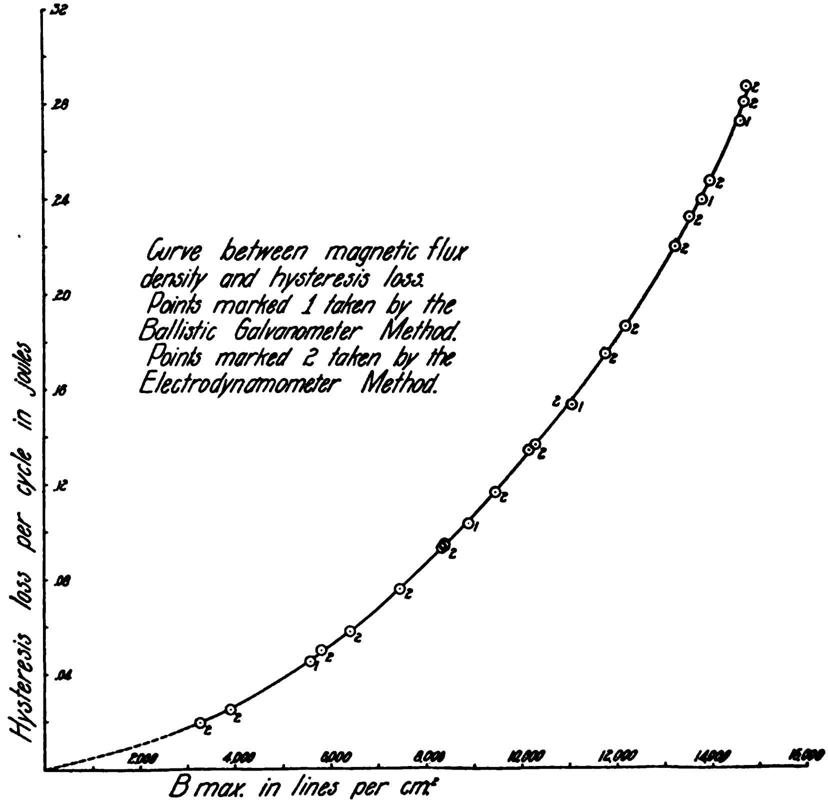

Fig. 3.

25 mils thickness was made up as was the preceding specimen, and its B–H curve carefully obtained by the ballistic galvanometer. Its hysteresis losses for various values of B were obtained by the ballistic galvanometer method and points plotted between hysteresis loss and B, as shown in the figure. Its hysteresis losses for various values of B were also obtained by the ballistic dynamometer method and points plotted between hysteresis loss and B, as shown in the

same figure. A smooth curve was then drawn among all points. An inspection of the curve and the individual points shows that practically none of the points are off the curve by as much as one per cent. in the value of hysteresis loss and most of them are considerably closer than one per cent. In the case of the ordinary sheet iron tested, the eddy current losses were ⅔ per cent., or less, of the total iron loss. Hence it is evident that for all ordinary commercial work with laminated iron, sufficient accuracy is obtained without correcting for eddy current losses.

The electrodynamometer used for this work should be a fairly sensitive instrument in order that the loss due to the current in the movable coil may be negligibly small. In the dynamometer used for these experiments the fixed coil system was made up of two coils of No. 12 copper wire each coil containing 200 turns. These coils were of about 12 centimeters mean diameter and were placed about 10 centimeters apart. The movable coil was a coil taken from an ordinary high sensibility D'Arsonval galvanometer of 1,200 ohms resistance. It was suspended between the two fixed coils by the ordinary galvanometer suspension with a fine spiral spring below. The complete period of swing of moving coil was about 40 seconds and it was found that the ballistic deflection was not appreciably diminished by lengthening the time of discharge to one and a half seconds. The switch *S* is an ordinary six point mercury pole changer and should be so made as to close the circuit on one side before opening it on the other, so that the circuit of the dynamometer and magnetizing coil is not opened during the taking of a reading. This insures that the dynamometer integrates the loss throughout the entire half cycle. The choke coil *D* is simply one coil of a transformer. In the experiments in question it was the 200 volt coil of a 2-K. W. transformer. When this coil is used in series with a magnetizing circuit of 5 or 6 ohms resistance, the eddy current losses and secondary copper losses are reduced to a fraction of one per cent. of the total losses.

Before concluding this paper, it may be helpful to others using the ballistic electro-dynamometer method for the first time, to emphasize some of the precautions that should be taken in any such work. In all work of this character, four readings should be taken for each point: two with fixed test coil connections, the primary or magnetizing current being reversed once in each direction; two with reversed and fixed test coil connections, the primary current being again reversed once in each direction as before. This procedure eliminates all effects of stray magnetic fields on the iron under test and on the movable coil of the dynamometer. It should be followed in both the ballistic dynamometer method and the ballistic galvanometer method, and for the same reasons. It is preferable to set the dynamometer in such a position that a current in its movable coil will not cause a deflection due to the magnetic field of the earth. All ballistic deflections, taken by either method, should be corrected for damping in the usual way.

If a series of readings is to be taken requiring a very wide variation of current in the magnetizing circuit, it is best to vary this current partly by changes in the voltage supplied rather than to depend entirely on changes in resistance, in order that the time constant of the magnetizing circuit can be kept large for all values of current used. This keeps the period of the hysteresis cycle sufficiently long to insure a small value of eddy current loss.

The movable coil of the dynamometer should be very carefully shielded or the heat radiated from the fixed coils even when only moderately warm may set up convection currents that will affect the stability of the instrument. Moreover even moderate variations of temperature cause corresponding variations in the resistance of the movable coil and in the tension of the spiral, thus changing the calibration of the instrument and shifting the zero. A double walled casing will generally give satisfactory results. All masses of metal such as brass tubes or cases should be rigidly avoided, in the construction of such an instrument, in order to avoid entirely any disturbing effects from eddy currents.

UNIVERSITY OF KANSAS,
February, 1909.

THE TEMPERATURE COEFFICIENTS OF GAS VISCOSITY.

By Willard J. Fisher.

IV. An Apparent Relation between Viscosity and Specific Heat.

IN a previous paper, Part I. of this series,[1] the writer plotted a curve-sheet, on which were graphically exhibited the relations of viscosity and temperature of a number of compound gases, Sutherland's equation being taken as a basis. At the time no approach to consistency or mutual relations was observed, but later it was found that the abscissa intercepts of these straight lines increased in an order which was the reverse of the order of the specific-heats ratio $\gamma = C_p/C_v$. This relation also applies to the elements, as exhibited in Fig. 1 of the same article, hydrogen and argon forming exceptions. No attempt is here made to explain these exceptions, though it is certain that hydrogen is a very difficult gas to work with, in many other relations beside viscosity. There is a smaller discrepancy between carbon dioxide and nitrous oxide; in regard to this it should be remarked that the experimental data on the former's viscosity relations are far from consistent, as a reference to Part I. will show. Table I. shows the relations of γ and the abscissa-intercept C/K.

It appears that we have here an indication of one of two things: either a new law of gases, connecting the viscosity with the complexity of the molecule, or else a result of approximation made in order to simplify the mathematical treatment of the gas-flow in the capillary tube. (The tube method is the easiest way to subject the gas to high temperatures under controllable conditions, and all the viscosity data of the table were obtained by it.) It is not easy to see how the complexity of the molecule can of itself, disregarding mass and volume, affect the transfer of momentum which, from the kinetic theory standpoint, constitutes internal friction; while the theory of the capillary tube is admitted to be incomplete.

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[1] Phys. Rev., 24, p. 400, 1907.

TABLE I.

Relation of Specific Heats and Sutherland Constants.

Substance.	C_p/C_v	Authority.	$C/K \cdot 10^{-6}$	Authority.
He	1.63	Behn and Geiger.[1]	5.42	H. Schultze.
Ar	1.6674	Niemeyer.	8.23	H. Schultze.
H_2	1.41	Cazin.	10.94	P. Breitenbach.
	1.384	Maneuvrier and Fournier.	12.50	H. Markowski.
	1.408	Lummer and Pringsheim.		
N_2	1.41	Cazin.	7.89	H. Markowski.
CO	1.4032	Wüllner.	7.55	v. Obermayer.
O_2	1.4025	Müller.	7.865	H. Markowski.
	1.3977	Lummer and Pringsheim.		
CO_2	1.3008	Fürstenau.[2]	15.18	P. Breitenbach.
			16.38	v. Obermayer.
N_2O	1.311	Wüllner.	17.94	v. Obermayer.
			18.40	W. J. F.[3]
C_2H_4	1.2455	Wüllner.	21.26	P. Breitenbach.
CH_3Cl	1.1991	Müller.	28.5	P. Breitenbach.

The values of C_p/C_v are taken from Landolt-Börnstein-Meyerhoffer, 1905, except as below. They have the temperature range 0°–20°.

The values of C/K are computed from the separate values of C and K given by the writer in Part I., except that for N_2O.

In 1866 O. E. Meyer[4] integrated the equations for the flow of a viscous gas through a uniform cylindrical capillary tube of circular cross section, making certain assumptions as to the different velocity-gradients, introducing the correction for slip on the walls, and using Boyle's Law as an equation of expansion. Poynting and Thomson[5] and M. Brillouin[6] give a more concise and direct treatment of the same problem, the former neglecting slip on the walls. The slip correction, as a function of the pressure, is given by A. Bestelmeyer[7] in his article on the viscosity of nitrogen, in a footnote; the same result is given, with derivation, in Part II.[1] of this series. In all of these the equation of expansion assumed is $pv = \text{const.}$; in other words, the gas expands isothermally throughout the tube, and the flow is assumed parallel to the axis. The recent paper of W. Ruckes[2] on the critical velocity of gases flowing in long, narrow tubes, contains measurements of a necessarily crude nature, which indicate that there is a decided temperature fall in the stream of gas, especially near the exit end. This makes it seem advisable to study the problem under the assumption of another equation of expansion.

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[1] U. Behn und H. Geiger, Verh. Deutsch. Phys. Ges., 9, p. 657, 1907.

[2] R. Fürstenau, Ann. d. Phys., 27, p. 735, 1908.

[3] PHYSICAL REVIEW, 28, p. 106, 1909. While these are not final, they cover a much greater range than v. Obermayer's.

[4] O. E. Meyer, Pogg. Ann., 127, p. 269, 1866.

[5] Poynting and Thomson, Properties of Matter, pp. 207, 210, 1902.

[6] M. Brillouin, Leçons sur la Viscosité, II., p. 36.

[7] A. Bestelmeyer, Ann. d. Phys., 13, p. 980, 1904.

The writer assumes $pv^n = k$ as an equation of expansion, and integrates with the same assumptions as to cylindrical flow, etc., as O. E. Meyer. The resulting equation is

$$F = \frac{\pi a^4}{8\eta l p^{1/n}\left(1 + \frac{1}{n}\right)}\left(1 + \frac{4\zeta}{a}\right)\left(p_1^{1+\frac{1}{n}} - p_2^{1+\frac{1}{n}}\right). \qquad (1)$$

In this F is the volume of gas passing per second, measured at pressure p, a is the radius, l the length of the tube, ζ the slip coefficient, p_1 the entrance, p_2 the exit pressure; k and n are assumed constant.

If $n = \infty$, the equation $pv^n = k$ represents an incompressible liquid, since then $dv/dp = 0$; and the flow equation reduces to the well-known form of Poiseuille,

$$F = \frac{\pi a^4}{8\eta l}\left(1 + \frac{4\zeta}{a}\right)(p_1 - p_2)$$

If $n = C_p/C_v$, the ratio of the two specific heats of a gas, the equation implies adiabatic expansion throughout the tube; conduction of heat through the capillary walls or the gas, and frictional production of heat in the gas, are both excluded from consideration. This is evidently a limiting case; *n cannot be as great as* C_v/C_v. Any introduction of heat into the gas will tend to diminish n.

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[1] PHYS. REV., 28, p. 73, 1909.

[2] W. Ruckes, Ann. d. Phys., 25, p. 983, 1908.

If $n = 1$, $pv =$ const., and the flow equation reduces to

$$F = \frac{\pi a^4}{16\eta l p}\left(1 + \frac{4\zeta}{a}\right)(p_1^2 - p_2^2) \tag{2}$$

the equation of O. E. Meyer. It implies perfect conductivity of both gas and walls, and that the latter are kept at a constant temperature. This also is a limiting case; *n cannot be as small as 1.*

As the transpiration method must actually be used, the gas expands neither adiabatically nor isothermally; there is a generation of heat through friction, a cooling by expansion against pressure, and there is imperfect conduction through the gas and walls, while outside the walls the temperature is maintained constant. Assuming that the production of heat by internal friction is too small to change the conditions of flow materially, then the value of n, while probably not the same for all points in the tube, must remain between 1 and C_p/C_v, and the true equation of flow is probably something between (1) and (2).

Supposing it possible for a gas to have $C_p/C_v = 1$; then there is no difference between adiabatic and isothermal expansion, and O. E. Meyer's equation represents the flow of such a gas, except as errors may follow in consequence of the assumed cylindrical flow. We would expect then that the accuracy of the accepted capillary tube determinations of gas viscosity is greater for the complex gases than for the simple, *e. g.*, greater for CH_3Cl than for He.

The integration of the equations of the gas-flow has not been performed even for the case of isothermal expansion, which is the simplest, without assumptions which may seriously affect the results obtained. No experimenter has made the attempt to apply a correction to his measured pressures, to allow for the kinetic energy of the entering or emerging gas; and this may be a serious matter when the pressure-difference between the two ends of the tube is as large as that used by the writer or by Bestelmeyer. The method used by the latter was originated by Holman, and consists in measuring the pressure differences between the ends of two tubes which are connected in series, so that three pressures are measured, one nearly atmospheric, one quite small, and one intermediate. The gas-flow being steady, the two tubes are immersed in melting ice, and from the ratios of the measured pressures the ratio of the tube resistances is computed, on the basis of O. E. Meyer's theory. On the same basis an accurate calibration with mercury was carried out, whence again the ratio of the tube-resistances was computed. The two values of this ratio differed by 1 in 200, too much to be explained by observational errors. Brillouin attributes this to noncylindrical flow. And certainly no one has attempted to make any allowance for cooling by expansion or conduction of heat through the glass walls of the tube. Until something has been done to solve more accurately the mathematical problem of the gas-flow, it is too early to say that we *measure* the viscosity of a gas by the transpiration method. The fact that Sutherland's equation applied graphically to the experimental data gives straight lines shows that the deduced values of the viscosity coefficients must be *linearly related* to the true values; the grouping of C/K in relation to the ratio γ indicates a strong possibility that the nonisothermal character of the expansion cuts a very important figure in the actual process.

A complete theoretical solution would involve consideration ot the law of viscous forces; heating by friction; the fact that there are *two* viscosity coefficients, as there are two of elasticity; the equation of state; cooling by expansion against pressure; conduction of heat through walls and gas; pressure lost corresponding to kinetic energy at the two ends; perhaps also the effect of temperature on the viscosity coefficient.

With regard to the two viscosity coefficients, one, λ, appears in the heating effect of friction as multiplied by the square of the divergence of the velocity, and hence determines the dissipation of energy by change of volume; the other, μ, is especially associated with shearing gradients. Stokes makes the hypothesis that $3\lambda + 2\mu = 0$, while $\mu > 0$, hence $\lambda = -\frac{2}{3}\mu$. Consequently the existence of the coefficient λ tends to diminish the waste of energy. (This is entirely apart from external work done on expansion.) This coefficient is entirely ignored in the equations presented above.

Brillouin (II., p. 37) writing of the forms of streamlines in the capillary, says: "chacque filet cylindrique s'épanouit de plus en plus à mesure qu'il avance dans le tube." It is more likely that the bundles of streamlines *contract* as they approach the exit end of the tube, until they are quite near it; the outer layers of gas *scale off*, so to speak, into the central stream. A small element of the gas in the neighborhood of the wall advances slowly into a region of lower pressure; it expands and presses in all directions; the walls do not yield at all, the gas behind is at a higher pressure, the gas in front can yield but slowly, owing to the friction; while the gas in the center, being freer to move than any other, goes forward, and the element expands mainly toward the center. Hence the streamlines must converge toward the axis.

Again, on the same page, in computing the correction to pressure due to kinetic energy, Brillouin deduces the correction to the entrance pressure from the kinetic energy of the emergent jet. The reasoning which he has previously employed (I., p. 132) in the case of liquids can hardly apply here; the velocity of the emergent jet is not due wholly, as for a liquid, to the initial pressure; a good share of it is due to work done inside the tube itself, especially if the expansion has been isothermal, and heat energy has been supplied all along the tube to keep up the temperature.

The difficulty of integrating the equations of the capillary tube could be avoided by an apparatus constructed on the plan of that used by Helmholtz and Piotrowsky,[1] or that of Gyözö Zemplén[2] or of J. L. Hogg,[3] all of which depend on the Helmholtz solution, of the problem of a sphere vibrating in or about a viscous fluid. Such an apparatus, constructed to give accurate results with controlable temperature over a considerable range (Gyözö Zemplén's apparatus can only be used at or near room temperature, and the uncertainty of his measurement for air, .0001794 at 20°.4 C., is 1 per cent.), and used in the study of two gases of considerably different complexity, *e. g.*, air or nitrogen and methyl chloride, would give a check on the capillary tube method which is much to be desired. The nearest approach to such a check at present available is found in the comparison which P. Breitenbach[4] gives of his own capillary tube results with those of J. Puluj, obtained by a

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[1] H. Helmholtz, Abhandlungen, I., p. 196.

[2] Gyözö Zemplén, Ann. d. Phys., 19, p. 783, 1906.

[3] J. L. Hogg, Contr. Jeff. Phys. Lab., 2, p. 611, 1904.

[4] P. Breitenbach, Wied. Ann., 67, p. 820, 1899.

	Puluj 14°.7	Breitenbach 15°.0.
CO_2	.0001468	.0001464
H_2	.0000893	.0000893

vibration apparatus in the laboratory of Kundt at Strassburg. This apparatus was of the disk type, which involves more mathematical difficulties than the sphere; but the agreement is interesting, and encouraging to experimenters who use the tube method

GÖTTINGEN,
March, 1909.

THE EFFECT OF ELECTRICAL OSCILLATIONS ON THE CONDUCTIVITY IMPARTED TO GASES BY AN INCANDESCENT CATHODE.

By Orin Tugman.

THE property of an incandescent carbon filament of emitting electrons in excess of positive electricity has been used by Fleming[1] for obtaining small direct currents from an impressed oscillatory potential difference of high frequency. Since the emission of negative electricity from the hot carbon tends to discharge a cold positively charged conductor near it a continuous current can be set up through the space between the cold conductor and the hot filament, if the charge on the cold conductor is continually renewed. In Fleming's experiment the carbon filament was surrounded by a metal cylinder as the cold conductor. This cylinder was connected in series with a galvanometer, a coil of wire of few turns and to one terminal of the carbon filament. The coil of wire was electromagnetically coupled to a primary coil in the discharge circuit of a Leyden jar. The induced secondary oscillations can pass in one direction through the vacuous space between the incandescent carbon and the cylinder but not to any great extent in the opposite direction. The direction of the rectified current is, according to the conventional way of stating, from the cylinder to the filament. The oscillations in the opposite direction were almost completely shut out. This device is called by Fleming an oscillation valve.

Lee DeForest[2] has inserted in the secondary circuit a battery of cells which will of themselves tend to send a current from the cold conductor to the filament. When the oscillations are induced we get the combined effect of the steady applied potential difference and of the oscillatory current. In this device called the "Audion" DeForest has the cold conductor made of metal plates in various shapes and positions. Recently the U. S. government has issued to DeForest a patent[1] on a wireless telephone receiver in which there are two metal plates, one on each side of the filament. Marconi[2] also has a patent device for a similar purpose in which the hot filament is surrounded by a metal cylinder.

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[1] Phil. Mag., p. 476. 1905.

[2] Amer. Institute Elect. Eng. Proc., December, 1906.

The investigation described in the present paper was begun at the suggestion of Professor Merritt, whom the author sincerely thanks for much timely advice throughout the work. The object of the experiment was to determine the currents which could be sent through the space between the cold conductor and the hot filament when the electrical oscillations were superimposed on direct applied potential differences. Also it is intended to show how this current varies with changes in the gas pressure and with the positions of the cold conductor.

The apparatus was set up as shown in Fig. 1. The carbon filaments kindly furnished by the General Electric Co., Harrison, N. J., were mounted at the end of glass tubes one and one quarter inch in diameter. The open end of the tube was fitted to another of the same size attached to an air pump. The joint of the two glass tubes was wrapped with sheet rubber securely wired on. The whole joint was then surrounded by a mercury seal. By this means different filaments could be mounted on the pump in a few minutes. As is shown in the diagram, a glass tubes extends vertically downward from the one holding the filament and ends in a deep cistern of mercury. A long steel rod carrying on its upper end the cold conductor, a brass plate as shown, passes down into the cistern of mercury and turns up on the outside of the tube. This affords a positive connection to the third terminal so its position can be changed at will. Electrical connection is made with the brass plate as shown through the platinum wire sealed in the glass at *c*. A vacuum tube with parallel flat electrodes is sealed on the tube leading to the pump. The discharge of an induction coil through this was used to indicate the pressure of the gas in the apparatus.

The electric oscillations were produced in a coil of eight turns and twenty centimeters diameter. One end of this coil was connected through a condenser to earth. The other end of the coil was connected to the third terminal at *c*. The primary coil consisted of three turns twenty centimeters in diameter, connected in series with a Leyden jar and zinc spark gap. The spark gap was always short so that the oscillations were large in number and of relatively small amplitude, otherwise sparking would occur between the third terminal and the filament. Throughout each series of readings the distance between the two coils and the length of the spark gap remained fixed.

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[1] U. S. Pat. No. 1427, 1908.

[2] U. S. Pat. No. 887, 1908.

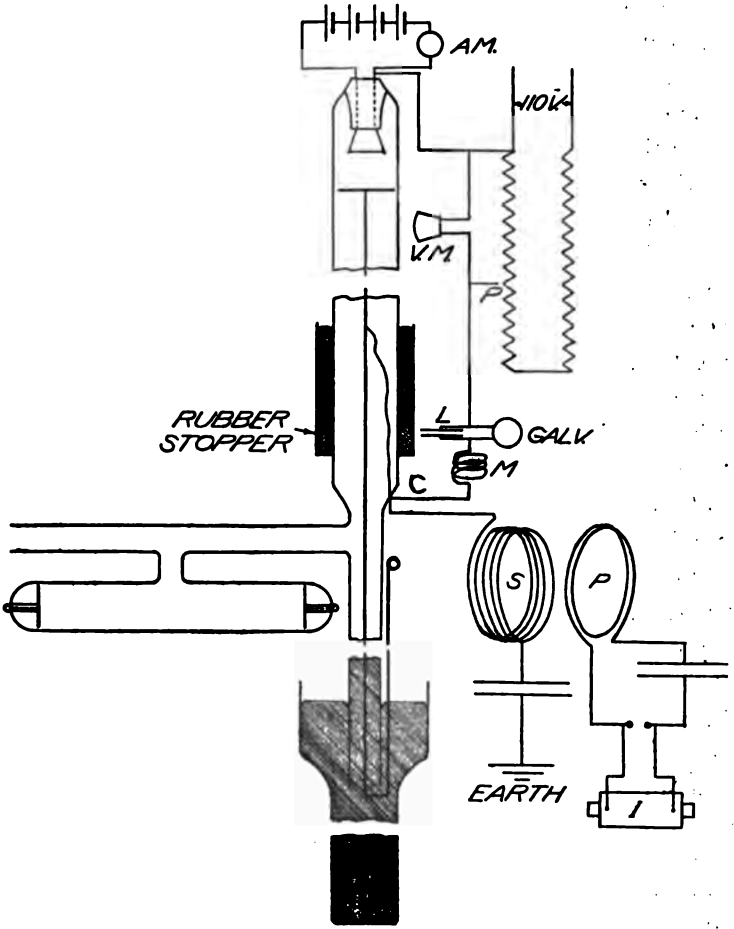

Fig. 1.

The direct potential difference could be regulated from the resistance connected to a dynamo circuit and was measured between points *A* and *B*. Since the currents through the galvanometer are small the fall of potential through it is negligible. The resistance of the galvanometer was 300 ohms and the sensibility 10^{-7} amperes per scale division. A variable shunt was used on the galvanometer so as to adapt it for a wide range. To prevent the oscillations from passing through the galvanometer an inductance was placed in series with it at *M*, Fig. 1. As a further protection to the galvanometer a Leyden jar was connected in parallel shown at *L*, Fig. 1. The carbon filament is heated by an independent storage battery insulated from all other sources of E.M.F.

The first series of observations were made on a horseshoe-shaped filament, burning on from five to seven volts. The third terminal consisted of a brass cup about 2 cm. deep and wide enough to nearly fill the tube. The cup was fixed in one position surrounding the filament and was not changed during this part of the experiment. All the connections were made as shown in the diagram. The air was pumped out until the vacuum gave a cathode ray discharge.

The observations were made in the following manner: The contact *D* was moved along the resistance *R* until the desired potential was produced. The current in the galvanometer due to the direct potential difference was then read off the galvanometer scale. After this the oscillations were started by operating the induction coil I. The galvanometer was again read, which reading gave the current through the vacuous space due to the direct potential and the superimposed oscillations.

The results of a series of observations of this kind are shown by the curves in Fig. 2. The abscissæ represent the direct potential difference between the negative end of the filament and the third terminal. The ordinates are the currents through the galvanometer. The continuous lines give the current through the vacuous space when the direct potential is on alone and the broken lines show the current obtained when the oscillations are added to the steady potential. Curves I., II., III., IV. and V. were made when the carbon filament was heated by currents of 2.5, 2.7, 2.9, 3.0 and 3.1 respectively.

The general form of the continuous line curves may be explained by considering the manner in which the potential difference between the third terminal and the filament varies as the contact D is moved along the resistance R, Fig. 1. When the contact D is placed at

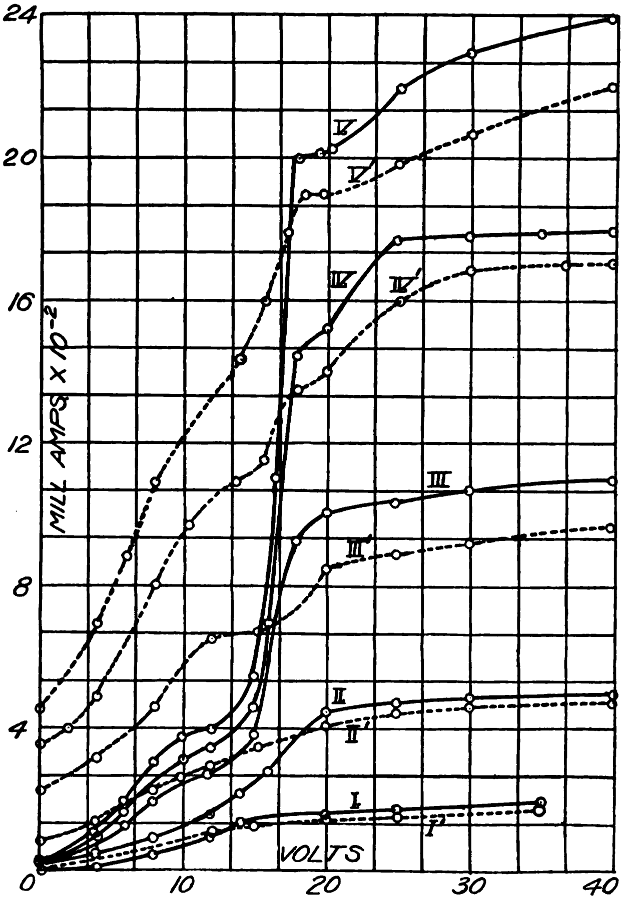

Fig. 2.

Curve.	Current in Filament Amperes.
I.	2.5
II.	2.7
III.	2.9
IV.	3.0
V.	3.1

In Curves I.′, II.′, III.′, IV.′, V.′ the same currents were in the filament as above respectively, but the electrical oscillations were added to the direct potential difference.

E the third terminal has the same potential as the negative end of the filament. The galvanometer showed practically no current with this connection. If the contact *D* is moved a short distance along *R* the potential of the third terminal will be equal to the potential of some point on the filament, a short distance from the

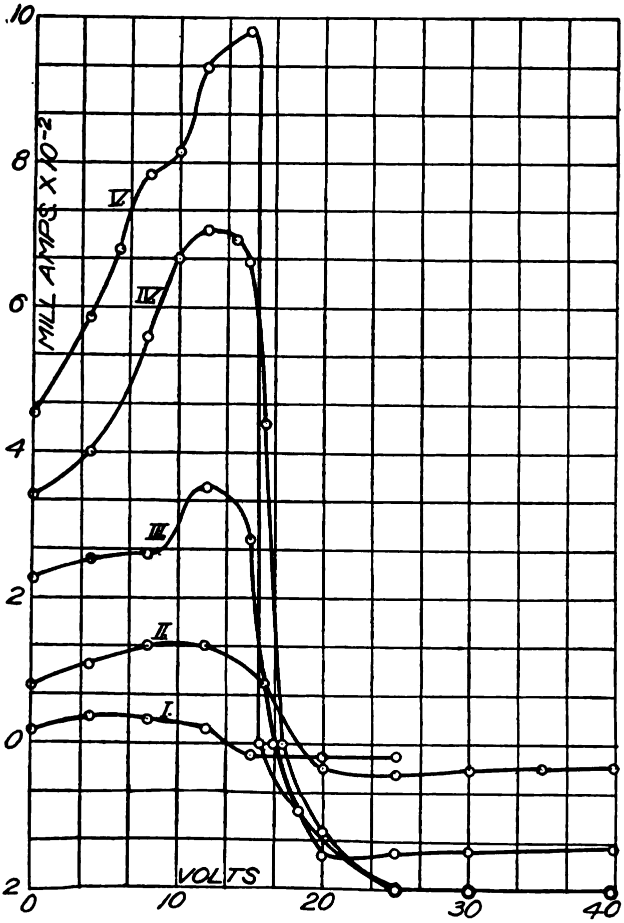

Fig. 3.

Curve.	Current in Filament Amperes.
I.	2.5
II.	2.7
III.	2.9
IV.	3.0
V.	3.1

negative end. Consequently electrons will now pass from the negative end of the filament to the third terminal. As higher potential differences are applied the third terminal will receive electrons from a greater portion of the filament until all the electrons emitted by the hot carbon reach the third terminal. When this state is reached the number of elections emitted by the hot carbon is not greatly influenced by any further increase in the potential difference between the third terminal and the filament. We see from the curves in Fig. 2 that the current through the galvanometer is saturated at about fifteen volts. The continuous line curves are similar to those obtained by Merritt and Stewart[1] with a metal cylinder surrounding a carbon filament.

The broken line curves in Fig. 2 show the results obtained by superimposing the electrical oscillation on the direct potential difference. It is seen from the curves that the electrical oscillations produce a current through the galvanometer even when the third terminal is at the same potential as the negative end of the filament. This is to be expected when we consider that only one half of the wave can pass through the vacuous space. When the third terminal is charged negatively by the oscillations the electrons are repelled and cannot take part in the conduction of electricity. Furthermore, since the number of positively charged particles emitted by hot carbon is so very small compared to the number of electrons, the negative half of the oscillation is cut out and current passes through the galvanometer only when the oscillations charge the third terminal positively. The potential produced by the oscillations is probably more than great enough to produce saturation. But this potential lasts for a short time and the duration of each train of oscillations is very small compared to the interval between each train of oscillations. Therefore it seems reasonable that the galvanometer current should not be saturated when the high oscillatory potential difference is superimposed on a small direct potential difference. The same line of reasoning will explain the decrease of the galvanometer current, produced by the oscillations, after saturation. When the direct potential difference has produced the saturation current, any farther increase in the potential difference cannot increase the number of electrons emitted by the hot carbon. But more that half of the oscillation wave which charges the third terminal negatively becomes effective in reducing the difference in potential between filament and third terminal. As a consequence of this reduced potential difference the galvanometer current falls below the saturation value.

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[1] Phys. Rev., p. 239, 1904.

By plotting the differences in the currents obtained by the direct potentials alone and that obtained by the resultant of the direct and oscillatory potentials we have the curves shown in Fig. 3. After

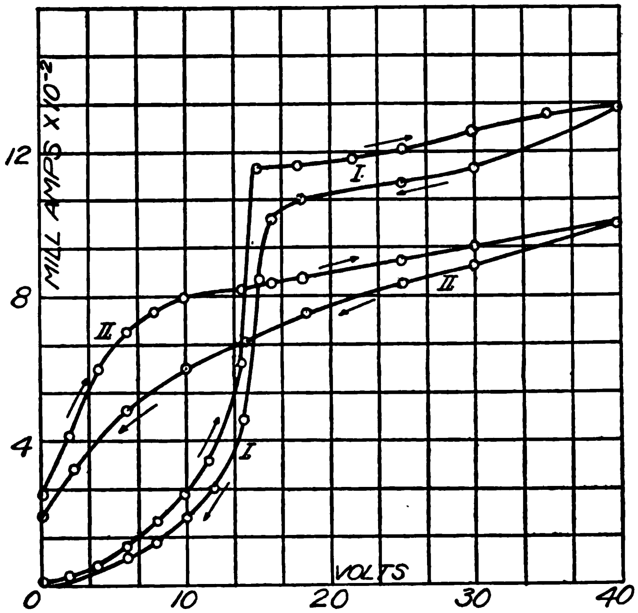

Fig. 4. Current in Filament, 3 amps.

reaching forty volts and the observations were made coming down the curve, it was found that the descending curve had shorter ordinates than the ascending curve. Such a curve is shown in Fig. 4. It is seen that the galvanometer current is greater with increasing voltages than with decreasing potential differences.

In all the above experiments the brass cup, as a third terminal, remained fixed in one position. It was thought that the position of the third terminal would have an interesting effect on the galvanometer current. In the remaining experiments the device previously described and shown in the diagram (Fig. 1) was used to vary the position of the third terminal. In this part of the investigation the third terminal was a brass plate three centimeters in diameter, mounted as shown in the diagram. The first experiment was made with a horse-shoe-shaped filament carrying 1.5 amperes on 60 volts. The brass plate was so adjusted that its distance from the bottom of the carbon loop was about half of a millimeter. The results of some experiments made without the electrical oscillations, but with a direct

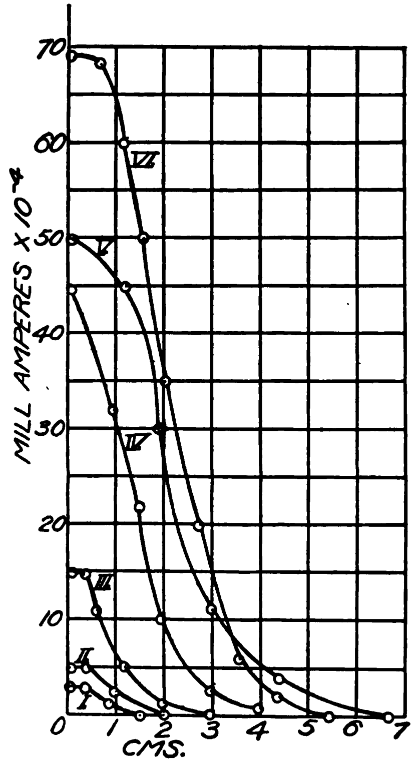

Fig. 5. Current in lamp filament for Curves I., II., III., IV., V. 1.6 amp. for Curves VI. and VIII.

Curve.	Potential Difference between Filament and Third Terminal Volts.
I.	0
II.	25
III.	50
IV.	75
V.	100
VI.	100

applied potential difference, are shown in Fig. 5. Curves I., II., III., IV. and V. were made when 0, 25, 50, 75 and 100 volts direct were impressed respectively, and in all five the current in the filament was 1.6 amperes. In Curve VI. the current in the filament was 1.8 amperes and the potential applied to the third terminal was 100 volts.

It is to be seen that in most of these curves the current in the galvanometer does not drop off quickly as the brass plate is removed until nearly half a centimeter intervenes. All the distances plotted along the *x*-axis were reckoned from the lowest point of the curved filament which hung in a vertical plane with its convex side down. Also the vacuum was high enough for the vacuum tube to give a good cathode ray discharge.

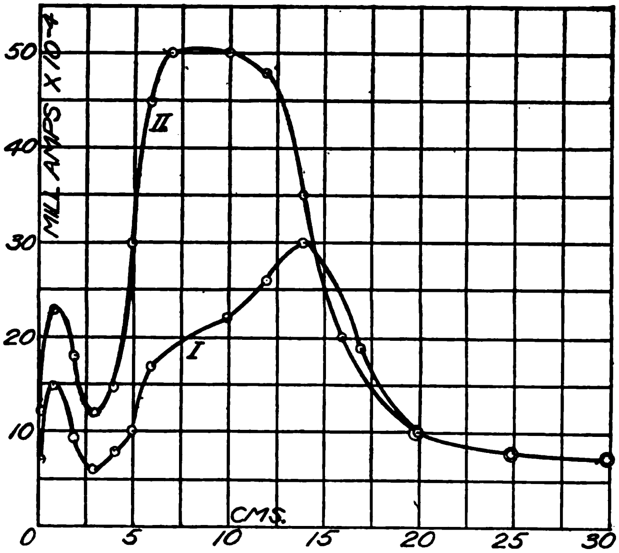

Fig. 6. 1.2 amp. in lamp filament; I., zero potential difference; II., 25 volts. potential difference.

More interesting results are obtained when a straight carbon filament is used in place of the curved one. This permits the brass plate to be the same distance from all parts of the carbon. Adjustments were made so that the brass plate could be brought up very close to the incandescent filament. Some results are shown in Figs. 6, 7 and 8. As the brass plate is moved away from the filament the current through the galvanometer at first increases with the distances. When the brass plate is nearly a centimeter off the galvanometer current is in some cases nearly twice as large as when the plate is a millimeter distant from the filament. Figs. 6 and 7 also show a decided increase in the galvanometer current when the plate is out about five centimeters. All of these curves were made with the oscillations on. The pressure was not as low as could be produced, the cathode of the discharge tube showing a large blue spot. After pumping for some time and repeating the observations curves like those in Fig. 8 were obtained. Here we have a maximum very pronounced at about one centimeter. After this point is passed the curve falls off rapidly at first and then gradually slopes off to zero. Continuous pumping was necessary to keep the second maximum from appearing. We see in Fig. 8, curve III. shows a slight tendency to increase after the first rapid decrease. From this it seems that the pressure of the gas determines the presence of the second maximum. Also it is noted that as the pressure is lowered the first maximum gets more pronounced.

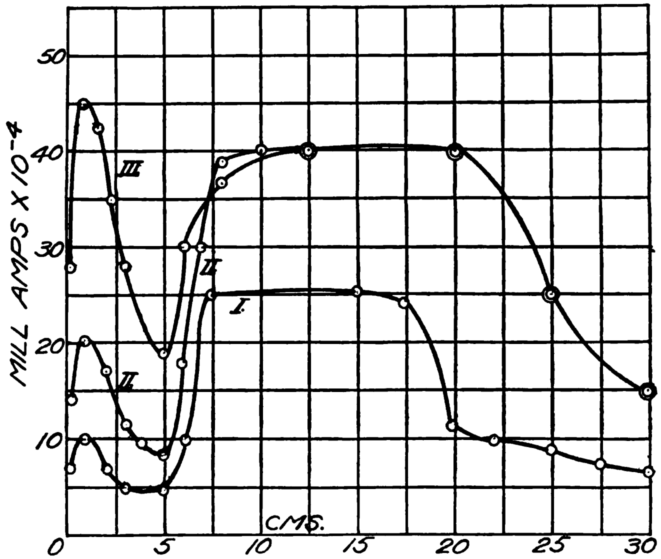

Fig. 7. 1 amp. in lamp filament; I., zero potential difference; II., 25 volts potential difference; III., 50 volts potential difference.

A series of observations was made using only the direct potential. A most decided maximum in some of these curves appears when the third terminal is about one centimeter from the filament. These are shown in Fig. 9. When the higher voltages are applied the per cent. of increase of current from the beginning to the highest value is greater than for the low voltages. Fleming made some experiments in which he measured the current obtained from the negative leg of a carbon loop to a metal plate. His curves correspond to those in Fig. 9 from the maximum point down.

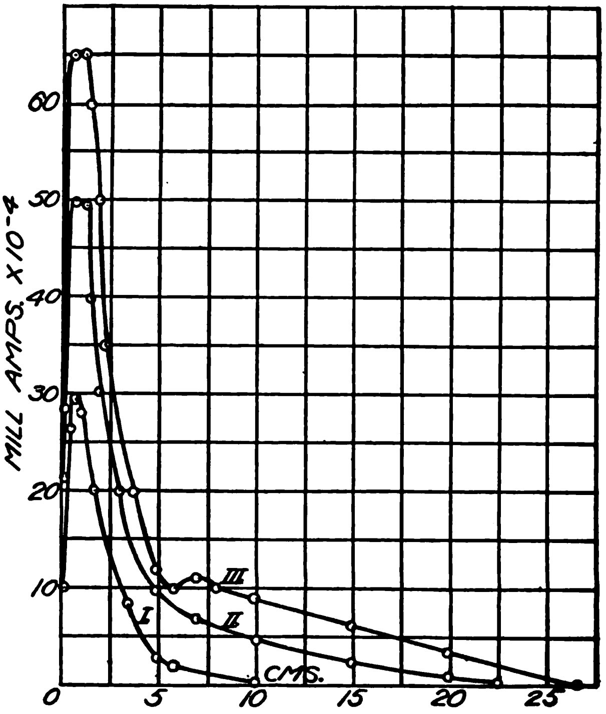

Fig. 8. 1.3 amp. in lamp filament; I., 20 volts potential difference; II., 30 volts potential difference; III., 40 volts potential difference.

If the filament current and the direct potential are maintained constant and the pressure varied some more data are given, which show under what conditions the second maximum appears when the oscillations are on. In Fig. 10, curve I. was made when the vacuum tube discharge was gray in color, just before cathode rays appear. Curve II. was made just as the first faint traces of cathode rays appeared. For curve III. the vacuum tube showed full cathode ray discharge, the blue spot on the cathode being a small blue point. Curves IV. and V. show a similar change as the air is pumped out. It can be seen here that the first maximum is increased with diminished pressure.

A reasonable explanation of the first maximum may be obtained from an analogy to the discharge through vacuum tubes. It is known[1] that the effect of cathode ray particle by impact on the gas molecules is small close to the cathode. The ionizing effect of the cathode rays increases rapidly as the negative glow is approached, reaches a maximum in the glow and quickly drops to a small

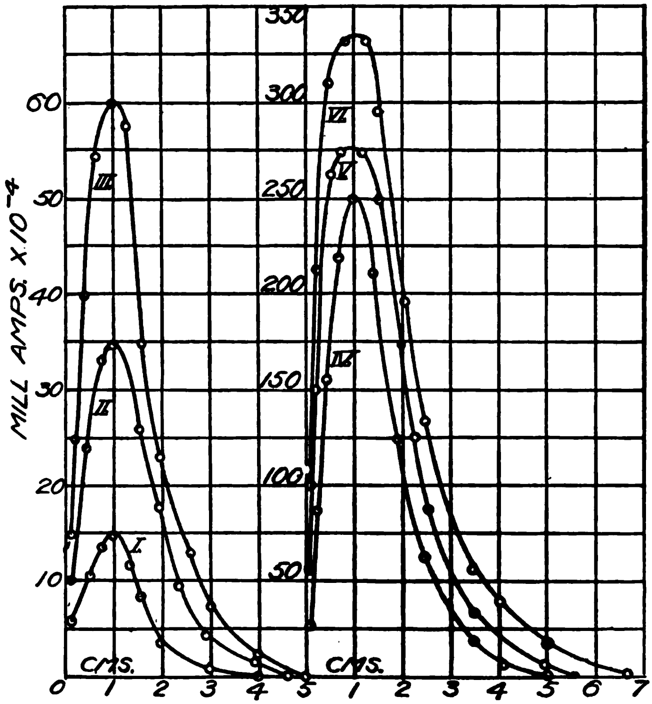

Fig. 9. 1.2 amps. in lamp filament; I., 40 volts potential difference without oscillations; II., 50 volts potential difference; III., 60 volts potential difference; IV., 70 volts potential difference; V., 80 volts potential difference; VI., 90 volts potential difference.

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[1] J. J. Thompson, Conduction of Electricity through Gases, p. 484.

value. In the present case the brass plate in the nearest position to the hot filament is within the cathode dark space. As the third terminal is moved away from the filament regions are reached where the electrons from the carbon have greater ionizing power. After passing this point the galvanometer current falls off.

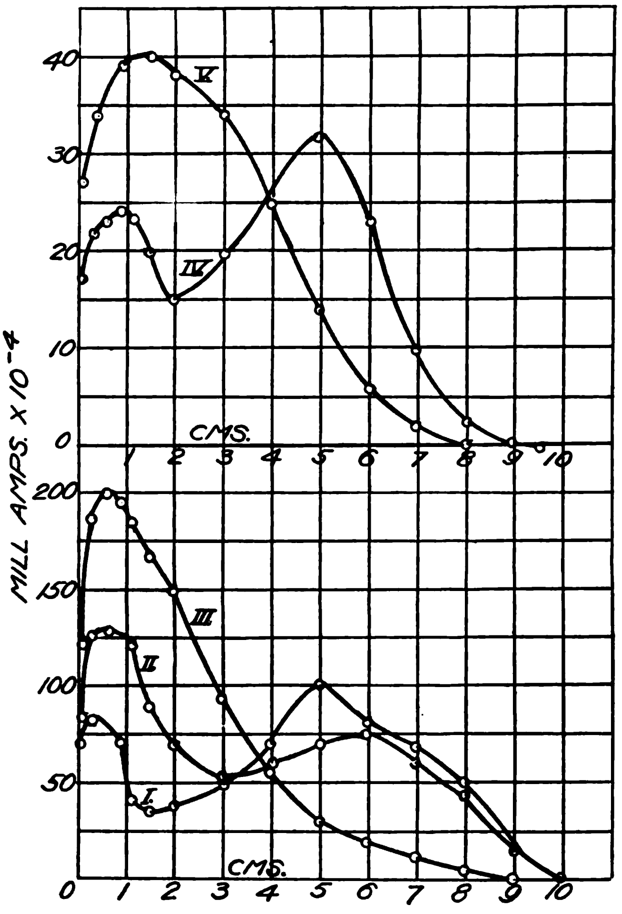

Fig. 10. I., II., III., current in lamp filament, 1.4 amp., 23 volts; IV., V., current in lamp filament 1.2 amp., 30 volts.

The second maximum must be due to the secondary ionization in the gas. As was cited above, this second maximum occurred only at relatively high pressures, *i. e.*, pressures greater than that required for good cathode rays. Furthermore, when the amplitude of the oscillation was small, the potential gradient was not sufficient to give the ions the required velocity for ionization.

In Fig. 11 are shown some curves made when the direct potential was comparatively high. It will be remembered that in Fig. 2

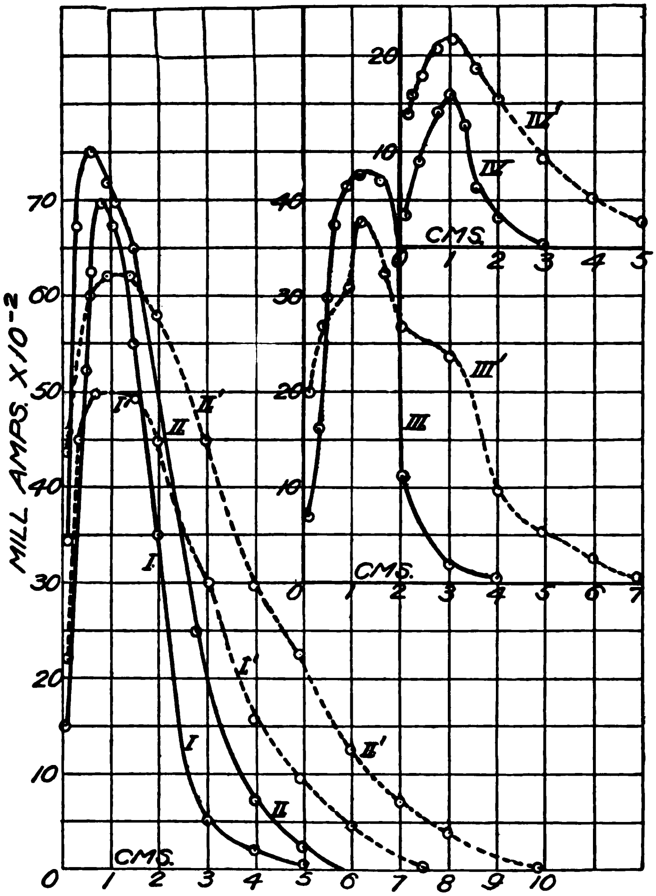

Fig. 11. 8 amp. in lamp filament; I., 50 volts without oscillations; I.′, 50 volts with oscillations; II., 60 volts without oscillations; II.′, 60 volts with oscillations; III., 70 volts without oscillations; III.′, 70 volts with oscillations; IV., 80 volts without oscillations; IV.′, 80 volts with oscillations.

the galvanometer current is decreased by the oscillations when saturation exists. In Fig. 11 we see that the oscillations produce a decrease in the galvanometer deflections when the brass plate is about one centimeter distant. After this distance is passed there is an increase of the galvanometer current due to the oscillations. The continuous line as before represents the current produced by the direct potential alone and the broken line when the oscillations are impressed on the direct potential. In all these curves the same current flowed in the filament and the pressure was low enough to prevent the second maximum from appearing. In curves *V*, *V′*, the direct potential is 40 volts. Under this condition the effect of the oscillation is always to increase the current through the gas. The highest maximum on this curve sheet is reached when about 80 volts is applied.

It appears that different filaments have not acted the same with regard to the potential required to produce saturation. In these last curves apparently over 50 volts must be used, whereas in Fig. 2 saturation is produced at about 15 volts.

SUMMARY.

When the direct potential between the third terminal and the hot carbon is not producing the saturation current, the effect of the electrical oscillations is to increase this current and to decrease it if the saturation is being already produced.

The distance between the third terminal has a marked effect on the current through the vacuous space. In these experiments the current reaches a maximum when the third terminal is about one centimeter distant from the filament. A second maximum occurs when the gas pressure is not too low and the oscillating potential high enough to produce ionization.

As the age of the filament increases with use the sensitiveness of all these effects decreases.

PHYSICS LABORATORY,
CORNELL UNIVERSITY.

ON THE POSSIBLE RADIOACTIVITY OF ERBIUM, POTASSIUM AND RUBIDIUM.

BY W. W. STRONG.

THE work of Thomson, Campbell, Wood, McClennan and others indicate that potassium and probably rubidium is radioactive. McClennan and Kennedy[1] have studied various potassium, sodium, lithium, rubidium, cæsium and ammonium salts and find that only potassium salts are appreciably active, the cæsium and rubidium salts giving but a very slight indication of radioactivity. The potassium cyanide salts investigated by them showed an activity roughly proportional to the potassium content (the potassium content was very small in some of the samples called potassium cyanide). Levin and Ruer[2] find by the photographic method that potassium salts emit radiations that are about a thousandth as effective in making a photographic impression as the β radiations from uranium. They find that metallic lead affects a photographic plate about as much as the potassium salts.

In work upon the radioactivity of ordinary substances the electrical method is sensitive and rapid but requires a great deal of attention. The photographic method requires but slight attention and long periods of time. For these reasons the photographic method has been used by the writer in making a preliminary examination of the possible radioactivity of a large number of salts. The effects upon the photographic plate may be due sometimes to chemical action as well as to radioactive emanations or radiations. But by the use of screens it is possible in many cases to decide between these two causes. When there is no photographic impression after a long exposure of at least six months it is certain that the substance does not emit any β-like radiations as strong as the radiations from most potassium salts.

Several years before his death Professor Rowland collected several minerals and salts of the rarer elements and through the kindness of Professor Ames these were placed at the writer's disposal. An investigation of the possible radioactivity of these salts has been made by the photographic method. The work is being continued by electrical methods.

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[1] Phil. Mag., No. 93, p. 377, 1908.

[2] Phys. Zeit., No. 8, p. 248, 1908.

The substances examined include salts of potassium, zirconium, cæsium, lead, bismuth, yttrium, tungsten, sodium, molybdenum, niobium, rubidium, erbium, tantalum, lanthanum, vanadium, neodymium, præsodymium and ruthenium. Most of these salts were collected by Professor Rowland at least fifteen years ago.

The method of investigation consists in exposing "Seeds' Dry Plates" for about six months to the salts. The salt is usually placed in a dish and covered with a screen containing several openings. Above this screen is the photographic plate, the film side being next the salt. As many of the salts had been in cork-stoppered bottles for years, there was usually not much danger of chemical action although in several instances this trouble occurred. The plates used were probably not as sensitive as those used by Levin and Ruer. Intense impressions were however obtained in 30 hours by exposing the plates to uranyl chloride or uranyl nitrate. The method of developing is the same as that used by Jones and Anderson.[1]

Potassium Salts. — Potassium cyanide (purchased for photographic purposes) gave a considerable impression in an exposure of 150 days. Thin aluminium foil (.00156 cm. thick) did not cut off much of the radiation. Thicker aluminium foil (.006 cm. thick) cut off at least half of the rays. A sample of potassium benzoate (Rowland collection) exposed for 150 days produced but a very slight impression. Potassium citrate (Rowland collection) exposed 160 days produced a slight effect which was not absorbed by the thin aluminium foil, but was by sheet iron (.04 cm. thick). Potassium urate (R. col.) shows practically no effect after an exposure of 150 days. Potassium carbonate (R. col.) gave a strong photographic impression after an exposure of 160 days. The radiations were cut off by sheet iron. On the other hand, potassium carbonate recently purchased for photographic work (Mallinckrodt) showed practically no activity. The same was true of a sample of potassium chromate (R. col.) which was exposed 150 days.

[1] Carnegie Publication, No. 110, p. 7.

Lead. — Several pieces of lead used by the writer[1] to screen electroscopes were tested. The length of exposure was 150 days. When the surface of the lead was bright and when near the photographic film, the action upon the plate was usually very intense and of a spotted nature. When the lead was at a centimeter's distance from the plate the impression was very weak. It seems that this effect cannot be produced by rays like the β rays from uranium. They may be δ-like rays or the effect may be due to chemical action, Lead chloride (R. col.) was found to be inactive. Lead peroxide (R. col. Merck.) gave a slight impression in 150 days. The radiations were cut off by thin aluminium foil.

Zirconium chlorate (R. col.) and zirconium oxide (R. col.) gave no photographic impressions. The former was exposed 160 days and the latter 100 days. Cæsium, bismuth chloride and sticks ot tellurium gave negative results after exposures of 160 days.

A plate was placed in a box containing mercury. The plate was quite black on developing, the part of the plate in the mercury being less black however than the part that was exposed only to the mercury vapor.

Three yttrium compounds belonging to the Rowland collection were tried — these being yttrium sulphate, yttrocerite and yttrium oxalate. The lengths of exposure were 110, 150 and 110 days respectively. No impression was produced upon the photographic film.

The following substances belonging to the Rowland collection were exposed for 110 days with negative results, oxide of tungsten, tungstate of ammonia, tantalic oxide, molybdenum metal (powdered), sodic molybdenate, ammonium molybdenate, niobic acid, Gadolenite (No. E_1G_2), vanadium chloride, cerium sulphate, neodymium oxide, præsodymium oxide, ruthenium "residues" and iridium metal. The quantity of several of these compounds was so small, however, that unless they were quite active, no impression upon a photographic plate would be made.

A sample of "erbium chloride" gave a very strong impression after an exposure of 110 days. This salt was in the Rowland collection. Several years ago the University purchased some "erbium metal" from the Harrington Brothers, London. This also gave a very strong photographic impression. The "erbium chloride" is known to contain a large amount of erbium as it has been used for spectroscopic work. The absorption spectra of the powdered "erbium metal" did not show any trace of the erbium spectrum. How much erbium is present the writer does not know.

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[1] PHYS. REV., p. 43, July, 1908.

Lanthanum chlorate and sulphate gave negative results. The absorption spectra of these salts showed the presence of neodymium in large amounts.

Samarkite "oxides from sodic sulphite" showed no radioactivity. Samarkite earth and monazite sand (Barker Co., N. C.) showed a very marked radioactivity. These minerals belonged to the Rowland collection.

Rubidium chloride after an exposure of 110 hours gave a slight photographic impression. Rubidium chlorate in 110 hours gave a very strong impression.

In several instances the photographic plates were supported by pine or oak blocks. In every case it was found that these darkened the plate and this darkening spread out as a cloud for several millimeters from the parts of the plate touching the plate. The effect of the wood was very small when it was a few millimeters from the plate.

SUMMARY.

Various "new" and "old" potassium salts have been found to give a photographic impression after long exposures. This action has been found to be quite small for other potassium salts.

Erbium and rubidium salts were found to act in a similar way to potassium salts. This indicates that erbium, rubidium and potassium may be radioactive. The fact that so many rubidium and potassium salts from widely different sources possess this activity seems to show that it is not due to an impurity.

Lead and wood also give photographic impressions but these seem to differ in their cause from the impressions produced by erbium and potassium.

JOHNS HOPKINS UNIVERSITY,
June, 1909.

ON THE PHOTOELECTRIC PROPERTIES OF SODIUM-POTASSIUM ALLOY.

BY JAKOB KUNZ.

IT has been shown by Elster and Geitel that when the incident light is plane polarized, the photoelectric effect for a given intensity of light and angle of incidence is greater when the light is polarized at right angles to the plane of incidence than when it is polarized in that plane. The same physicists have also determined how the amount of light absorbed by the metal varies with the angle of incidence for light polarized in and at right angles to the plane of incidence and their results show that there is a close connection between the absorption of light and the photoelectric current, a connection which has been accounted for by certain theoretical considerations in a pretty satisfactory way. As however the photoelectric current or leak depends upon the number and velocity of the electrons emitted from the metal under the action of the light, the question arises as to the influence of the plane of polarization and the angle of incidence on the initial velocity of the electrons and on the number of electrons emitted from unit surface in unit time. If the units of electromagnetic energy were not susceptible of polarization, we should expect the plane of polarization to have no influence on the initial velocity of the electrons. In order to decide the question by means of experiments I introduced between the arc and the sodium-potassium alloy a Nicol prism so that the long diagonal of the end surfaces was horizontal and thus the light incident on the metal polarized in a plane at right angles to the plane of incidence. The deflection of the electrometer was exactly the same as that produced without the Nicol prism. This is what we should expect from the fact that the velocity of the electrons is not affected by the intensity of the incident light. The angle of incidence was 55°. If the Nicol prism from this initial position was gradually turned into the position 90°, where the plane of polarization was parallel to the plane of incidence, the deflection decreased as shown by the following figures:

0	410.9	50	368.0
5	409.9	55	359.4
10	407.9	60	349.5
15	404.8	65	338.2
20	401.1	70	325.7
25	397.1	75	312.5
30	392.3	80	296.8
35	387.8	85	285.0
40	381.9	90	270.4
45	375.2		

The deflection 410.9 mm. corresponds to a positive potential of the metal of 1.10 volts, the deflection 270.4 to a potential of 0.723 volt. Thus the positive potential decreases 34.3 per cent. when the plane of polarization from the position where it is perpendicular on the plane of incidence is turned into the position where it is parallel to the plane of incidence. The same experiment was repeated for a special wave-length of violet light. The result was the same. On the electromagnetic theory there is in light polarized at right angles to the plane of incidence an electric force with a component normal to the reflecting surface; when the light is polarized in the plane of incidence the electric force is parallel to this surface. Thus electrons which receive the energy from the electromagnetic units, acquire an amount of kinetic energy greater than that due to thermal agitation; at every collision with a molecule of the metal, the electron loses a part of its kinetic energy, and if it is to escape from the metal, it should make as few collisions as possible before reaching the surface, or it should move approximately at right angles to the surface. Such an electron will acquire an additional amount of kinetic energy from the light polarized at right angles to the plane of incidence, having an electrical component at right angles to the surface. Under these conditions an electron can easier escape than when acted upon by an electric force parallel to the surface of the metal. The decrease of the positive potential acquired by the metal, when the plane of polarization is turned from the first into the second position, is very small in comparison with the decrease of the photoelectric current measured by Elster and Geitel under the same conditions.

For an angle of incidence of 60° the currents are represented by the numbers 28.7 and 1.51, or the current for the first position of the plane of polarization is 19 times as large as for the second position; it follows from this result, combined with the result of the above experiment, that the number of electrons emitted must be affected very strongly by the plane of polarization.

Of great importance for the theory of the photoelectric phenomena is the effect of temperature. The results of previous investigations are by no means concordant; many of them are not conclusive, as secondary phenomena disturbed the pure effect.

Mr. Millikan,[1] who carried out a careful investigation of this subject, found that the positive potentials acquired by different metals under the influence of ultra-violet light are wholly independent of temperature, and furthermore that the discharge of negative electricity from metals or the photoelectric current is independent of temperature. The alkali metals are not among those investigated in this research. Elster and Geitel, J. J. Thomson, and recently J. A. Fleming on the other hand, find that the discharge of negative electricity from potassium and sodium-potassium alloy increases very considerably with increasing temperature. I studied the influence of temperature upon the positive potential acquired by the alloy and found that it does not increase with increasing temperature; there was rather a slight decrease of the deflection, due probably to increasing conductivity of the surface of the glass wall of the photoelectric cell.

This preliminary investigation was carried out in the physical laboratory of the University of Michigan. I wish to express my best thanks to Professor H. S. Carhart for the facilities afforded for this research. My thanks are also due to those students of the university in Ann Arbor, who were always ready to help me in arranging the experiments and in taking the readings.

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[1] R. A. Millikan and G. Winchester, The Influence of Temperature upon Photoelectric Effects in a very high Vacuum, Phil. Mag. [6], vol. 14, p. 188, 1907.

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THE

PHYSICAL REVIEW.

ON THE SECONDARY RADIATION PRODUCED FROM SOLIDS, SOLUTIONS, AND PURE LIQUIDS, BY THE β RAYS OF RADIUM.

By S. J. Allen.

THE secondary radiation, however produced, is of the greatest interest and importance, since it affords a possible means of deciding whether or no the atom can be disintegrated by an external agency. The aim of experimenters has been to accumulate facts which would tend to prove or disprove this important point. In order to account for the production of secondary radiation by γ and Roentgen rays, two theories at the present time are in prominence.

J. J. Thomson considers the wave front of a beam of X-rays to consist of an amount of energy concentrated in bundles, the space in between containing no energy. As these bundles of concentrated energy pass over the atoms of a substance enough of the energy is absorbed to destroy in some of the atoms the equilibrium, and an expulsion of corpuscles results. On this theory the secondary radiation is derived from the bombarded atoms and at the expense of the energy of the primary beam.

I have great difficulty myself in getting a conception of what concentrated bundles of energy mean, and of the mechanism whereby such a system could be propagated; consequently the above theory of secondary radiation does not appear to me very explanatory of the known facts. It certainly does not lead to an explanation in terms of more simple ideas.

On the other hand, Bragg has advanced a totally different theory[1] of γ and X-rays to account for his own experimental results on the secondary radiation, and criticizes quite strongly J. J. Thomson's theory. On his view the secondary rays are derived from the primary beam and not from the bombarded atoms. He assumes the beam of γ or X-rays to be made up of a number of electric doublets travelling at high speed, and each doublet as composed of a negative and positive election of the same charge and mass, and held together by a bond which under ordinary circumstances is strong enough to prevent rupture. When this doublet penetrates among the atoms of a substance they may in some cases be broken up, the positive parts remaining at rest in the substance, while the negative parts are scattered in all directions and form the secondary radiation.

In the first place, in order to make this theory workable, Bragg has to assume the existence of a positive electron of the same mass as the negative, which is a conception that is not at the present time borne out by any experimental evidence. Next, he assumes in the case of γ rays that the doublet is expelled entire from a radioactive atom, whereas in ordinary matter it can be ruptured quite readily. If there is a positive electron of the same mass as the negative, then it must obey, according to all our present views on electrical charged bodies, the same laws that the negative does, the only difference being one of direction. In the determination of the velocity, and ratio e/m, for the positive particle by the electric and magnetic deviations the same formulas are used as in the case of the negative electron, no distinction being made except that the direction of deviation is opposite. It is then very difficult to see how the positive part of such a doublet could behave in any way different from the negative. The positive electron would not be brought to rest after rupture, but would on the average have the same velocity as the negative.

A metal plate struck by a beam of doublets ought therefore to show both on the incident and emergent sides the presence of secondary or scattered positive corpuscles as well as negative. This we know is contrary to the known experimental facts.

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[1] Phil. Mag., December, 1908.

The doublet theory of Bragg, while it may agree closely with his experimental facts, does not seem therefore justifiable on our present views of positive electricity.

The secondary β radiation from solids has been studied by a number of experimenters and a great deal of valuable information has been obtained, but as far as I can learn no work has been done on salt solutions, and pure liquids, a field which ought to be productive of important results.

The present work of the author on solutions and liquids was undertaken to see if the results obtained would agree in any way with those already obtained for the pure elements, and especially to test whether the atomic weight law would hold for such complex systems of atoms as we there have to deal with.

A number of experiments were also made on the radiation from pure metals and powdered metallic salts in order to compare their results with those from solutions and liquids. The experiments on the metals and salts to a great extent parallel those of other investigators, and wherever possible due credit will be given.

Before describing the experiments it may be well to discuss the principal facts in regard to the secondary β radiation.

It was early shown that by far the greater proportion of secondary β rays consisted of electrons traveling with speeds only little inferior to those of the primary. The author[1] showed that the velocities of the secondary rays varied with different materials being greatest for lead. The velocity in the case of lead was measured, and found to be of the order 2×10^{10} cm. per second for the highest speed rays, which is only slightly less than that of the primary electrons. It is also well known that the secondary rays in turn produce tertiary rays of the same nature but a little less velocity.

It has recently been shown by McClelland[2] that the amount of secondary radiation from a given material can be considered as made up of two parts, the first representing what he calls the reflected radiation and the second the true secondary radiation. The reflected part does not vary much with the atomic weight of the bombarded and is considered by McClelland to be some of the primary rays which have bent backward without entering or causing any disintegration of the atom. The true secondary radiation is a function of the atomic weight of the material struck rapidly decreasing with decreasing atomic weights. The amount of secondary rays also varies with the angle of incidence of the primary beam, being least for normal incidence. The true secondary rays are thought by McClelland to be due to a disintegration of the bombarded atoms.

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[1] Phys. Ser., August, 1906.

[2] Phil. Proc., July, 1908.

Crowther[3] has shown that the primary β rays undergo a scattering when they penetrate into a material but that this scattering is complete after a certain distance has been traversed.

There seems to be a great deal of evidence therefore that a portion at least of the secondary rays from solid substances is due to scattering of the primary. Whether the remainder is derived from the atoms of the substance, or from the primary beam, does not appear so conclusive.

We are certain, however, that almost, if not quite all, the total secondary radiation from a solid is of the same nature as the primary. Part at least of the amount from any given pure element seems to be a function of its atomic weight.

Experimental Arrangement.

It has been pointed out by several investigators that the method of measuring the secondary radiation used by the early experimenters was not suitable for obtaining the true relative values of the radiation from different materials, on account of the variation in the velocities of the rays. The lower the velocity the greater the ionizing power of the rays, and consequently the radiation from the atoms of low atomic weight would be exaggerated over that for the high atomic weight ones.

The ionization chamber ought therefore to be such a size that the rays could be completely absorbed before striking the walls. Now such an arrangement would be very difficult to obtain, since it would necessitate a very large ionization chamber, a very high potential to saturate the enclosed air, and a narrow beam of the secondary rays so that they could not strike the side walls of the chamber in their flight.

[1] Phil. Mag., October, 1906.

The best that can be done is to approximate to the ideal arrangement by making the chamber as large as possible consistent with the available conditions of the experiment.

Bragg[1] has sought to overcome this difficulty in the following manner: Two concentric hemispherical shells were used to form the ionization chamber instead of the customary rectangular or cylindrical chambers with a central electrode. The outer hemisphere was connected to an electrometer, while the inner one constructed of very thin aluminum foil was joined to a battery. The radium was placed in a block of lead fastened to the inner side of the lower electrode at the top. The metal plate to be examined for the secondary rays was placed in a horizontal plane at the diameter of the two hemispheres. The beam of β rays fell at nearly normal incidence upon the metal plate, and the secondary rays there produced passed in all directions through the thin aluminum shell into the space between the two hemispheres.

By this arrangement practically all the secondary rays produced get into the ionization chamber, and therefore a much larger amount can be measured than by any other method. Sheets of tinfoil are then placed over the thin aluminum foil and the ionization measured with the increasing thickness until the secondary rays are practically all absorbed. If we assume that the absorption in air follows the same general law as it does in tinfoil the above experimental process would be practically the same as removing the outer hemisphere to greater and greater distances from the inner, since the solid angle would be the same in all cases.

If the readings are therefore plotted in the form of curves, in which the ordinates represent ionization, and the abscissæ thickness of absorbing layer, the area of such curves will represent numbers proportional to the total secondary radiation. The ratios of these areas for various radiating substances will then represent more correctly the relative radiating powers than the old method.

In the present investigations the author has made use of two methods, which will be referred to as method *A* and method *B*.

Method A. — The arrangement of this method is shown in Fig 1, and is almost the same as that used by Bragg, differing only in the position of the radium, and consequently in the angle of incidence of the primary beam.

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[1] Phil. Mag., October, 1908.

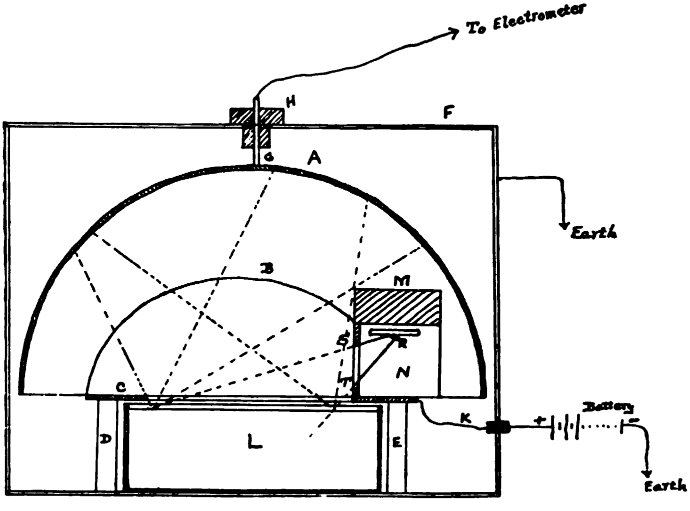

Fig. 1.

It may be briefly described as follows :

A is a hemispherical shell fastened to a metal rod *G* which passes through and is insulated from the metal case *F* by means of the ebonite block *H.* A wire from this hemisphere is joined to the electrometer. Supported on the insulating ebonite pillars *D* and *E* is the circular brass plate *C* which is joined to the battery by means of the wire *K.* Fastened to this plate is a black of wood *N*, which supports the small glass tube *R* containing 5 milligrams of radium bromide. The front of this wooden block is covered with a lead plate in which an opening is cut to allow a well-defined pencil of β rays to emerge. *B* is the inner shell made of thin aluminum foil stretched on a number of fine wires joined to the plate *C.* In the plate *C* is cut an opening which is considerably larger than the area of the beam of β rays at that point, thus insuring that none of the β rays shall strike the sides of this opening. A block of lead *M* is placed over the radium so as to cut down the γ radiation as much as possible.

L represents a cylindrical glass vessel in which is placed the substance to be tested for secondary rays.

The upper surface of the substance, whether solid or liquid, was always placed at the same height, and level, so that all readings would be comparable with each other.

The secondary rays emerge from the substance bombarded in all directions and passing upward through the thin aluminum ionize the air between the hemispheres.

The electrometer used in these experiments was the null reading instrument devised by the author and already described in several papers. The charge of electricity communicated to the quadrants from the shell *A* is balanced by an equal amount of opposite sign furnished by the standard. In the present case the quantity of charge was too great to be balanced by a layer of uranium oxide and so a uniform layer of rather weak radium bromide was used.

A small amount of radium was dissolved in water and a few drops spread on a metal plate, and rubbed over the surface uniformly. After drying the surface of the plate was rubbed over with a fine emery cloth until the amount of radium left was just sufficient for the experiment in hand.

This standard was found to act excellently, and no appreciable variation in its activity was observed, and no trouble was experienced from the very small amount of emanation which may have been liberated.

The balance part of such an instrument is independent of changes in the potential of the electrometer needle, which was a very advantageous feature in the present experiments, since the γ rays of the radium passing through the walls of the testing vessel and through the case of the electrometer discharged the needle quite rapidly. In fact the needle had to be recharged about every 20 minutes.

With a electrometer using the "rate of leak" method great difficulty would have been experienced from this cause.

Method B. — The arrangement of this method is shown in Fig. 2. *A* is a cylinder of zinc 60 cm. long by 15 cm. in diameter. This formed the outer electrode and was joined to one pole of the battery. The central electrode consisted of a brass rod *B* passing through the ebonite insulating block *D*, and joined to the electrometer by a wire as indicated. A guard ring *E* connected to earth was fastened between *A* and *B*.

The bottom of the cylinder was closed with a thin sheet of aluminum. The cylinder was supported on two insulating blocks *H* and *F*, and had its axis inclined at a considerable angle to the vertical.

The tube containing the radium was fastened inside *F* and the rays allowed to emerge in the same manner as already described in method *A*. The size of the beam and the angle of incidence was the same as in method *A*.

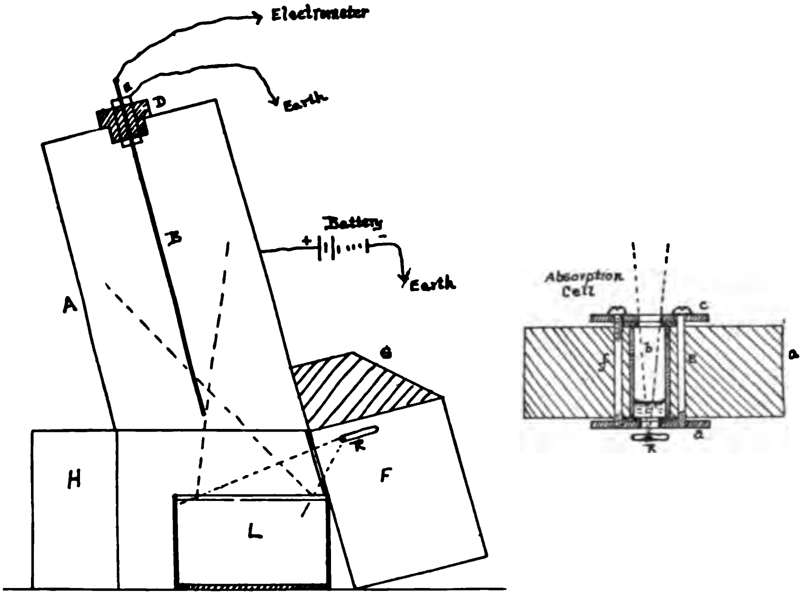

Fig. 2.

A block of lead *G* was placed over the radium so as to cut down the γ rays as much as possible.

The method of operation was precisely the same as in method *A*.

Radiation from Pure Metals.

These experiments to a certain extent parallel those of McClelland and Bragg, but it was thought advisable to perform them in order to be able to compare their results with those of the author, and so obtain a check on the accuracy of the various results. The department was in the fortunate possession of a number of thick plates of pure metals which were at my disposal.

The plates were placed directly under the opening in the brass plate *C* (Fig. 1) and always at a fixed distance away from it. The area of the plates with the exception of the gold, platinum and palladium, was greater than the area of the incident beam of β rays. In all cases the thickness of the plates was more than sufficient to absorb all the incident rays.

In order to make the readings for the gold, platinum and palladium plates comparable with those for the other metals, a plate of lead of the same area was tested; and by means of this reading and that of the large lead plate, the readings for the gold, platinum and palladium could be corrected for their smaller area. On this account the results for these metals are perhaps not quite so accurate as the others.

The ionization between the two hemispherical shells is due to γ radiation, secondary rays caused by the γ rays, secondary β rays from the metal plates, and possibly a small amount of secondary β rays produced where the β rays strike the edges of the opening in the lead plate. In fact it is impossible to narrow down a beam of β rays without introducing at the same time a small amount of secondary rays.

Two readings were taken: (*a*) with a sheet of lead placed over the opening *ST* of sufficient thickness to cut off the β radiation, (*b*) without this sheet of lead.

The difference ($b - a$) between these two readings was taken as a measure of the secondary rays from the plate under test.

A sheet of tinfoil was then placed over the aluminum foil hemisphere and the readings (*a* and *b*) again taken.

This operation was repeated until the secondary rays were practically all absorbed by the tinfoil. The reading (*a*) decreased only a very little as successive layers of tinfoil were added.

The results of these experiments are shown in Table I. The reading for lead is taken arbitrarily as 1,000 and the others calculated in terms of it.

The third column represents the radiation through the thin aluminum foil which absorbed extremely little of the radiation. The next six columns represent the radiation through the various thicknesses of tinfoil.

If we examine the second and third columns we see that, with the exception of palladium, the radiation follows the order of the atomic weight of the struck substance. The values for palladium and silver are very close to one another, and the difference is probably due to experimental errors, since as already stated the area of the palladium plate was less than that of the silver.

If the results are plotted, having as ordinates the radiations and as abscissæ the atomic weights, the resulting curve is fairly regular but it is far from being a straight line, the radiations for the low atomic weight atoms being much greater in proportion than those from the high atomic weight atoms.

When compared with the results of Bragg, it is seen that his results for the light atoms are much less relatively to the high ones than are those of the author. In Bragg's experiments the primary rays fell upon the plate at nearly normal incidence, whereas in the author's arrangement the angle of incidence was between 70° and 80°.

Table I.

Substance.	Atomic Weight.	Radiation Uncovered.	adiation .00105 cm.	Radiation .00210 cm.	Radiation .00315 cm.	Radiation .00725 cm.	Radiation .0155 cm.	Radiation .0237 cm.	Area of Curve.
Lead	207	1,000	760	570	460	191	57	27	1,000
Bismuth	209	1,005							
Gold	197	970	733	557	451	189	56	27	954
Mercury	200	990	755	570	460	191	57	27	985
Platinum	195	966							
Antimony	120	895							
Cadmium	112	870							
Silver	108	835	600	435	347	143	37	18	725
Palladium	107	839	620	446	363	160	37	18	740
Zinc	65	742	500	373	310	114	30	14	600
Copper	63	732	484	361	298	110	29	14	593
Iron	56	660	440	335	260	93	24	12	464
Sulphur	32	578							
Aluminum	27	500	343	245	190	64	13	6	353
Carbon	12	347	220	156	110	43	8		220

Now the work of McClelland has shown that when the angle of incidence was increased the ratio between the radiation for a light atom and a heavy one was much reduced. For example, at 75°

incidence the radiation given by him for lead was 58, while that for aluminum was 27, a ratio of $\frac{27}{58} = .466$. The ratio in the case of the author's results is .500.

It thus seems very probable that the total secondary radiation from a plate struck by β rays depends to a great extent upon the angle of incidence of the β rays, as stated by McClelland.

It is also clear from the author's results that the secondary radiation increases with the atomic weight.

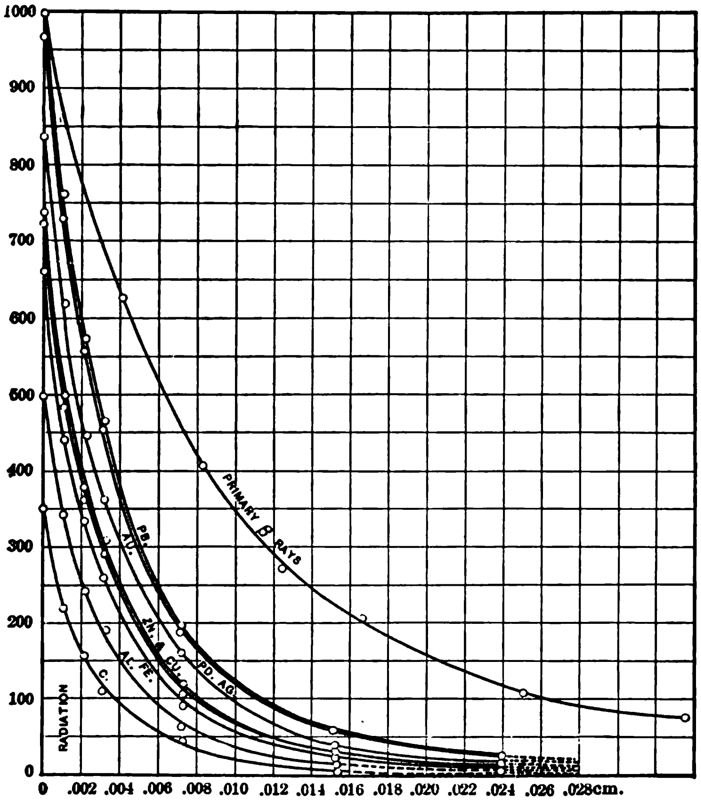

Fig. 3.

In Fig. 3 the results from the above table are plotted in the form of curves, the ordinates representing the radiation and the abscissæ the thickness of the tinfoil. The absorption curve for the lead is not very far different from the values given by Bragg for the same metal, but when we come to examine those for the lighter atoms we find a great difference.

In Bragg's results the radiation from aluminum, for example, decreases very much more rapidly than it does in the results of the author.

It is thus evident that when a large angle of incidence is used the average penetrating power of the rays from substances of low atomic weight is much greater than when the angle of incidence is normal.

If we we examine the figures in columns 8 and 9 of Table I. it is seen that after the rays have gone through .0155 cm. of tinfoil the remaining portion is of about the same penetrating power for all the substances, being cut to half value in .008 cm. of tinfoil. All the substances observed give off a small proportion of rays of the same penetrating power.

In the last column of Table I. are expressed the relative areas of the surfaces below the absorption curves of Fig. 3.

These areas were obtained by means of a planimeter. The figures thus obtained represent the total secondary radiation of the various substances relative to that of lead, and they decrease much more rapidly with decrease of atomic weight than those in column 3.

This is in agreement with the results of Bragg, and show that the old method of using a shallow ionization chamber does not give true relative values for the secondary rays.

RADIATION FROM POWDERED SALTS.

A short series of tests were made on powdered salts, to see if the radiation bore any relation to the molecular weights of the salts. If the radiation from a pure element is a function of the atomic weight, we should expect as a first approximation the radiation of a salt to be a function of the sum of the atomic weights of its chemical constituents, that is, of its molecular weight.

The salts were powdered very fine in a mortar, and then placed in a shallow cardboard box, the surface of the salt being pressed down as smooth as possible by a brass weight. The area of the salt was the same as the metal plates in the previous experiments and the method of testing the same.

The results are represented in Table II., the readings being expressed in terms of lead equal to 1,000.

An examination of this table shows that the radiation from powdered salts does not follow in general the order of the molecular weights. Lead carbonate has a greater radiation than lead nitrate although the molecular weight is less. Potassium chloride also has a greater radiation than its molecular weight would warrant. The same is true of lithium and sodium chlorides. The effect of an atom of high atomic weight can, however, be readily seen by an increase in the radiation. Lithium chloride has a greater radiation than lithium carbonate though of much less molecular weight.

This is perhaps due to the presence of the chlorine atoms giving more radiation than the carbonate.

Table II.

Substance.	Chemical Formula.	Molecular Weight.	Intensity of Radiation. Pb = 1,000.
Lead carbonate	$PbCO_3$	327	954
Lead nitrate	$Pb(NO_3)_2$	331	892
Lead acetate	$Pb(C_2H_3O_2)_2$	325	885
Mercuric chloride	$HgCl_2$	271	878
Barium chloride	$BaCl_2$	208	754
Strontium chloride	$SrCl_2$	159	606
Copper chloride	$CuCl_2$	134	576
Manganese chloride	$MnCl_2$	126	500
Potassium chloride	KCl	75	555
Calcium chloride	$CaCl_2$	111	480
Sodium chloride	NaCl	59	500
Lithium chloride	LiCl	43	490
Lithium carbonate	Li_2CO_3	74	362
Sugar (cane)	$C_{12}H_{22}O_{11}$	342	330

Let us examine the bichlorides more in detail, arranging them in the following table.

Substance.	Molecular Wt.	Radiation.	
		Total.	Total—Cl_2.
$Hg+Cl_2$	200+71	878	638
Ba + "	137+ "	754	514
Sr + "	88+ "	606	366
Cu + "	63+ "	576	336
Mn+ "	55+ "	500	260
Ca + "	40+ "	480	240

All these salts have the same amount of chloride which we will assume to give a constant radiation. The difference between this constant radiation and the total radiation for any salt ought to be due to the presence of metal. We have no means of knowing just what this constant amount would be, but in the case of calcium chloride I have assumed it to be half the total, which is probably somewhere near the truth, since the atomic weights of calcium and chloride are nearly equal. In the last column are represented these differences and they show, if the above reasoning is true, that the radiation due to the presence of a metal in a bichloride salt is a function of the atomic weight of the metal. The same conclusion may possibly hold for other groups of metallic salts, such as the carbonates, nitrates, sulphates, etc.

When we compare, however, the salts regardless of their chemical grouping and consider only the molecular weights we do not get results which are very consistent. Cane sugar although having a high molecular weight has only a low radiation.

Radiation from Aqueous Solution of Salts.

In these experiments the salts were dissolved in distilled water, the solutions being made up to so many grams of salt per 1,000 grams of water. The number of salts which are readily soluble in water is not great, and the number of tests made were therefore somewhat limited, since for many of the salts in order to get a fairly large radiation the concentration had to be great.

The solution to be tested was poured into the glass cylindrical cell described above, the surface of the liquid always being at the same level and depth.

The reading for the pure water was taken frequently during a series of tests on the solutions, so as to make correction in case of any changes. The solutions were carried in all cases to saturation.

The results are shown in Table III. Under concentration we have three columns, *a*, *b*, *c*. In column *a* the concentration is expressed in grams of dissolved salt per 1,000 grams of water, in column *b* the same divided by the molecular weight of the salt, in column *c* the values of column *b* corrected for the valency of the metal part of the salt. The radiations are expressed both in terms of lead as 1,000 and in term of water as 100. When a salt is dissolved in water there is an increase of volume of the solution, and therefore the amount of water in the fixed volume of the glass testing cell will get less and less as the concentration is increased. The increase of radiation due to a solid substance dissolved in water will consequently always be a differential effect of the increase caused by the salt and the decrease caused by the water displaced. If by chance the molecules of a liquid dissolved in water should have the same specific radiation as the molecules of the water, then we would expect the total radiation from the solution to remain constant, since the increase due to the dissolved liquid would be just balanced by the decrease due to the water displaced. That is, a cubic centimeter of the solution would always contain the same number of molecules of both kinds.

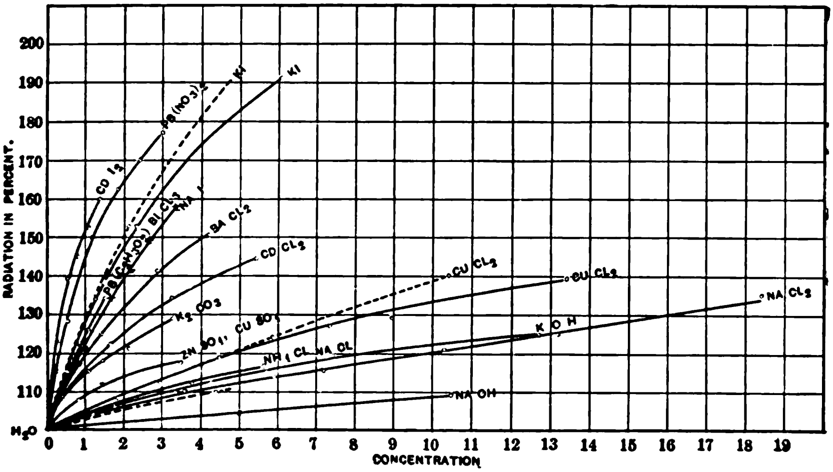

Fig. 4.

The results given in Table III. are plotted in the form of curves in Fig. 4, the ordinates representing the total radiation of the solutions, with water alone taken as 100 per cent., and the abscissæ the concentration in gram molecular weights per 1,000 grams of water. In the figures here given pure water has a radiation of 320 in terms of lead taken as 1,000.

If now we examine these curves in detail we can I think bring out the following facts:

Table III.

Substance.	**Chemical Formula.**	**Concentration.**			**Radiation Pb=1,000.**		**Radiation Water=100.**	
		a	*b*	*c*	**Salt Alone.**	**Total.**	**Salt Alone.**	**Total.**
Lead nitrate	$Pb(NO_3)_2$	100	.30	.60	88	408	28	128
" "	(331)	200	.60	1.20	156	476	50	150
" "		295	.90	1.80	200	520	62	162
" "		395	1.20	2.40	229	549	70	170
" "		495	1.50	3.00	247	567	77	177
Lead acetate	$Pb(C_2H_3O_2)_2$	96	.30	.60	47	367	15	115
" "	(325)	161	.50	1.00	67	387	21	121
" "		240	.74	1.48	108	428	34	134
Mercuric	$HgCl_2$	22	.80	.16	24	344	7	107
chloride	(271)	45	.167	.334	49	369	15	115
Bismuth tri-	$BiCl_3$	34	.11	.33	32	352	10	110
chloride	(316)	69	.22	.66	63	383	20	120
Potassium	KI	62	.40	.40	42	362	13	113
iodide	(166)	126	.80	.80	71	391	22	122
"		214	1.30	1.30	109	429	34	134
"		380	2.30	2.30	170	490	53	153
"		1,000	6.06	6.06	290	610	91	191
Sodium iodide	NaI	75	.50	.50	30	350	9	109
" "	(150)	125	.83	.83	56	376	18	118
" "		166	1.11	1.11	83	403	26	126
" "		250	1.67	1.67	109	429	34	134
" "		333	2.22	2.22	132	452	41	141
" "		400	2.67	2.67	158	478	49	149
" "		500	3.34	3.34	185	505	58	158
Cadmium	CdI_2	50	.14	.28	74	394	23	123
iodide	(366)	100	.28	.56	121	441	39	139
"		150	.41	.82	144	464	45	145
"		200	.54	1.08	168	488	52	152
"		250	.68	1.36	190	510	60	160
Cadmium	$CdCl_2$	66	.36	.72	38	358	12	112
chloride	(183)	150	.82	1.64	64	389	22	122
"		300	1.64	3.28	109	429	34	134
"		500	2.73	5.46	142	462	44	144
Barium chloride	$BaCl_2$	50	.24	.48	27	347	9	109
" "	(208)	150	.72	1.44	79	399	25	125
" "		300	1.44	2.88	133	453	41	141
" "		436	2.09	4.18	159	479	50	150
Copper chloride	$CuCl_2$	200	1.48	2.96	38	358	12	112
" "	(134)	300	2.24	4.48	60	380	19	119
" "		400	2.96	5.92	73	393	23	123
" "		500	3.68	7.36	83	403	27	127
" "		600	4.48	8.96	91	411	29	129
" "		900	6.72	13.44	125	445	39	139

TABLE III.—*Continued.*

Substance.	Chemical Formula.	Concentration.			Radiation Pb=1,000.		Radiation Water=100.	
		a	*b*	*c*	Salt Alone.	Total.	Salt Alone.	Total.
Calcium	$Cacl_2$	245	2.20	4.40	30	350	10	110
chloride	(111)	400	3.60	7.20	47	368	15	115
"		570	5.13	10.26	67	387	21	121
"		730	6.60	13.20	80	400	25	125
"		1,020	9.20	18.40	112	432	35	135
Sodium	Nacl	100	1.69	1.69	18	338	6	106
chloride	(59)	200	3.38	3.38	33	353	10	110
"		255	4.32	4.32	40	360	12	112
"		350	5.93	5.93	52	372	16	116
Ammonium	NH_4Cl	100	1.88	1.88	26	346	8	108
chloride	(53)	200	3.77	3.77	40	360	12	112
"		300	5.65	5.65	53	373	16	116
Potassium	K_2CO_3	100	.73	.37	26	346	8	108
carbonate	(138)	200	1.45	.73	40	360	12	112
"		300	2.18	1.09	50	370	15	115
"		400	2.90	1.45	58	378	18	118
"		600	4.36	2.18	66	386	21	121
"		900	6.54	3.27	91	411	29	129
Potassium	KOH	200	3.6	3.6	30	350	10	110
hydrate	(56)	420	7.5	7.5	63	383	20	120
"		720	12.8	12.8	80	400	25	125
Sodium	NaOH	200	5.0	5.0	12	332	4	104
hydrate	(40)	420	10.5	10.5	27	347	9	109
Zinc sulphate	$ZnSO_4$	140	.87	.87	25	345	8	108
" "	(161)	233	1.45	1.45	40	360	12	112
" "		350	2.16	3.50	54	374	17	117
Copper	$CuSO_4$	225	1.41	1.41	34	354	11	111
sulphate	(159)							

For those solutions which contain only atoms of low atomic weight the increase of radiation due to solution is nearly proportional to the concentration, even for high concentrations. In the case of solutions containing high atomic weight atoms the radiation is approximately proportional to the concentration for low concentrations, but as the concentrations increase the curves depart more and more from a straight line and seem to be approaching a limiting or saturating value. This is quite noticeable in the curves for lead nitrate and potassium iodide, which are very soluble in water.

If these results are plotted in terms of normal solutions on a volume basis the curves approach even more to a straight line. For example, potassium iodide and copper chloride are so plotted in the dotted curves in Fig. 4. Copper chloride is now approximately a straight line. Now copper chloride in the solid form (Table II.) has a radiation of 180 in terms of water 100, and 140 for a concentration equal to 10 normal. The radiation of the solution is thus seen to be approaching very rapidly to the limiting value of the solid salt, which would seem to indicate that in solution the molecules may give more specific radiation than in the solid state.

When a salt contains an atom of high atomic weight it can be readily seen from the above results that the radiation is greatly increased, and, other things being equal, that the radiation follows the order of the atomic weight.

In order to bring this fact out clearly let us consider the bichlorides in the same manner as was done in the case of the powdered salts. The values for these are expressed in the following table.

Substance.	Molecular Weight.	Radiation for Concentration = 4.		Radiation (Atomic Wt.)$_2$.
		Total.	Total—Cl$_2$.	
Ba + Cl$_2$	137 + 71	50	43	.00228
Cd + "	112 + "	38	31	.00248
Cu + "	63 + "	17	10	.00252
Ca + "	40 + "	11	4	.00250
Hg+ "	200 + "	X	105 (extrapolated)	.00262

In this table, the radiation for each salt is taken for equal concentration (in this case 4) and the radiation for pure water subtracted from it. The remaining radiation will be that due to the dissolved salt, less a small amount due to the water displaced.

It is readily seen from the figures in the fourth column of the above table that the radiation from the metal part of the salt increases very rapidly with the atomic weight of the metal. If this radiation is divided by the square of the atomic weights the result in each case is nearly a constant, as is shown in the last column. The greatest concentration obtainable with mercuric chloride was only .344 and therefore it cannot be compared directly with the others for a concentration of 4. I have however compared its radiation at a concentration of .344 with that of barium chloride at the same concentration. From these two values the radiation of mercuric chloride at a concentration of 4 can be extrapolated and compared with the other bichlorides to a rough degree of approximation. This is shown in the last set of figures at the bottom of the table. The ratio of radiation divided by the square of the atomic weight in the case of mercuric chloride is thus in general agreement with the other chlorides. Whether this result is accidental, or really means something, cannot be decided with any degree of certainty as yet, but at any rate it can be definitely stated. I think that the radiation from the metal part of the salt (positive ion) does increase very rapidly with the atomic weight.

In the case of iodides the radiation also increases rapidly with the atomic weight of the positive ion; for a concentration = 1 the figures are, sodium iodide 22, potassium iodide 28, and cadmium iodide 52. In these the greater part of the radiation is undoubtedly due to the presence of the iodine. This fact can be shown by comparing the results for cadmium iodide and cadmium chloride. For equal molecular concentration both these salts contain the same amount of cadmium. The radiation for cadmium iodide for a concentration = 1.35 is 60, and for cadmium chloride for the same concentration, 19.

I have calculated the relative radiation due to the cadmium alone by taking the same proportional value as was used in the above table of bichlorides, and then subtracted this value from the total. The remainder in the case of cadmium chloride is 3.5, while for cadmium iodide it is 44.5. The ratio radiation/(atomic wt.)^2 for cadmium chloride is .00278, and for cadmium iodide .00276.

This interesting result is in agreement with that of the bichlorides and the ratio is of that same order.

We thus see that in all cases the presence of atoms of high atomic weight is responsible for a large radiation, and moreover, in a group of salts which have a common constituent of the same amount, the radiation due to the other constituent increases very rapidly with the atomic weight, being in the cases considered nearly proportional to the square.

If we have two salts which have a common constituent and the other constituent in both of nearly the same atomic weight we would expect the radiations for equal concentrations to be practically the same. Copper sulphate and zinc sulphate are of this form, and we see from Table III. that for equal concentrations the radiations in the two cases are nearly equal, that of zinc sulphate being if anything a little greater. Copper sulphate could not be dissolved to a very high concentration, and therefore the figures for it are somewhat approximate.

For equal concentrations potassium hydrate gives a radiation which is much greater than that of sodium hydrate.

In the cases of lead nitrate the radical $(NO)_3$ seems to be a much more efficient radiator than the radical $(C_2H_3O_2)$, assuming that in each the lead gives the same amount of radiation.

The fact that in these salt solutions the various ions are in a very different degree of dissociation does not seem to have any influence on the general results.

Cane sugar, acetic acid, glycerine, and tartaric acid crystals, dissolved in water give very interesting results.

No change in the radiation from that of pure water could be detected for any concentration. Now cane-sugar and glycerine have a specific radiation the same as that of water, acetic acid a radiation slightly less, while tartaric acid was not measured, but is probably of the same order as water.

This result might be explained in the case of a liquid such as glycerine, acetic acid, by the fact that in any proportion whatever of the liquid and water the total number of parts of both kinds remains the same, and therefore also the total radiation, since the specific radiation in each is the same. In other words the increase of radiation due to the dissolved substance is just balanced by the decrease due to the solvent displaced.

This explanation however does not seem to fit in with the case of sugar and tartaric acid, as these substances are in the solid state to begin with, and when dissolved in water the number of both kinds in a given volume of the solution is continually increasing.

The molecular weights of sugar and water are widely different, and yet the radiations are the same, which shows that nothing can be based on mere molecular weights, even when these molecular weights are made up of the same atoms. This fact will be brought out very clearly when we come to examine the results for the pure liquids in the next section.

RADIATION FROM PURE LIQUIDS.

For this series of experiments apparatus *B* was used in place of apparatus *A*. A preliminary set of readings for the radiation from metals was first tried to see how close the readings of one apparatus agreed with those of the other. The readings for the various metals by apparatus *B* relative to lead were always a little lower than by apparatus *A*. Aluminum, for example, gave with apparatus *B* a reading of 480 relative to lead as 1,000. The reading by apparatus *A* from Table I. is 500. The readings by apparatus *B* are therefore a little nearer to the true values for the secondary radiation, but the difference is not very great.

Over 50 liquids in all were tested, nearly all of them being organic. Over 20 of these were in sufficient quantity to test in the same cylindrical cell as was used for the aqueous solutions. The remainder of the liquids being very expensive and not present in the chemical department of the University in large quantities, a very much shallower cell had to be used. The surface of the liquid in this was the same as in the large cell but the depth was only 8 mm. The readings for the deep cell are therefore more correct than those for the shallow cell. Whenever possible readings were made with the liquid in both cells.

The observations were made in exactly the same manner as in case of the aqueous solutions of the salts. Pure water was taken as the standard and its radiation put equal to 100, the other liquids being calculated in terms of it. On this same scale the radiation for lead was 340 for the deep cell and 360 for the shallow one.

The results for all the liquids tested are shown in Table IV. The first column represents the name of the liquid these being arranged according to their chemical grouping. The second column gives the chemical formulæ of the liquids. The third column expresses the molecular weights, and the fourth the densities. In the fifth column are placed the values for the secondary radiation from the liquids using the deep cell, and in the sixth column the same using the shallow cell.

TABLE IV.

Liquid.	Chemical Formula.	Molecular Wt.	Density.	Secondary Radiation Water = 100.		Relative Absorption of β Rays Water = 100.
				Thick Layer.	Thin Layer.	
Water Group—						
Water	H_2O	18	1.000	100	100	100
Methyl alcohol	CH_3OH	32	.790	74	80	70
Ethyl "	C_2H_5OH	46	.800	74	80	70
Amyl "	$C_5H_{11}OH$	88	.812	74	80	70
Allyl "	C_3H_5OH	58	.858	125	—	—
Tertiary butyl alcohol	C_4H_9OH	74	.786		78	70
Iso " "	C_4H_9OH	74	.803		77	74
Ether	$(C_2H_5)_2OH$	74	.736	75	—	—
Acetic anhydride	$(C_2H_3O)_2OH$	102	1.073	74	—	74
Acid Group—						
Formic acid	CH_2O_2	46	1.227	131	—	—
Acetic "	$C_2H_4O_2$	60	1.057	89	—	87
Valeric "	$C_4H_9(COOH)$	102	.958	122	—	—
Propionic acid	$C_3H_5(CO_2H)$	74	.996	—	81	100
Butyric "	$C_3H_7(CO_2H)$	88	.962	—	81	100
Caprylic "	$C_7H_{15}(CO_2H)$	144	.929	—	81	80
Ester Group—						
Amyl ester	$CH_3(CO_2C_5H_{11})$	130	.857	75	—	—
Formic "	$H(CO_2C_2H_5)$	74	.936	—	77	62
Acetic "	$CH_3(CO_2C_2H_5)$	88	.905	—	77	70
Propionic ester	$C_2H_5(CO_2C_2H_5)$	102	—	—	77	70
Methyl acetate	$CH_3CO_2CH_3$	74	.956	—	77	71
Iso butyric ester	$C_4H_9COOC_2H_5$	116	.890	—	77	80
Hydrocarbons—						
Benzene	C_6H_6	78	.879	74	—	70
Toluene	$C_6H_5CH_3$	92	.822	74	—	70
Ethyl benzene	$C_6H_5C_2H_5$	106	.866	—	80	86
Amylene	C_5H_{10}	70	—	—	71	45
Halide Group—						
Methyl iodide	CH_3I	142	2.199	280	329	285
Ethyl "	C_2H_5I	156	1.944	—	326	295
Amyl bromide	$C_7H_{11}Br$	151	1.543	170	197	175
Brom benzene	C_6H_5Br	157	1.495	198	196	196
Chlor "	C_6H_5Cl	113	1.106	100	—	101
Propyl chloride	C_3H_7Cl	79	.916	—	80	65
" bromide	C_3H_7Br	123	—	—	230	141
Tertiary butyl bromide	C_4H_9Br	137	—	—	189	118
Iso amyl "	$C_5H_{11}Br$	151	1.666	—	276	280
Iod benzene	C_6H_5I	204	1.833	—	294	330

TABLE IV. —*Continued.*

Liquid.	Chemical Formula.	Molecular Wt.	Density.	Secondary Radiation Water = 100.		Relative Absorption of β Rays Water = 100.
				Thick Layer.	Thin Layer.	
Halide Group—						
Ethylene chloride	$C_2H_4Cl_2$	99	1.280	—	284	400
" bromide	$C_2H_4Br_2$	188	2.178	—	245	140
Chloroform	$CHCl_3$	119	1.526	172	190	175
Carbon tetrachloride	CCl_4	154	1.630	172	190	175
Tetra brom acetelene	C_2Br_4	344		—	290	560
Amine Group—						
Aniline	$C_6H_5NH_2$	93	1.022	74	—	88
Methyl aniline	$C_6H_4NHCH_3$	107	.976	79	—	88
Di methyl aniline	$C_6H_4N(CH_3)_2$	121	.955	78	80	—
Ortho chlor aniline	$C_6H_4NH_2Cl$	128	1.234	101	—	100
Meta chlor aniline	$C_6H_4NH_2Cl$	128	1.243	103	—	100
Mono ethyl "	$C_6H_5NHC_2H_5$	121	.951	—	77	88
Meta brom "	$C_6H_4NH_2Br$	172	—	—	189	230
Di methyl toluidine	$C_6H_4CH_3N(CH_3)_2$		--	—	77	88
" " amine	$(CH_3)_2NH$	45	—	—	77	71
Ortho toluidine	$C_6H_4CH_3NH_2$	107	1.003	—	77	—

Let us now examine these results in detail taking them by groups.

Water Group. — In this group besides water we have a number of alcohols, ether, and acetic anhydride. The secondary radiation from these, with the exception of allyl alcohol, is the same and about 25 per cent. less than that of water. The results for the shallow cell, as can be seen from the table, give values, which are higher relatively to water than those for the deep cell. This result is common to all the liquids and is probably caused by the different arrangement of the liquid in the two cases. It is, however, seen that if two liquids have the same radiation in one cell they also have equal radiation in the other, and therefore comparisons can be made of the various liquids without introducing any appreciable error. In fact one can calculate very approximately from the values obtained by the shallow cell what the radiation would be from the deep cell.

In this group water has a much larger value than one would expect judging by the atomic weight of its atoms. The hydrogen atoms cannot of themselves give a very great radiation, at least on any atomic hypothesis of secondary radiation, and the large radiation given by water must therefore be due to the oxygen atoms, or to a special arrangement of these atoms.

Acetic anhydride has a much greater molecular weight than water, and contains more atoms of higher atomic weight, and yet the radiation from it is much less than that of water. We see at once from this that the molecular weight of the molecules of the liquid has in general no deciding effect on the amount of radiation given off. Even the substitution of an atom of higher atomic weight does not always result in a higher radiation, though it does in general, other things being equal.

This very interesting phenomenon will be much more markedly brought out in some of the other groups.

Density of the liquid does not have any apparent effect in general on the radiation, the density of acetic anhydride for example being greater than that of the water.

Allyl alcohol forms a remarkable exception on this group giving a radiation which is 25 per cent. greater than that of water and 66 per cent. greater than that of the other alcohols. Its molecular weight is less for example than that of amyl alcohol, it has a smaller number of carbon and hydrogen atoms, and yet a much greater radiation. What is the cause then of this anomalous result? Allyl alcohol has what chemists term a double bonding of the carbon atoms and this may have some very marked effect on the secondary radiation. This is the first indication I know of that the interatomic relations can have any effect on the secondary radiation, the phenomena being due, according to the present theories, to a subatomic action. Before discussing this further it will be better I think to examine the other groups.

Acid Group. — In this group are a number of organic acids. Propionic, butyric and caprylic acids show nothing unusual and have about the same radiation as the alcohols, though the number of atoms differ largely. Acetic acid has a higher radiation than alcohol, but still not quite as great as water.

The two great exceptions occur in the case of valeric and formic acids, the radiation in these exceeding that of water by a considerable amount. Formic acid which has the greatest radiation has the least number of atoms. In this group also the chemical arrangement of the molecules must have a considerable effect on the radiation.

Ester Group. — The esters all have about the same radiation as alcohol, and show no unusual features. Here again the molecular weights vary between wide limits without causing any change in the radiation.

Hydrocarbons. — Benzene, toluene, nitro benzene and ethyl benzene have the same radiation as alcohol. Here we have a very simple arrangement of the carbon and hydrogen atoms, and yet the radiation is the same as for the more complicated. The addition of a number of oxygen atoms to the carbon and hydrogen does not therefore in many cases add any more radiation.

Amylene is an exception to this group and its radiation is the lowest of any of the liquids yet examined, the value being about 65 in terms of water 100.

Amine Group. — This consists for the most part of aniline and its various derivatives of the form of ammonia compounds. Aniline methyl aniline, dimethyl aniline, mono ethyl aniline, dimethyl, toluidine, dimethyl amine and ortho toluidine have all the same radiation as alcohol. The presence of a nitrogen atom has apparently not changed the amount of the radiation.

Ortho chlor aniline and meta chlor aniline have a greater radiation than that of alcohol, being about the same as water. This increased radiation is probably caused by the presence of the chlorine.

Meta brom aniline shows very clearly by its large increased radiation the presence of the bromine atom.

Its radiation is over double that of aniline.

One atom then of high atomic weight may be in many cases of more importance in producing secondary radiation than a greater number of atoms of lower atomic weight.

Halide Group. — This is the most interesting group of all, and includes those liquids which contain the highest atomic weight atoms and have the greatest density. The liquids are all compounds of carbon and hydrogen in various proportions, and chlorine, bromine and iodine.

In general, as can be seen from the values given in the tables, the radiation follows the orders of the atomic weight of the chlorine, bromine and iodine constituents, but these are several remarkable exceptions.

Let us consider first those which contain chlorine. Propyl chloride has the lowest radiation of any, about the same as alcohol. Chlor benzene is next in order, having a radiation about the same as water. Both these contain the same number of chlorine atoms and their only difference is in the amount of carbon present.

Chloroform and carbon tetrachloride have the same radiation, which is nearly double that of water. The adding of ∞ chlorine atoms does not always make a difference in the radiation.

Ethylene chloride is the most remarkable one in this group, the radiation being nearly three times as great as water. Comparing it with propyl chloride we see that the amount of carbon and hydrogen is less and the chlorine just double. At first sight we might think that the increased radiation is due to the extra amount of chlorine present, but then chloroform and carbon tetrachloride contain even more chlorine and have a yet smaller radiation. The large radiation cannot therefore be due entirely to the chlorine, and it certainly is not caused by the carbon and hydrogen.

Before discussing this exception further let us look at the results for the iodides and bromides. The three iodides, ethyl, methyl and benzene, all have a very high radiation, the greatest yet measured for any liquid. Methyl iodide has the largest radiation of the three, being over 90 per cent. that of lead, and greater than the value of antimony in Table I. In fact, if we consider that the radiation here is almost entirely due to the iodine present, the value of this radiation will fall very close to that for pure iodine from Table I. These three liquids contain each the same amount of iodine and exhibit no anomalous effects.

The radiation from the bromides is intermediate in value between that of the chlorides and that of the iodides, though tetra brom acetelene has almost as great a value as iod benzene. The results for the bromides are almost as irregular as those for the chlorides. In no case, however, is the radiation very low.

Comparing propyl bromide with propyl chloride, we see that the radiation in the case of the former is nearly three times as great as that of the latter. In both the carbon and hydrogen is the same, but the substitution of a bromine atom for a chlorine one causes the radiation to increase very rapidly. On the other hand ethylene bromide has a smaller radiation than ethylene chloride. Here the effect of substituting the bromine for the chlorine is to decrease the radiation.

This remarkable and totally unexpected result, together with the others of similar nature, in this and the other groups, lead very clearly I think to the conclusion that there is present in some of these liquids phenomenona which to a great extent control the secondary radiation, and which cannot be explained entirely on any simple atomic theory. It seems more likely to depend to a large extent on the molecular arrangement of the constituent parts of the liquids. Hitherto we have been accustomed to consider the secondary radiation as being controlled entirely by the atom of the bombarded substance, but in view of the above results we are forced to conclude that the molecule or groups of associated molecules have also very large influences. Many of the above liquids are believed by chemists, from experiments on the freezing and boiling points, to be in a state of a molecular association, but unfortunately no definite information can be had which is of much use to the physicist. Therefore nothing definite can be said of the association of those liquids in Table V. which show anomalous effects. It is difficult for me to see how they can be explained entirely on any subatomic theory such as has been advanced hitherto.

TABLE V.

Substance.	Radiation in Per Cent.			
	Uncovered.	.0011 cm. Foil.	.0021 cm. Foil.	.0042 cm. Foil.
Lead	100	77	60	35
Copper	100	75	59	34
Carbon	100	73	54	—
Chloroform	100	—	68	40
Water	100	—	70	42
Benzene	100	—	70	47
Lead nitrate	100		67	36
Potassium iodide	100		66	37
Copper chloride	100		69	39

It must not be forgotten that in this entire research only one angle of incidence of the primary rays has been used. It is possible that like McClelland's results for metals,[1] there is present in the total secondary radiation from liquids two constituents varying in individual values with the angle of incidence. One of these constituents might depend entirely upon the atomic weight of the atoms, while the other would be independent of atomic weight but be controlled by molecular arrangement.

In the author's experimental arrangement the latter constituent might be at a maximum, and in some liquids mark or overcome the other.

Before this point can be considered with any degree of certainty experiments at different angles of incidence will have to be made. It is the intention of the author to carry these out as soon as possible.

I do not think, however, that the angle of incidence will be able to account entirely for such an extreme change as takes place in the ethylene chloride.

In looking over the results in the above table one is struck with the great number of compounds of carbon, hydrogen, oxygen and nitrogen in all proportions which has the same radiation. The only exceptions observed are allyl alcohol, formic and valeric acids. In most cases then the radiation from a liquid composed of carbon, hydrogen, oxygen and nitrogen is independent of the number of atoms present, and has a constant value, about 75 per cent. of that of water.

ABSORPTION OF THE SECONDARY RAYS BY TINFOIL.

In order to determine whether the secondary rays from solids, solutions and pure liquids differed much in velocity, a short series of tests on their absorption by tinfoil were made.

Apparatus *B* was used in the same manner as already described. Sheets of tinfoil were interposed in the path of the rays and the decreased ionization measured. From these values, and the reading without any tinfoil, the absorption in per cent. was calculated in the usual manner. The values so obtained will serve very well to compare the average penetrating power of the different rays.

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[1] *Loc. cit.*

The results are expressed in Table V. The values for the three metals — lead, copper and carbon — are of the same nature as obtained by apparatus *A*, the penetrating power of the rays decreasing slowly with decrease in atomic weight.

In the case of pure liquids the results are quite different, the penetrating power of the rays not differing much for the three liquids observed and being in all cases greater that that of the metals. The velocity of the rays from pure liquids thus seems to be on the average greater than from pure metals, and the lighter the atom is the greater is this velocity, though the difference is very small.

The solutions — lead nitrate, potassium iodide and copper chloride — show similar results to the liquids, though not quite to so marked an extent.

Since the velocities of the rays from the various liquids and solutions do not apparently differ very much in value, the readings which have been obtained for the secondary radiations from these will represent more nearly the real values than is the case for the pure metals.

The fact that the penetrating power of the rays from liquids is greater than for those from the pure metals is difficult to explain on the disintegration theory of the atom.

ABSORPTION OF THE β RAYS BY PURE LIQUIDS.

If the second radiation from liquids is due to a scattering of the primary beam, then the difference between the total incident radiation and the total transmitted radiation will represent the total absorption. This absorption will be made up of two parts, the true absorption of the rays in the liquid and the scattered radiation which gets turned completely back and appears on the incident side as so-called secondary radiation.

Whatever the mechanism of the liquid which affects the secondary radiation, the same mechanism should affect the absorption in the same degree. There ought therefore to be a definite relation between the amount of secondary radiation from a given liquid and the absorption of the primary rays by the same liquid.

It was therefore of the greatest interest to see if those liquids which showed remarkable radiating power would also exhibit anomalous results for the absorption of the primary rays.

The accessory apparatus used for this experiment is shown in Fig. 2, and may be briefly described as follows:

The absorption cell consists of a thick block of lead (*a*) in the center of which a hole (*b*) is drilled. A glass tube with carefully ground ends fits closely in this hole. The length of the tube is slightly greater than the thickness of the lead block. On the top of the glass tube rests a washer of lead, and over this a brass plate *C*. There is the same arrangement at the bottom of the tube, and the two brass plates *C* and *d* are firmly pressed together by screw bolts. Between the bottom of the tube and the lead washer is placed a thin mica sheet. This arrangement makes a very liquid tight joint.

The tube of radium *R* is placed beneath the opening in the lower plate *d*, which is made small so as to confine the β rays to a fairly narrow beam.

The cell is placed directly underneath the ionization chamber of apparatus *B*.

In making an absorption test of any liquid the standard of the electrometer was always set at a definite position for which the ionization was considerably less than that due to the β rays. The liquid under test was then poured into the cell by means of a fine capillary pipette and the level carefully adjusted until the ionization due to the β rays was cut down to the same value as that of the fixed position of the standard. The initial reading for every liquid was therefore the same. This procedure insured that in every liquid we were concerned with the same quantity of rays initially, and also that these rays were of the same average velocity. Moreover, the disturbances due to the capillary action and non-uniformity of the thickness of the initial layer were thus eliminated.

Each liquid in this way was tested under exactly the same conditions, and therefore any error due to difference in velocities would be the same for all.

After this reading was obtained a known quantity of the liquid was poured into the cell and the new ionization measured. The shape of the meniscus in both cases would be the same, and therefore the pouring in of a known quantity of liquid would mean a known increase in thickness of the liquid.

The beam of β rays was quite small in area at the bottom of the cell and therefore passed almost entirely through the center of the meniscus.

The results for a number of liquids are shown plotted in the curves of Fig. 5, the ordinates representing the ionization and the abscissæ the thickness of absorbing layer in mm.

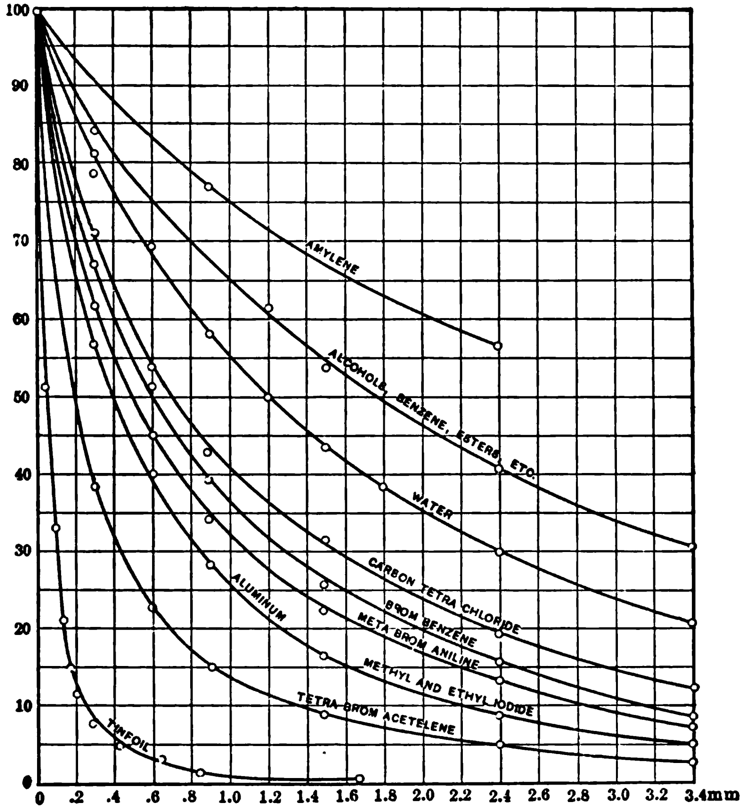

Fig. 5.

The relative absorbing power of these liquids was compared in the following manner: Water was taken as the standard and the thickness of water necessary to cut down the radiation to a definite amount (*e. g.*, 50 per cent.) was put equal to 100. The thickness necessary to reduce the radiation to the same amount for any other liquid was then read off from the curves and expressed in terms of water 100.

By an examination of curves it is seen that this ratio is nearly constant throughout a large portion of the curves, especially for the low absorption ones.

The average values over a considerable range were taken for the various liquids and are placed in the last column of Table IV., where they can be conveniently compared with the relative secondary radiations for the same liquids.

A number of these liquids were so volatile that it was difficult to obtain a very correct reading as the thickness of the liquid would change before the electrometer could be balanced.

In such case the relative absorption as taken from curves is too low.

Looking through this last column we see a remarkable parallelism between the relative secondary radiation from a liquid and its relative absorption. Those liquids which have a radiation less than water have in nearly all cases an absorption less than that of water, while those which have a high radiation have, with a few exceptions, also a high absorption. Those compounds of carbon, hydrogen and nitrogen which have a nearly constant radiation, give a relative absorption, which, considering the errors of observation, is also quite close to a constant.

Amylene, which gave the lowest radiation, shows the smallest amount of absorption, though the value obtained is, on account of the great volatility of the liquid, abnormally low.

In the group of halides there is, with two or three exceptions, very good agreement between the radiation and absorption. Those liquids which show anomalous results for the secondary radiation exhibit the same for the absorption of the primary rays. Propyl chloride is much less than propyl bromide, while ethylene chloride is much greater than ethylene bromide, the same as was the case for the secondary radiation. Chloroform and carbon tetrachloride have the same radiation and also about the same absorption. Tetra brom acetelene shows a very high absorption, much greater than one might expect from its radiation, though this is high also.

We thus see, from these tests on the relative absorption of the primary rays by the liquids, that there is an intimate relation in most cases between the secondary radiation and the absorption. Whatever mechanism of the liquid is responsible for the production of secondary radiation, that same mechanism must affect the absorbing powers in the same way.

It can be seen from Table IV. that the absorption is in no way dependent on the density of the liquid, though as a general rule those liquids which have a high density have also a high absorption.

N. Campbell has very recently published a paper[1] on the absorption of β rays of uranium by liquids. He examines water, alcohol, aniline, liquid paraffine and aqueous solutions of NaI, KNO_3 and K_2CO_3, using a thick piece of filter paper to hold the liquids. The absorption curves which for solids are exponential, show for liquids great irregularities, and no definite conclusions for them can be deduced. At any rate it is evident from his results that λ/ρ is not a constant for liquids (λ being the absorption coefficient and ρ the density).

The filter paper method of examining the radiating and absorbing power of liquids and solutions seems to me open to objections on account of the non-homogeneity of both thickness and material.

The absorption curves given by the author for liquids are not exponential, but the rays in this case are not homogeneous, as would be the case for uranium.

The absorption curves for tinfoil, aluminum, silver and platinum were also taken at the same time and under exactly the same conditions as those for the liquids. The curves for aluminum and tinfoil are plotted in Fig. 5. Aluminum happens to coincide with the curve for methyl and ethyl iodide, which shows that the absorption of aluminum is the same as that for the iodides. The atomic weights of the atoms are, however, in the two cases entirely different.

The radiation of aluminum is very much lower than that of the iodides.

The solids then are more efficient absorbers than the liquids for equal atomic weights, but the same is not true with regard to radiation.

ABSORPTION OF γ RAYS BY LIQUIDS.

A short series of tests on the absorption of γ rays by liquids was made. The radium was covered with a sheet of lead of sufficient thickness to cut out all the β rays. A cylindrical glass vessel was used to hold the liquids and was large enough in diameter to include all rays which could possibly enter the ionization chamber.

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[1] Phil. Mag., January, 1909.

The rays here used are of the soft kind and are clearly not homogeneous.

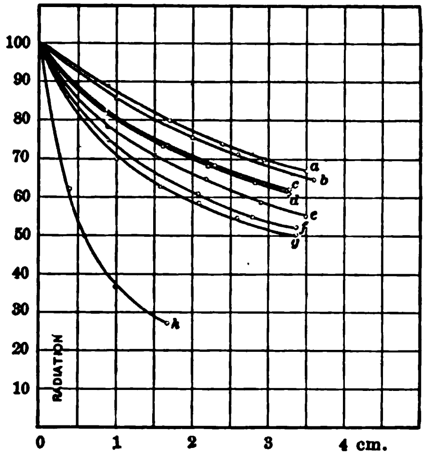

Fig. 6.

The results obtained are shown in Fig. 6, where the ordinates represent radiation, and the abscissæ the thickness of the liquid. Curve *a* is that for water, alcohol, benzene and acetic acid, which all have about the same absorbing power.

Curve *b* is that for nitro benzene and glycerine; curve *c* that for chloroform; curve *d* that for carbon tetrachloride; curve *e* brom benzene; curve *f* ethyl iodide; and curve *g* that for methyl iodide. Curve *h* represents the absorption in the case of mercury.

The absorption of the γ rays by liquids follows in general the order of the atomic weight of the heaviest atom. The density of the liquid seems however in certain cases to have a large effect. In those liquids of low density the absorption is nearly the same.

Mercury is very much greater in absorbing power than any of the rest. Its atomic weight and density are also much greater.

Conclusion.

It is not the intention of the author in this present paper to advance any new theory of secondary radiation, or to reconcile the facts therein described with the present theories. The results here stated, especially those with regard to the pure liquids, are entirely too complicated and anomalous to be satisfied with any simple theory of atomic disintegration, and a great deal more work will have to be done on the subject before the true explanation is arrived at.

Although in a great many cases the secondary radiation does seem to be a function of the atomic weight, yet there are, as has been shown, many exceptions which are undoubtedly in some way closely associated with the way in which the atoms are combined with one another chemically. The arrangement of the molecules is in all probability a very important factor in the production of secondary radiation from pure liquids.

At any rate it is too early yet to put forward any hypothesis to account for the author's results, even if a logical one were at hand.

We are perhaps too liable at the present time to advance incomplete and illogical theories, and it is an open question whether our true scientific knowledge is increased by such a procedure, if indeed we do not in many cases delay the truth.

As can be readily seen by a careful consideration of this paper, there are a great many interesting points brought up which need further investigation, and it is the intention of the author to take them up as soon as convenient in the hope that more light may be thrown on the abnormal results here described.

The secondary radiation from various substances is of the greatest importance in deciding whether or no the atoms can be disintegrated by outside agency.

The present research, while not deciding this question, certainly shows that the process is not in general simple, and in a great many cases that there are other very important factors coming in besides the atoms.

In conclusion I wish to express my obligation to Professors Jones and Fry of the chemical department of the University, for placing at my disposal the many expensive chemicals used in this research, and for valuable chemical advice. The research has also been greatly aided by apparatus and supplies purchased by a grant from the Bache fund of the National Academy of Sciences.

University of Cincinnati,
April, 1909.

ON THE PHOTOELECTRIC EFFECT OF SODIUM-POTASSIUM ALLOY AND ITS BEARING ON THE STRUCTURE OF THE ETHER.

By Jakob Kunz.

THE velocity of electrons emitted by a plate of metal exposed to ultra-violet light or Roentgen rays does not depend upon the intensity of the light; while the number of electrons emitted per unit time from unit surface increases with the intensity of the rays. This fact seems to indicate that the forces which expel the electrons from the metal are not the electric forces in the light wave incident on the metal; but that the electrons are expelled by a kind of explosion of some molecules of the metal, which under the action of the light, have become unstable in their equilibrium. If this view of the nature of the photoelectric effect were right the velocity of the electrons emitted would not at all depend on the light applied but would be determined by the properties of the atom of the metal. For each metal there would be a characteristic velocity with which the electrons escape; and one special wave-length or a small interval of wave-lengths would be required to produce the explosion in the same way as, for instance, a particular wave-length of light is required to excite the explosive decomposition of the compound NBr_3.

The mechanism of the photoelectric effect might also be similar to a process of resonance. The incident light sets the electrons of the metal into forced vibrations, the amplitude of which may become so large that the electron is emitted from the atom with a certain velocity characteristic for each element; the nearer the period of the incident light is to that of the free vibrations of the electrons, the greater will be the amplitude of the forced vibrations; and when a given value of the amplitude has been reached the electron will be shot out of the atom. In this case of resonance again one special wave-length would produce a maximum velocity of the electrons, while the velocity of electrons emitted under the action of light of smaller or greater wave-length would decrease very rapidly. Some experiments made by E. Ladenburg[1] make the theory of explosion or of simple resonance improbable. E. Ladenburg found that the velocity of electrons emitted from plates of platinum, copper and zinc, under the action of ultra-violet light of wave-lengths from 270–220 $\mu\mu$, increases proportionally to the frequency of the light. It may be shown, however, that the numbers obtained by E. Ladenburg agree with the assumption that the square of the velocity is proportional to the frequency. However this may be, the velocity of the electrons is independent of the intensity of the light and depends only and in a continuous way on the wave-length of ultra-violet light. This would be very improbable if the electrons were expelled by an explosion of the atoms. It is more probable that the kinetic energy of the electrons emitted is transformed energy of radiation of the incident light, and yet as we have seen, the kinetic energy is independent of the intensity of the light. How can these two facts be reconciled?

Sir J. J. Thomson[2] pointed out a long time ago that the small fraction of the molecules of a gas ionized by Roentgen rays and ultra-violet light suggests that these rays possess a structure. The electromagnetic energy in these rays is localized in special regions and the great majority of the molecules of the gas are not ionized because they are not struck by those regions of intense electric and magnetic forces. The front of the electromagnetic disturbance in a Roentgen pulse is not a surface of uniform illumination, but resembles rather a series of bright specks on a dark background. The same conclusion may be drawn with regard to the ultra-violet light producing ionization in a gas or the emission of electrons from metal surfaces. The structure in the wave front implies that the ether through which light is travelling has also a structure.

According to Maxwell's theory of electrodynamics, the distribution of the energy in the electric, magnetic and electromagnetic field is continuous. The lines of force are a mere mathematical fiction. Faraday, however, attributed to the tubes of force some physical meaning. The properties of the ether inside and outside the tubes are different and the electric energy is localized along these tubes while the space free of tubes contains no energy. Thus in a charged condenser the energy is localized along the individual lines of force in a discontinuous way (see Fig. 1). Let us suppose also that the number of Faraday tubes connected with the electron is a limited one. If the electron that was moving with a velocity v, comes into collision with a solid obstacle, each individual tube will be disturbed during the time of collision τ and the disturbance consisting in a tangential electric force T in the direction PQ of Fig. 2, and a new magnetic force perpendicular on the plane OPQ will travel out in the direction OPP' with the velocity of light; t seconds after the electron began to be stopped, the disturbance of the tubes will be found in the shell of thickness δ between two spheres and the total amount of energy in the Roentgen pulse will be equal to

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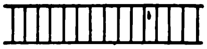
[1] Erich Ladenburg, Physikalische Zeitschrift, 8, p. 590, 1907; 9, p. 821, 1908.

[2] J. J. Thomson, "On the Ionization of Gases by Ultra-violet Light and on the Evidence as to the Structure of Light Afforded by its Electrical Effects," Proceedings of the Cambridge Philosophical Society, Vol. XIV., Pt. IV., p. 41 f., 1908.

Fig. 1.



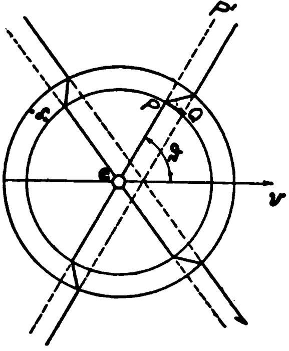


Fig. 2.

$$E = \frac{2}{3}\frac{e^2v^2}{\delta}.$$

This energy is spread continuously through the shell with variable density according to ordinary electrodynamics, while it is localized in a certain number n of isolated bundles of intense electric and magnetic forces, the average energy of such a region would be $\frac{2}{3} \cdot e^2v^2/\delta n$. Thus the front of the disturbance corresponding to the Roentgen rays is not a surface of uniform electromagnetic illumination, but resembles rather a series of bright specks on a dark background. The energy appears to be divided up into units of invariable character. If in the old emission theory of Newton we replace the moving particles by isolated Faraday tubes set in vibration we get a picture of this new form of the theory of light.

This structure of light or the distribution of the electromagnetic energy in distinct units would just account for the facts that the number of the molecules of a gas when struck by Roentgen rays is exceedingly small and that the velocity of the electrons emitted from metals under the action of Roentgen rays and ultra-violet light is independent of the intensity of the light, that the velocity increases with decreasing thickness δ of the Roentgen pulse and that the number of electrons emitted from unit surface in unit time increases with increasing number of units of electromagnetic energy passing the same surface in the same time, or the number of electrons emitted from a metal must increase with the intensity of the Roentgen rays and the ultra-violet light. Indeed, if a unit of electromagnetic energy colliding with a molecule can at any place make it liberate an electron it will do so, and start the electron with the same velocity, whatever may be the distance from the source when the unit impinges on a molecule. Thus the velocity would be independent of the intensity of the light. A given source of Roentgen rays, or of ultra-violet light, emits per unit time a certain number of units; the fraction of these units passing through unit surface at a certain distance from the source will be inversely proportional to the square of the distance; the further we recede from the source the smaller the number of units passing through unit surface, and therefore the smaller the number of electrons emitted from the metal.

According to the electron theory of radiation, the waves of heat and light emitted by a hot black body is of the same origin as the Roentgen radiation. The electrons like an exceedingly thin gas come into collision with the molecules of the substance and produce Roentgen pulses. The electromagnetic disturbance in the pulse can be resolved into simple harmonic vibrations by means of Fourier's theorem. And when such a disturbance goes through a prism the elementary oscillators resolve according to their own period of vibration the pulse arriving in the beam of light.

The thickness δ of the pulse is equal to $\delta = c\tau$, where τ is the time of collision between an electron and an atom and c is the velocity of light. In order to account for the laws of radiation we have to admit that the time of collision τ is inversely proportional to the square of the velocity of the electron or inversely proportional to the absolute temperature of the body. If c_1 is a proportionality factor, we have

$$\tau = \frac{c_1}{v^2},$$

hence

$$\delta = \frac{cc_1}{v^2} = \frac{c_2}{v^2},$$

$$E = \frac{2}{3}\frac{e^2v^2}{\delta} = \frac{2}{3}\frac{e^2v^4}{c_2},$$

or

$$E = \frac{2}{3}\frac{e^2c_2}{\delta^2}.$$

The energy in a Roentgen pulse is proportional to the fourth power of the velocity of the electron. It would be important to test this relation. From the last equation we see that the energy in the pulse is inversely proportional to the square of the thickness of the pulse. Thin pulses will have more energy than broad pulses. Therefore we should expect that the velocity of electrons produced by hard rays is greater than that of electrons produced by soft Roentgen pulses. This conclusion agrees with the results of experiments.

When a Roentgen pulse passes over an atom containing electrons, the electrons will be acted upon for a very short time by a strong electric force. The electrons will be subject to a very strong acceleration and will emit secondary Roentgen pulses of a certain thickness δ_2. If the forces between the electron and the rest of the atom are weak the electron may be ejected, accompanied by a secondary pulse. If these forces between the electron and the atom are strong the electrons after the displacement by the pulse may vibrate through their positions of equilibrium and produce disturbances of a periodic character or ordinary light. The first process constitutes the ejection of electrons and the production of secondary Roentgen radiation, while the second process gives rise to the phenomena of phosphorescence.

As the energy in each pulse is inversely proportional to the square of the thickness, we conclude that the thickness of the secondary pulse cannot be less than that of the primary radiation.

If E_1 and E_2 represent the energy in the primary and secondary radiation and δ_1 and δ_2 the thickness of the pulses, we have

$$E_1 \geqq E_2,$$

$$\delta_1 \leqq \delta_2.$$

This agrees with the results of experiments made by Barkla on secondary radiation. The rule derived in this way corresponds exactly to Stockes' rule of phosphorescence, when we replace the thickness of the Roentgen pulse by the wave-length of light.

$$\frac{cc_1}{v_2} = \delta = \frac{c_2}{T},$$

where T represents the absolute temperature of the radiating substance. Now the velocity of the electrons in the discharge tube is much larger than the velocity of the electrons in a metal at ordinary temperature; the pulses from metals at ordinary temperature and even at high temperature will therefore be much thicker than the electromagnetic pulses emitted from a Roentgen tube. The energy in the disturbance that constitutes ordinary light will be smaller than the energy contained in the Roentgen pulses themselves, and we should therefore expect that the velocity of the electrons emitted from metals struck by Roentgen rays is far greater than the velocities of the electrons produced by the impact of ultra-violet light. Indeed, the velocity of the electrons emitted under the action of Roentgen rays is of the order of magnitude 10^{10} cm./sec., while the velocity of those emitted by ultra-violet light is less than 10^8 cm./sec.

Let us suppose that an electron comes at O of Fig. 3 into collision with an atom, and that its velocity falls from its constant value v to zero, during the time interval $\tau/2$. The Faraday tube t seconds after the beginning of the collision will have the position $OP_1P_2P_3$. If, however, the electron begins to move immediately after the stoppage in the opposite direction along the same line as before, and if during the next interval $\tau/2$ it acquires its original velocity v again, then the shape of the Faraday tube t seconds after the beginning of the accident will have the form $O_1P_2P_3P_4$. There will be a tangential component of the electric force $P_1Q_1 = P$ of given direction in the point P_1 and in all points along the line P_1Q_2 and the same tangential component of opposite direction in the point Q_2 and along the line Q_2P_3. The constitution of the pulse represented by the two parallelograms $P_1A_1A_2Q_2$ and $Q_2P_3A_4A_3$ of Fig. 4 is now more complicated than it was before, and we see from Fig. 4 that half of the thickness of the pulse corresponds to half a wave-length of light, or the thickness δ of the complicated Roentgen pulse due to the type of collision as it occurs in the metals, will correspond to a wave-length λ of ordinary light. The energy in the pulse is inversely proportional to the square of the thickness of the pulse; from the analogy drawn between the two processes we should expect that the energy of a light wave will be inversely proportional to the square of the wave-length λ of the light.

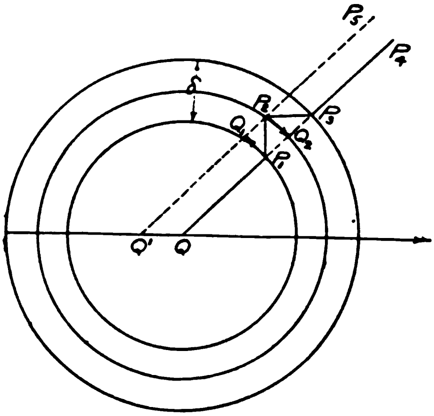


Fig. 3.

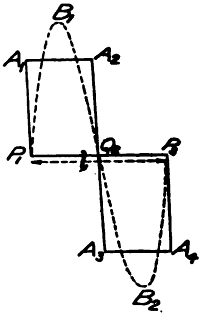


Fig. 4.

If in the case of a collision the velocity of an electron instead of being reduced to zero is diminished by dv during the interval dt we find that the magnetic force H in the corresponding pulse is equal to

$$H=\frac{e}{c}\frac{dv}{dt}\frac{\sin\vartheta}{r}$$

and the tangential electric force T is equal to

$$T=\frac{e}{c^2}\frac{dv}{dt}\frac{\sin\vartheta}{r}.$$

But the acceleration f of the electron is equal to

$$f = -\frac{dv}{dt},$$

hence

$$H = -\frac{ef}{c}\frac{\sin\vartheta}{r},$$

$$T = -\frac{ef}{c^2}\frac{\sin\vartheta}{r},$$

and the energy dE emitted from the electron during dt will be given by the expression

$$dE = \frac{2}{3}\frac{e^2f^2}{c}\,dt.$$

Let us suppose that during a collision the velocity of the electron changes according to

$$v = v_0 \cos(2\pi nt),$$

that τ is the interval of time of a collision, where the velocity changes from v_0 to o and to $-v_0$, and that $n\tau = 1$, then we have

$$f = -\frac{dv}{dt} = v_0 2\pi n \sin(2\pi nt).$$

The energy radiated away from the electron during the time τ of a collision will be equal to

$$E = \frac{2}{3}\frac{e^2}{c} v_0^2\pi^2n^2 \int^{\tau} \sin^2(2\pi nt)dt,$$

$$= \frac{2}{3}\frac{e^2}{c} v_0^2\pi^2n^2 \frac{1}{2}\tau,$$

$$= \frac{4}{3}\frac{\pi^2}{c}\frac{v_0^2e^2}{\tau}.$$

The disturbance will be a simple harmonic vibration and correspond to a wave of monochromatie light of wave-length Λ.

$$c = \frac{\lambda}{\tau},$$

$$\tau = \frac{\lambda}{c},$$

$$E = \frac{4}{3}\frac{\pi^2e^2v_0^2}{\lambda}.$$

But

$$\tau = \frac{c_1}{v_0^2}; \quad v_0^2 = \frac{c_1}{\tau} = \frac{c_1 c}{\lambda},$$

hence

$$E = \frac{4}{3} \frac{\pi^2 e^2 c_1 c}{\lambda^2} = \frac{C}{\lambda^2}.$$

The energy radiated away from an electron during a collision is inversely proportional to the square of the wave-length of the simple harmonic radiation. This energy radiates away from the electron in form of a simple harmonic wave with the velocity of light.

Let n be the frequency of the light whose wave length is λ, then we have

$$n\lambda = c$$

and

$$E = Cn^2.$$

When the wave travels out into the space the density of the energy in the wave will become smaller and smaller according to the ordinary conception of continuous distribution of the electromagnetic energy. If, however, we assume again, as in the case of Roentgen rays and with exactly the same right, that the energy in the wave front is localized into distinct regions, *i. e.*, along the N Faraday tubes that start from the elementary charge, we find that the average energy of such an electromagnetic unit in the wave of light is equal to

$$E_1 = \frac{C_1 n^2}{N} = kn^2,$$

or the unit of electromagnetic energy in a wave of light is proportional to the square of the frequency.

The theory of thermal radiation as developed by Planck leads to another result. In this theory also the electromagnetic energy appears to be divided up into well separated units; and the unit of energy of radiation is proportional to the frequency of the light.

$$E_2 = hn,$$

$$h = 6.5 \cdot 10^{-27}.$$

In either case these units travel out with invariable properties.

Let us suppose that the units E_1 enter the surface of a metal and are partly converted into kinetic energy of the electron and that the energy of one unit is imparted to one electron. Another part of the energy of the unit may be transformed into other forms of radiations or into heat in the same or opposite way as cathode rays when impinging on different metals produce Roentgen rays of different intensity. We shall suppose that the fraction of the energy of the unit, characteristic for each element, is transformed into kinetic energy of an electron.

$$E = akn^2.$$

The kinetic energy of the electron will be so large that it may overcome the attraction of the positive electricity left behind. The electron may come into collision with molecules before it reaches the surface of the metal and lose a part w of its kinetic energy. Thus the electron leaves the metal with the kinetic energy $akn^2 - w$. When the process is going on the metal will acquire a positive charge and a positive potential P, sufficient to prevent the electrons from escaping. In the case of equilibrium we have

$$Pe = akn^2 - w,$$

e being the change of the electron.

Neglecting w we have

$$Pe = akn^2,$$

$$P = \frac{ak}{e} n^2,$$

$$\tfrac{1}{2}mv^2 = Pe,$$

$$v = \sqrt{\frac{2e}{m} P},$$

$$v = \sqrt{\frac{2ak}{m}}\, n,$$

v being the initial velocity of the electrons. The velocity of the electrons would be proportional to the frequency of the incident light. Planck's theory, however, would lead to the expression

$$v = \sqrt{\frac{2ah}{m}} \sqrt{n},$$

the velocity would be proportional to the square root of the frequency of the incident light.

The experiments made by E. Ladenburg for a small interval of wave-lengths of ultra-violet light agree quite as well with the first as with the second of these two expressions.

In order to extend the interval of wave-lengths I studied the velocity of electrons emitted from an equimolecular liquid alloy of sodium potassium, that emits electrons even when struck by daylight. The vacuum tubes that were used are shown in Figs. 5 and 6. The tubes are connected with the mercury pump by

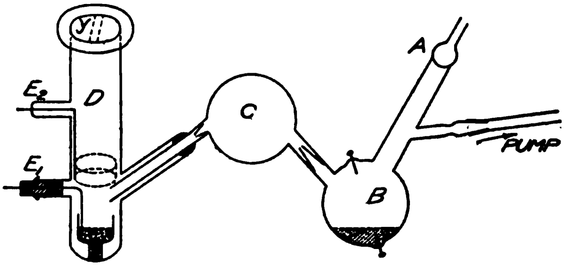


Fig. 5.

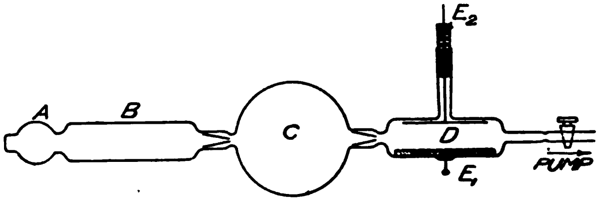


Fig. 6.

means of a ground-joint. When the air has been pumped out the tubes are filled with dry nitrogen, then opened at *A*, where 3 gr. potassium and 1.77 gr. sodium were introduced. The tubes that have been carefully cleaned with a solution of potassium chromate and with distilled water are then sealed off at *A* either by means of sealing wax or by melting the glass directly. The vacuum will be established again and then the metals melted. The surface of the alloy appears to be covered with a dark crust of impurities. The glass walls of the system of tubes are also carefully heated; they give off a considerable amount of gas. The degree of the vacuum can be estimated by the character of the electrical discharge between the two platinum electrodes in B. It was easy to diminish the pressure so far as to make the discharge very difficult or impossible. The heating of the tubes and the removing of the gas will be repeated several times during two days. The tube of Fig. 5 will then be turned 180° in the ground-joint so that the liquid alloy flows over in the tube C. The tube of Fig. 6 contains a stop-cock so that it could be removed from the pump and the alloy was allowed to flow into the tube C. The crust is held back by means of little funnels of glass between the bulbs B and C. The alloy is almost perfectly clean in the bulb C; the system of tubes is heated again and the rest of the gas developed from the glass walls is removed. In the same way as before, the alloy will be allowed to flow into the tube D, where it arrives perfectly clean. In the tube D of Fig. 5 there is a little glass cup insulated from the outer glass wall by means of amber. The platinum electrode E_1 connecting the alloy with the electrometer, goes through a piece of amber, so that the electrical insulation is as perfect as possible. The electrode E_2 of Fig. 5 consists of a platinum wire connected with a ring of wire gauze touching the glass wall. Two strips of platinum are arranged parallel to one another, leaving free a slit similar to the slit L through which the beam of light passes. The tube of Fig. 6 is held in its final position by means of well insulating paraffine. The electrode E_1 is directly connected with the alloy through the glass, while the negative electrode E_2 is insulated by a piece of amber. The electrode E_2 is a slip of platinum foil about 1 cm. wide and 4 cm. long. By turning the tube round its axis the distance between the two electrodes E_1 and E_2 could be chosen at liberty. It was usually about 0.5 cm. Thus prepared the tubes were sealed off from the pump.

The general arrangement of the experiment is shown by Fig. 7. A parallel beam of light produced by an arc and a condenser, passed through a small slit L_1. A system of lenses produced a sharp image of this slit on the screen L_2 which was endowed with a millimeter scale. The beam was then deflected by means of a total reflecting prism and decomposed in the spectrum by an ordinary prism. The tube of Fig. 6 was mounted in a wooden box, one side of which possessed a metallic window that could be moved up and down so that the light struck the metal to be tested. The whole box could be displaced also along the scale L_2. The electrode E_1 was connected to a commutator through a long wire insulated by pure paraffine. The commutator was made of paraffine and little glass cups. One pair of the quadrants of the electrometer was charged with the positive electricity of the alloy, whereas the other pair through the commutator was connected to the electrode E_2 and to earth. The wooden box T_1 containing the glass tube, the tube T_2 and the box of the electrometer were covered with tin foil connected to earth. The quadrant electrometer had a fine quartz suspension and amber insulation. The needle was charged from a storage battery to a potential of about 12 volts. In spite of the care that was taken in insulating the system, there was always a small leakage, the amount of which could easily be taken account of. The electrometer was calibrated by means of accumulators and resistances. The electromotive force of the storage battery was determined with a potentiometer.

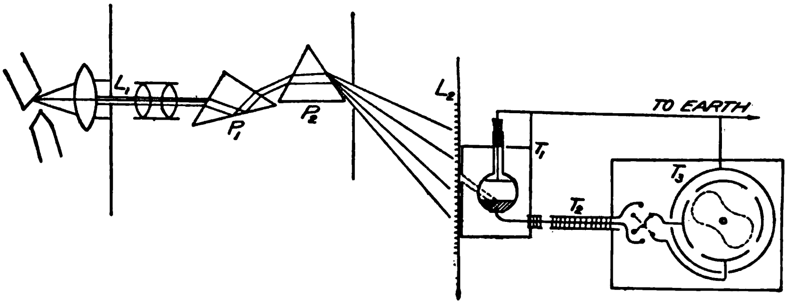


Fig. 7.

If the system of tubes of Fig. 5 was used the two prisms P_1 and P_2 were turned so that the beams of the spectrum fell in a nearly perpendicular direction upon the alloy. As in this case the distance between the metal and the negative electrode E_2 was about 2 cm.; the potential measured appeared to be smaller than that measured with the other tube. Moreover, the surface of the alloy in the tube of Fig. 5 oxidized in about four or five days after its preparation, so that for further measurements the alloy had to be replaced, whereas the vacuum in the tube of Fig. 6 remained constant and the surface of the alloy perfectly clean for an apparently unlimited time. Nor could I in the last case discover any trace of photoelectric fatigue.

The first question to answer was the influence of the intensity of the light. I found that the potential difference between the two electrodes E_1 and E_2 was constant and independent of the intensity of the light. If the alloy was struck by a very strong beam of white light from the arc, or by daylight, or by the weak light that remained after the strong beam from the arc had passed through a sheet of snow and water of about 1 cm. thickness placed between the arc and the alloy, the deflection of the electrometer was always the same corresponding to a potential difference of 1.12 volts. In addition the potential difference thus determined was that due to the action of the shortest violet rays in the spectrum. It should be mentioned, however, that the deflection became decidedly smaller when the intensity of the light fell below a certain value. When, for instance, I darkened the room of the experiment, or when the beam from the arc was forced to go through a glass plate blackened with lamp black, the deflection of the electrometer became considerably smaller. This is probably due to the fact that the number of electrons emitted per unit time depends on the intensity of the light. If now the number developed per unit time becomes very small, saturation of the metal and of the electrometer becomes impossible and the deflection of the electrometer drops down. There is another phenomenon whose nature is not yet plain, in which the intensity of the light plays a part, namely, the red end of the spectrum beyond the yellow region where the photoelectric effect begins, produces also a small and irregular effect, depending on the intensity of the light and on the heat of the rays in this part of the spectrum.

J. A. Fleming[1] measured the photoelectric effect by means of a galvanometer and found that the galvanometer deflections increase very rapidly with the intensity of the incident light. This observation is not in contradiction to the constant deflection of the electrometer over a large range of intensities, because the galvanometer deflexions depend not only upon the velocity of the electrons but also upon the number emitted per unit time.

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[1] J. A. Fleming, "A Note on the Photoelectric Properties of Potassium Sodium Alloy," Phil. Mag., 6th s., Vol. 17, p. 286, 1909.

On reversing the potentials on the quadrants of the electrometer by means of the commutator I was able to take the readings to the right and left, and I took as a rule four readings on either side. The mean values λ of eight readings for each position of the tube in the spectrum are given in the following table as a function of the wave-length λ of the incident light.

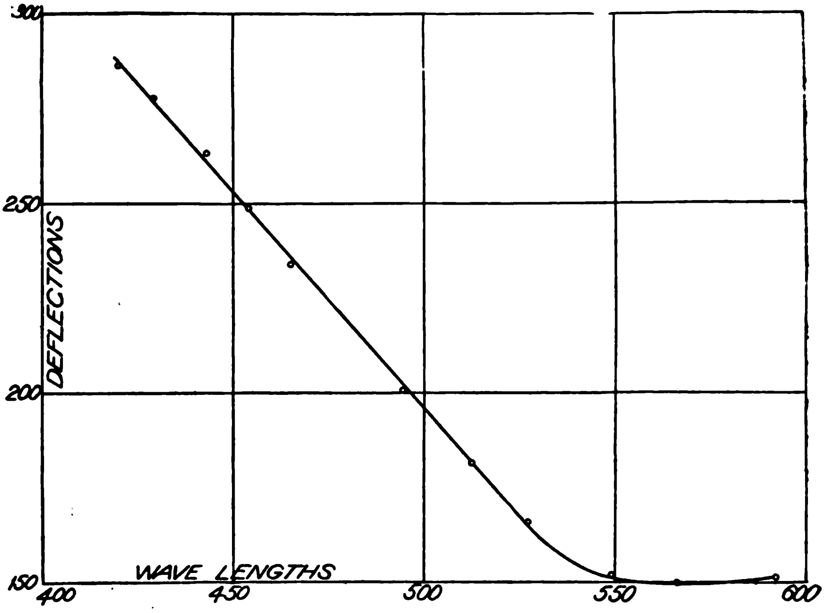

Fig. 8.

λ	λ Observed.	λ Calculated.
592	151.8	—
566	149.9	—
549	152.1	—
527	166.0	166.0
712.5	181.5	180.7
494.5	200.9	201.4
465.0	234.03	236.1
454	248.4	249.0
443	263.2	261.1
429	279.0	278.0
420	286.5	288.3

The numbers are represented in Fig. 8. The experiment has often been repeated with the same result. The spectrum was calibrated by means of the following lines:

Li	670.8	Ag	546.6
Li	610.4	Ag	521
Na	589.0	Li	497.2
Hg	577	Li	460.2
Hg	546	Hg	436

The potential difference between the electrodes of the vacuum tube appears to be proportional to the wave-length and decreases with increasing wave-length. The figures calculated on this assumption are given in the table. The difference between the deflections calculated and observed exceeds in no case 2 per cent.. This law seems to hold good only to wave-lengths of about 520 $\mu\mu$. From this point up to the wave-length of 582 $\mu\mu$ the connection between the potential of the alloy and the wave-length is almost a straight line. Beyond this point the deflections become small very rapidly and irregular at the same time. These irregular deflections depend partly on a heat effect, when a layer of water is introduced in the path of the light, the deflections decrease but they do not quite disappear. So far as these preliminary experiments go they seem to indicate that the velocity of the electrons is independent of the intensity of the light and that the kinetic energy is proportional to the frequency of the incident light.

$$Pe = ahn,$$

$$n\lambda = 3.10^{10},$$

the maximum deflection was found for a wave-length of 420 $\mu\mu$, corresponding to a potential difference $P = 1.12$ volts.

$$\lambda = 4.2 \cdot 10^{-5} \text{ cm.},$$

$$n = 0.715 \cdot 10^{15},$$

$$e = 1.55 \cdot 10^{-20} \text{ absolute electromagnetic units,}$$

$$P = 1.12 \cdot 10^{8} \text{ absolute electromagnetic units,}$$

$$Pe = 1.74 \cdot 10^{-12},$$

thus

$$ah = 2.425 \cdot 10^{-27},$$

$$h = 6.5 \cdot 10^{-27},$$

$$a = 0.37.$$

The velocity of the electrons emitted under the action of violet light of wave-length 420 is given by the equation

$$\tfrac{1}{2} mv^2 = Pe\,,$$

$$v = \sqrt{2\frac{e}{m}P},$$

$$\frac{e}{m} = 1.77 \cdot 10^7\,,$$

$$v = 6.3 \cdot 10^7 \text{ cm./sec.}$$

Many problems spring up in connection with the present investigation, which was carried out in the physical laboratory of the University of Michigan. I shall extend the research in different directions. A small addition to this paper has already been published in the August number of this journal.

COUNTER ELECTROMOTIVE FORCE IN THE ELECTRIC ARC.

By C. D. Child.

IN a somewhat recent article by Becknell[1] it was shown that there was an E.M.F. of approximately .7 volt between the terminals of an arc after the impressed E.M.F. was removed. This may be explained as being due to the difference of temperature between the two carbons, and a few simple tests of such an explanation have been tried by the present writer and are described in the following article.

This subject has been so intimately connected with the discussion of the existence of a counter E.M.F. in the arc, that some remarks on this latter subject may not be out of place. It may seem like bringing back to life what has been decently buried to discuss this again, but the subject has one phase which I believe has never been properly considered, namely, the exact meaning which different writers give to the expression "counter E.M.F." when applied to the arc.

Meaning of Expression "Counter E.M.F." — It would appear as if this term should mean the same when applied to the arc as when applied to a motor, or a storage cell. In those cases we find: first, that the potential difference between the terminals of a cell or motor does not obey Ohm's law; secondly, that there is an E.M.F. remaining after the impressed E.M.F. has been removed and that this is of sufficient magnitude to account for the deviation from Ohm's law; and thirdly, that in the majority of cases the electrical energy is changed into something besides heat energy.

In the arc only the first of these properties is found to exist. In fact the arc deviates from Ohm's law much as other forms of discharge through gases deviate, where the expression counter E.M.F. has never, I believe, been applied.

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[1] Phys. Rev., 21, 181, 1905.

In the beginning of the investigation on the arc it was not known that this was the only point of similarity between the arc and a cell, and writers using the word no doubt had in mind a counter E.M.F. identical with that of the cell. Later writers have used the term merely to mean that the potential difference of the arc deviates from Ohm's law in a definite manner, and have so defined it. Yet no one seems to have pointed out that the meaning of the word as defined by Duddell, for example, is not the same as the meaning which Edlund had in mind, nor the meaning generally given to the term in physics. This has led to more or less confusion in what has been written on the subject during the past few years. For example, one occasionally sees the statement that it has at last been definitely shown that there is a counter E.M.F. in the arc. The fact is that on the one hand it has neither recently nor at any other time been shown that the arc has more than the first of the three peculiarities mentioned above, and on the other it has never been doubted but that there was this one point of similarity between the arc and the cell.

Because of this confusion in the use of the term it seemed to the present writer that a brief account of the different types of experiments on this subject and of the different meanings of the words used which have accompanied these experiments will be of some value.

The discussion had its origin in an attempt by Edlund[1] to explain the relation between the "apparent resistance" of the arc and its length. By "apparent resistance" he meant the ratio between the potential difference at the terminals of the arc and the current. This ratio he found equal to $a + bl$, where l is the length of the arc and a and b are constants depending on the kind of carbons used. The first term of this expression he considered to be due to a counter E.M.F. equivalent to 23 Bunsen cells.

Others questioned the existence of such an E.M.F. and several different methods have been used to test the question. These methods may be divided into two general classes: First, methods which attempt to find a counter E.M.F. by measuring the resistance of the arc, and second, those attempting to find it by measuring the E.M.F.

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[1] Pogg. Ann., 131, 595, 1867; 133, 353, and 134, 250, 1868.

remaining between the carbons after the impressed E.M.F. has been removed, or what we may call the residual E.M.F. of the arc.

Those who attacked the problem by measuring the resistance of the arc have assumed that the counter E.M.F. was the difference between the potential difference at the terminals of the arc and that part of it which appeared to obey Ohm's law. In other words that $E' = E - RI$, where E' is the counter E.M.F., E the potential difference at the terminals, R the resistance of the arc, and I the current.

Meaning of "Resistance of the Arc." — We find, however, that "resistance of the arc" has meant four different things with different experimenters, and with these different meanings there are corresponding differences in what is meant by counter E.M.F.

First of all it has been used to mean the ratio between the potential difference at the terminals and the current. This, however, has usually been called the "apparent resistance."

Secondly, it has been used to mean the term bl in the expression $r = a + bl$, which was used by Edlund, namely, the part of the apparent resistance which was proportional to the length of the arc. This Edlund called the "true resistance."

At a later time it was used to mean the ratio between a small increase in E and the corresponding increase in I, or what we may call dE/dI. An approximation to this was made by those who used some modification of the Wheatstone bridge.

Finally it was pointed out by Luggin[1] that any change in the current through the arc produced a corresponding change in both the vapor of the arc and the shape of the carbon ; so that one does not have the same arc after the current has been increased as before The methods which are modifications of the Wheatstone bridge do not correctly measure either the resistance of the arc in its first condition, nor its resistance after the current has been increased. This difficulty has been clearly stated by Mrs. Ayrton,[2] Duddell[3] and others. They have, therefore, defined the true resistance of the arc as the ratio between a small change in potential difference and the corresponding change in the current, when the change is made so small, or for so short a time, that the condition of the arc itself does not vary. In other words the resistance as thus defined is the partial derivative of the potential difference with respect to the current which we shall indicate by $\partial E/\partial I$.

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[1] Centralbl. f. E. T., 10, 591, 1888.

[2] The Electric Arc, p. 400.

[3] Phil. Trans., 203, A, 306, 1904.

This would undoubtedly be the correct way of defining the resistance, if it were not for the doubt which must exist in one's mind as to whether it is possible to have a change in current without having a change in the condition of the arc. If the current in the arc is carried by ions, as we have reason to believe it is, it is not possible to change the current without changing either the number of the ions or their velocity. In either case there would be a change in the condition of the gas. However, this definition may properly be considered as giving a limiting condition toward which a close approximation may be made.

Resumè of Experiments on Resistance and Counter E.M.F. — Since Edlund's time there have been many experimenters who have examined the relation between the potential difference at the terminals of the arc and the length and current, and as far as their experiments were correct they gave results similar to those of Edlund.[1]

As has been shown by different experimenters, these results prove nothing definite concerning the existence of a counter E.M.F., and we need not consider them further in this connection.

If we pass to other methods of attacking the problem we find that Swendler[2] gave the value of the counter E.M.F. as 2 volts, but since he did not divulge his method of arriving at this conclusion we are unable to judge of the importance of his work.

As far as we know the first experimenter to use a method different from that of Edlund's was von Lang,[3] who endeavored to measure the resistance of the arc by a modification of the Wheatstone bridge. He found the resistance to be 1.82 ohms and concluded that the counter E.M.F. was 38.9 volts. This method gave approximately the value of the resistance as defined the third way, namely, dE/dI.

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[1] Ayrton and Perry, Proc. Phys. Soc., 5, 197, 1882. Frohlich, Elektrot., Z. S., 4, 150, 1883. Peukert, Elektrot., Z. S., 7, 443, 1885. Nebel, Centralbl. f. E. T., 8, 517 and 619, 1886. Cross and Shepard, Proc. Amer. Acad. Science, 86, 2, 1886. Uppenborn, Centralbl. f. E. T., 9, 663, 1887. Luggin, Centralbl. f. E. T., 10, 567, 1888.

[2] Lond. Elec., 2, 107 and 117, 1879.

[3] Centralbl. f. E. T., 7, 443, and Wied. Ann., 6, 145, 1885.

Attempts were then made to find the resistance when the change in the current was made much smaller, that is, to get the value indicated by the expression $\partial E/\partial I$. Arons[1] attempted to do this by using a different modification of the Wheatstone bridge. The arc with the battery producing it was used as one branch of the Wheatstone bridge. Instead of the battery ordinarily used with the bridge a transformer was used and instead of a galvanometer a dynamometer. The three arms of the bridge were adjusted until no current was shown by the dynamometer. The resistance of the arc could then be determined. The resistance was found to be approximately 2 ohms and the counter E.M.F. 40 volts. Luggin[2] used a different modification of the bridge and Firth[3] still another, both getting results similar to those of Arons.

Negative Resistance. — In 1895 Ayrton[4] concluded from the result of a few simple experiments that the resistance of the arc was negative. What he measured was dE/dI. It had been previously pointed out by Luggin that this quantity was negative, and it has since been shown even more clearly by the curves given by Mrs. Ayrton.[5] If one is to define resistance, as meaning dE/dI, there is no question but that the resistance of the arc is a negative quantity, but of all the uses of the word this is perhaps the least justifiable, and to speak of a negative resistance is, to say the least, misleading.

The word resistance means primarily something which hinders the movement of some object. An electrical resistance means something which hinders the flow of an electric current, and the most natural meaning of the expression "negative resistance" would be something which helps the flow of the current. It is needless to say that the resistance of the arc does not help the flow of current.

Firth and Rogers[6] superimposed an alternating current on the direct current through the arc, and the resistance of the arc to this alternating current was taken as its resistance. Frequencies between

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[1] Wied. Ann., 30, 93, 1887.

[2] Centralbl. f. E. T., 10, 43, 1888.

[3] Mem. and Proc. Manchester Lit. and Phil. Soc. (4), 9, 139, 1895.

[4] The Electric Arc, p. 75.

[5] The Electric Arc, p. 113.

[6] Phil. Mag. (5), 42, 407, 1896.

7 and 250 complete alterations per second gave the same resistance. They, therefore, believed that the changes in the current were so rapid that there were no corresponding changes in the arc. The value of the resistance which they found varied much with different carbons and with different currents. Values ranging from 2 to − 2 ohms are given by them. The corresponding values of the counter E.M.F. were in the neighborhood of 40 volts.

Following the investigation of Firth and Rogers was a careful examination of the counter E.M.F. and resistance by Duddell.[1] His principal advance over the work of others consisted first in devising a method by which one could test whether the arc remained constant during the change in current or not, and secondly, in using much higher frequencies than those which had previously been used.

To decide whether the arc changed any or not he imposed a small alternating current on the direct current which was passing through the arc, and determined whether this alternating current was in phase with its E.M.F. or not. Ordinarily, when an alternating current is thus used, it lags behind its E.M.F. Duddell assumed that if the two were in phase the condition of the arc had not changed during the change in current. He found that for low frequencies with solid carbons the oscillations were 180° out of phase, and that to have the two in phase with such carbons it was necessary to have frequencies of 100,000 or more per second. For cored carbons 26,000 alternations per second were needed.

When the alternating current was in phase with its E.M.F. the ratio between the E.M.F. and the current was taken as the "true resistance" of the arc, and the counter E.M.F. was found from the formula $E' = E - RI$. The value of E' as found by him varied from 11.2 to 18.5 volts with different currents and different carbons.

"*Forward E.M.F.*" — He also endeavored to find the part of the arc where this counter E.M.F. was situated. For this purpose he inserted an exploring pencil in the arc and found the potential difference between this and the anode, both for the direct and for the superimposed alternating current. From this data he computed the counter E.M.F. near the anode, using the same method as that given above. This he found to be approximately 17 volts. A similar set of readings gave a "forward E.M.F." near the cathode of 6 volts, the two accounting for the 11 volts which was found in this particular case for the whole arc.

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[1] Phil. Trans., 203, A, 305, 1904.

This again is a use of words which in one sense is entirely correct, and yet is entirely misleading. Duddell undoubtedly found the value of $E-RI$ near the cathode to be a negative quantity, and since he had called this a counter E.M.F. when it was positive, it is not surprising that it should be called a "forward E.M.F." when it was negative. Yet to do so gives one the impression that there is an E.M.F. at the cathode of the order of six volts, which changes some other form of energy into electrical energy. In reality there is no reason whatever to suppose that there is any such E.M.F. at that point. All that Duddell's experiment showed was that $\partial E/\partial I$ was greater than E/I between the gas of the arc and the cathode. In other words, that the form of the curve showing the relation between the cathode drop and the current was that shown in Fig. 1.

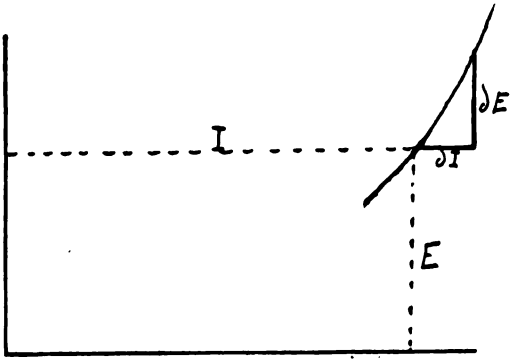

Fig. 1.

This is the same kind of curve that one finds when examining the relation between the potential difference and the current for discharge produced by Röntgen rays. No one ever speaks of a "forward E.M.F." with such discharge and there would appear to be no more reason for using that expression in connection with the arc.

Reinke[1] compared the counter E.M.F. from observations made on the whistling arc. His method was in principle the same as that of Duddell's, but the alternating current was produced by the arc itself, and not by a separate dynamo. The current in a whistling arc is equivalent to a small alternating current superimposed on a direct current. He says concerning it that "only about one eighth of the observed potential difference, namely, in this case about 6 volts, was used in overcoming the ohmic resistance of the arc, while the remainder, namely, about 40 volts, is to be considered a polarized potential difference, or counter E.M.F. (cathode and anode drop in potential)."

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[1] Verh. d. V. z. Beford. d. Gewerbfliesses, 83, 403, 1904.

We may sum up these results by saying that they show that the potential difference between the terminals of the arc varies when the current is varied in much the same way as the potential difference between the terminals of the cell of a storage battery, but they show nothing, of course, concerning the phenomena occurring after the impressed E.M.F. is removed. For this purpose we must examine the second group of experiments.

Residual E.M.F.—The second method of testing for a counter E.M.F. in the arc was to measure the residual E.M.F. after the impressed E.M.F. was removed. There is no question of the meaning of terms involved in these measurements and there is apparently no great experimental difficulty, and yet there has perhaps been as great disagreement among experimenters concerning such an E.M.F. as concerning any question in physics.

Edlund[1] not only suggested a counter E.M.F. as explaining the relation which he found between the potential difference at the terminals of the arc and its length, but endeavored to find a corresponding residual E.M.F. He passed a current from a number of cells through the vapor of the arc after the impressed E.M.F. had been removed, passing it first in the direction of the supposed E.M.F. and then in the opposite direction. He found the current in the first direction to be the greater. To compare the E.M.F.'s in the two cases he eliminated the resistance of the circuit by using Ohm's law, but Ohm's law does not at all apply to the case of discharge through hot vapors, and the supposition in this case introduced a very large error. He was led to believe that the counter E.M.F. was equivalent to 10 or 15 Bunsen cells.

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[1] Pogg. Ann., 134, 250 and 337, 1868.

But eight years before this Wild[1] had measured this same quantity by throwing the arc with a double-throw switch from the batteries to a high resistance galvanometer. He was not able to determine the E.M.F. accurately, because the resistance of the vapor between the carbons was not known to him, but he concluded that it was more than 100 times as great as the E.M.F. produced by a copper-German silver couple having the same difference of temperature at its terminals. This statement, indefinite though it is, is more accurate than any statement of residual E.M.F. made for several decades thereafter.

After Edlund's experiments there were a series of observations by different experimenters, giving all kinds of values from zero to two volts. The fact that many of the earlier experimenters found no residual E.M.F. is less surprising, since they were looking for one comparable with the supposed counter E.M.F. of the arc, and we can now say definitely that there is none of this order of magnitude. But that some of the later experimenters should have differed so decidedly as to a small residual E.M.F. is hard to understand. Lecher,[2] Luggin,[3] Corbino and Liga[4] concluded that there was no residual E.M.F. Strenger[5] concluded that if there was one it was small compared with 10 volts. Hertzfeld[6] thought at first that he had found one, but afterwards concluded that the observed effects were due to thermo-junctions outside the arc. Le Roux[7] found a residual E.M.F. but did not give its magnitude.

Arons[8] thought that the vapor remaining between the carbons after the impressed E.M.F. was removed was not conducting for small voltages, and that it was necessary to apply 18 volts in order to get any current through it.

Granqvist[9] showed that Arons's failure to get a current with lower voltages was due to using a non-sensitive galvanometer. Granqvist himself was able to pass a current from one Daniell cell. His method was similar to Edlund's, except that he used a rotating switch which closed the galvanometer circuit .0009 second after the main circuit was broken. He also assumed Ohm's law to hold, but his voltages were much smaller than those of Edlund's, so that the error which was introduced in his calculations was much smaller. He found a residual E.M.F. of .23 volt in the opposite direction to that of the impressed E.M.F.

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[1] Pogg. Ann., 111, 624, 1860.

[2] Wien. Sitzungsberichte, 95, 11, 992, 1887.

[3] Wien. Sitzungsberichte, 98, 1 and 192, 1889.

[4] Monash, Der Elektrische Lichtbogen, p. 134.

[5] Wied. Ann., 45, 33, 1897.

[6] Wied. Ann., 62, 435, 1897.

[7] Lum. Elec., 3, 285, and C. R., 92, 709, 1881.

[8] Wied. Ann., 57, 185, 1896.

[9] Beib., 22, 243, 1898.

Blondel[1] using a method almost identical with that of Granqvist concluded that there was no E.M.F. 1/600 second after the arc was interrupted as great as .16 volt. He used both solid and cored carbons, long and short arcs, changed the time between the disconnection of the arc from the main circuit and the connection to the galvanometer circuit, and in other ways varied the conditions as far as possible.

On the other hand Hotchkiss,[2] using an oscillograph having a period of .0002 second, found indications of an E.M.F. This was less than than .66 volt, but was still clearly discernible. Milkiewicz[3] found a residual E.M.F. of from 1.5 to 2 volts. And Becknell[4] found an appreciable E.M.F. existing for several seconds after the impressed E.M.F. was removed. He determined the amount by balancing it against a known potential difference by means of a sensitive galvanometer. The maximum was .63 volt, which occurred .3 second after the removal of the impressed E.M.F. When the arc was first extinguished the resistance was small, but increased very rapidly with the time. He showed that the residual E.M.F. was not due to the thermal junctions outside the arc.

It is very difficult to reconcile the work of Blondel with that of the others. Furthermore, it is hard to see how it could be otherwise than that there is such a residual E.M.F. It was shown by Dubs[5] and Olivette[6] that the current flows from a cooler carbon to one very hot. This may amount to several volts. All the phenemena observed by Becknell are such as would be expected from the difference in temperature of the carbons. There is probably a very small area on the cathode which is as hot as the anode, so that when the impressed E.M.F. is first removed the difference in temperature between the electrodes is small. The area of this high temperature on the cathode is small. It will, therefore, cool more quickly than the anode, so that the difference in temperature between the electrodes at first increases. This would correspond to the increase in the residual E.M.F. observed by Becknell. The observed residual E.M.F. is in the right direction, since the anode is hotter than the cathode after the arc is extinguished.

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[1] Lond. Elec., 39, 615, 1897.

[2] Trans. Amer. Phys. Soc., 2, 1901.

[3] Science Abs., 7, 360, and Beib., 27, 465, 1903.

[4] Phys. Rev., 21, 181, 1905.

[5] Centralbl. f. E. T., 10, 749, 1888.

[6] Lond. Elec. Rev., 31, 728, 1892.

Test of Explanation. — This explanation can be tested by changing in some way the temperature of the electrodes. The greatest change in the residual E.M.F. may be expected when the anode is cooled, for under ordinary conditions it is hotter than the cathode, and when cooled we should expect the E.M.F. to be diminished or reversed.

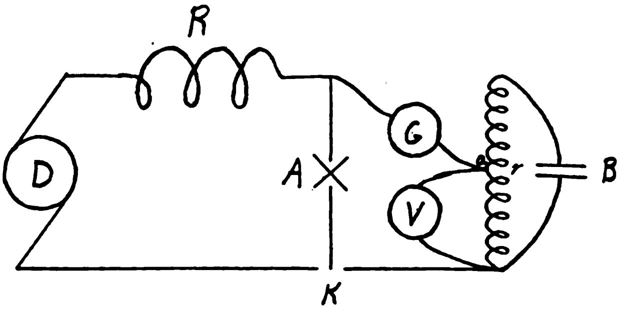

Fig. 2.

In the first experiments performed by the writer to test this explanation a hollow carbon was used for the anode and water was caused to flow through this. The method of determining the E.M.F. was the same as that used by Becknell. The connections are shown in Fig. 2. *A* is the arc, the current coming from the dynamo *D* through a regulating resistance *R*. *K* is the key for switching the arc from the dynamo circuit to the testing circuit. This circuit consists of the galvanometer *G* and the variable potential difference which is balanced against the E.M.F of the arc. The galvanometer used gave a deflection of 1 mm. with the scale 60 cm. from the mirror, when there was a potential difference of 3.5×10^{-5} volt. The potential difference in *r* was maintained by two cells at *B*, and could be varied by changing the point *a* on the resistance *r*. It was measured by a Weston voltmeter at *V*. A pendulum was used to change the connections at *K*. This had no advantge over the rotating wheel used by Becknell, except that there happened to be a pendulum arranged for this purpose in the laboratory. The time between opening the dynamo circuit and closing that to the galvanometer could be varied from .03 to .5 second.

It was found very difficult to balance the residual E.M.F. against a potential difference in *r*, because the E.M.F. never appeared to be twice the same. Becknell found it necessary to run the arc a minute before making observations and also to have it so that it did not run over the crater, but in the experiment here described it was not practical to fulfill either of these conditions. The arc could not be allowed to run for a minute, for if it was, the carbon was apt to be burnt away, so as to allow the water to leak through and put out the arc. And this difficulty could not be remedied by having thicker walls for the carbon tube, for with thick walls no effect at all could be detected when the water was sent through it.

Neither could the crater be properly formed, for the carbon was liable to be burnt through before this could be done. Moreover, with a cooled anode the arc is much more variable than with a hot anode. As a result no attempt was made to take a series of observations, but only to show definitely that the residual E.M.F. was reduced by cooling the anode.

The apparatus was first tested with ordinary cored carbons with results very similar to those of Becknell's. Then the cooled carbon was substituted for the hot anode. The outer diameter of this before the arc had burnt any of the carbon away was 12 mm. and the inner diameter was 4.4 mm. This was cooled by allowing water from the tap to run through it. The current was approximately 10 amperes and the length of the arc 2 mm. With the cooled carbon the E.M.F. was approximately .27 volt .03 second after the impressed E.M.F. was removed, decreasing gradually to .22 volt .17 second after the removal of the E.M.F. The corresponding numbers given by Becknell are .37 volt and .52 volt respectively. After .17 second the space between the electrodes had lost its conductivity to so great extent that further measurements were difficult.

The cathode was then cooled, but no change could be definitely shown. Since the hot point on the cathode is smaller than that on the anode and consequently must be expected to cool more rapidly than the anode, it would not be expected that cooling the cathode artificially would produce as large an effect as cooling the anode.

It was desired to show the change more strikingly, if possible, and for this purpose a hollow brass tube was used for the anode and this was kept cool as before by causing water to flow through it. A new difficulty was here met. The region between the electrodes was found to be nearly non-conducting after the arc was extinguished. It was only by having the electrodes very near together and by making the observation as soon after the impressed E.M.F. was removed as possible, that sufficient current could be passed between the electrodes to affect the galvanometer. Even then there was not always a movement of the galvanometer needle, but whenever there was, the deflection was such as to indicate a residual E.M.F. in the opposite direction to what it is normally. The time between opening the dynamo circuit and closing the galvanometer circuit was .03 second. The length of the arc was approximately 1 mm.

When the cooled brass tube was used for the cathode it quickly became coated with a layer of carbon and did not then behave differently from what it would if it had been a cooled carbon tube.

Residual E.M.F. in a Vacuum. — The arc was then tried in a vacuum of approximately .02 mm. of mercury pressure. Under these conditions the anode is not heated as quickly as in the air, nor does it appear to reach as high a temperature as in the air. The residual E.M.F. in this case is somewhat smaller than in air, but it depended so much on the length of time that the arc had been running and on the way that it happened to jump from one point to another that no measurements were made when both electrodes were carbon.

It was found that conditions could be much improved by using mercury instead of carbon for the anode. This, of course, never reaches a high temperature, and this arrangement showed most successfully of all the effects produced by having the anode at a comparatively low temperature. With this there was a residual E.M.F. of nearly one volt in the opposite direction to the ordinary residual E.M.F. The exact value depended very largely on where the cathode end of the arc happened to be at the time the switch *K* was changed; .03 second after the impressed E.M.F. was removed the residual E.M.F. was .8 volt or more, if the cathode had been at the bottom of the carbon, and somewhat less than this if it had been at the side.

As near as could be determined the residual E.M.F. did not decrease appreciably during the brief time that the region between the electrodes remained conducting. No current could be passed of sufficient amount to affect the galvanometer used, .14 second after the impressed E.M.F. was removed.

Although it is well known that a hot carbon in a vacuum gives off negative ions and charges bodies in its neighborhood negatively, it seemed well to test this explanation further by substituting a carbon pencil that could be heated by an electric current for the cathode of the arc in the preceding experiment. This was done and it was found that there was an E.M.F. between the hot carbon and the mercury in the same direction as before.

Summary.

A review of the literature on the subject of the counter E.M.F. of the electric arc shows that this expression has been used with several different meanings and the most recent meaning is not the same as that given to the expression when applied to a cell or an electric motor.

A review of the literature on the residual E.M.F. of the arc shows that the majority of experimenters have found such an E.M.F. whose value was somewhat less than one volt. This E.M.F. has been explained as being due to the difference in temperature of the electrodes. This was tested by cooling the anode by three different methods and the results were in harmony with such an explanation.

Colgate University,
July, 1909.

ON THE ELECTRICAL CHARACTER OF THE SOURCE OF LIGHT RADIATION.

By T. P. Irving.

THE experimental and theoretical studies of Lenard, J. Stark, Fredenhagen and others, concerning the source of light radiation, have led them to conclusions that differ rather widely.

Lenard[1] drew the following conclusions from his examination of the spectra and electrical wandering of substances volatilized by the electric arc and by the Bunsen flame in an electrical field: (1) "The emission centers of the principle series are electrically neutral metal atoms." (2) "The emission centers of the subordinate series are positively charged metal atoms, that is, such as have lost elementary quantities of negative electricity."

Fredenhagen[2] from his study of substances volatilized in oxygen and non-oxygen flames, in the divided Bunsen flame, and in the vacuum lamp arrived at conclusions that may be summed up as follows: (1) The principal series comes from an oxygen compound of the metal, and are due to the processes of combination between the metal and oxygen. (2) The subordinate series come from the metal itself, and are due to the process of change which a metal atom undergoes while acquiring or losing one, two or more negative electrons.

J. Stark[3] from his study of the Doppler effect in canal rays comes to the following conclusions: (1) "The carriers of the line series are positive ions." (2) "The centers of emission of the line spectrum are negative electrons." (3) "The carrier of the band-spectrum is a neutral atom constituting a system made up of a positive atom (Restatom) and a negative electron."

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[1] Drude Ann., IX., p. 642, 1902; XI., p. 636, 1903; XVII., p. 197, 1905.

[2] Drude Ann., XX., p. 133, 1906. Phys. Zeit., VIII., pp. 404, 729, 1907.

[3] Drude Ann., V., p. 421, 1901; VIII., p. 257, 1902; IX., p. 666, 1902; XIII., p. 669, 1904. Wien and Planck Ann., XXI., p. 401, 1906. Phys. Zeit., VII., p. 355, 1906. Die Elektricität in Gasen, Leipzig, 1902.

J. J. Thomson[1] from his experiments concludes: "The 'candle spectrum' is the spectrum of carbon when the atom is charged with negative electricity, or of some compound of carbon in which its atom is negatively charged." Further he finds the red and green hydrogen lines from both negative and positive hydrogen. W. J. Humphreys[2] thinks that possibly "the only origin of spectrum lines is the shock of 'restatoms' by swiftly moving negative corpuscles." Pringsheim[3] and Smithells[4] look upon chemical processes as simultaneous with the production of light radiation, and Pringsheim favors the view that the coloration of a flame is due to a reduction process. Hartley[5] holds that only the *D*-lines are present when metallic Na is volatilized in a flame, whereas the spectrum derived from some salt of Na gives not only principal lines, but also the subordinate series of lines. A. S. King,[6] C. de Watteville[7] and Paschen[8] think that light-emission is largely a thermal effect. R. W. Wood[9] has obtained a resonance spectrum from Na vapor by a method which precludes chemical action.

Thus it appears that some ascribe the light-radiation to matter in an electrically positive, negative or neutral condition; others ascribe it to chemical processes; and others to thermal, resonance, or other phenomena.

It might be suggested that the electrical condition and the chemical and other processes are but diverse causes of one and the same final state of a substance, and that such final state is necessary to the production of the radiation. In one case this state may be brought about by oxidation or reduction, in a second by light excitation, in a third by electrical condition, and so on. If the radiation arises from such a final state due to different causes acting singly or in combination, the interpretations given by the different investigators to their results are probably right so far as attempted, but are incomplete. A theory, broader than the present rather special and conflicting theories, is needed to harmonize the interpretations of experimental evidence. The formulation of such a theory requires further knowledge. The various conditions under which the radiation is obtained must be examined and compared; its supposed different causes must be studied more thoroughly not only in themselves, but also in relation to one another.

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[1] Proc. Roy. Soc., LVIII., p. 244, 1895.

[2] Astrophys. Jour., XXVII., p. 200, 1908.

[3] Weid. Ann., XLV., p. 459, 1892.

[4] Phil. Mag., 5, XXXIX., p. 133, 1895.

[5] Proc. Roy. Soc., A, LXXIX., p. 242, 1907.

[6] Astrophys. Jour., XXVII., p. 353, 1908.

[7] Phil. Trans., A, CCIV., p. 139, 1905.

[8] Weid. Ann., L., p. 409, 1893; LI., pp. 1, 40, 1894; LII., p. 209, 1894.

[9] Phil. Mag., 6, XV., p. 581, 1908.

The author of this paper has attempted to acquire some of the needed knowledge, through studying simultaneously the electrical and spectroscopic characters of the divided Bunsen flame, both in its simple state, and when colored by metals or salts.

(*A*) DESCRIPTION OF APPARATUS AND METHOD OF INVESTIGATION.

The burner used to separate the cones of the Bunsen flame consisted of an inner brass tube (*a*) 22 cm. long and 1.4 cm. in diameter and an outer glass tube (*b*) 15 cm. long, 5.1 cm. in diameter at the bottom and 3 cm. at the top, fitted over the brass tube by means of a bored plug. This form of glass tube was used by Fredenhagen. It is much more satisfactory than the straight tube, since it gives an outer cone so steady and regular that any bending of this cone, due to an electrical field, can be detected with certainty. So far as could be ascertained no vapors were introduced into the cones by the substances of the tubes. The device for mixing the gas, air and vapor consisted of a glass bulb (*c*) 6.7 cm. in diameter, fitted over the bottom of the tube (*a*) and connected at its side with the neck of the retort (*d*) which contained the solution. The gas entered at the bottom of the bulb through the tube (*e*), 1.5 cm. in diameter, drawn out so as to leave an opening .08 cm. in diameter for the issuing of the gas, and extending up into the brass tube a few mm. By this arrangement of burner and mixing device the cones could be kept burning steadily for a long time.

The method used for introducing the vapor into the burner is that described by Beckmann.[1] An acid solution of a salt and a few small pieces of coppered zinc are placed in the retort (*d*). The metallic vapor given off together with the proper quantity of air

[1] Zeitsch. für Angewandte Chemie, p. 561, 1907.

admitted through the loosely stoppered tubular of the retort, pass into the bulb (*c*) and mix in the tube (*a*) with the gas issuing from (*e*). The gas used was the illuminating gas from the mains of the city of Washington.

When the supply of air and gas has been carefully regulated, the inner cone burns at the top of the brass tube (*a*) and the outer cone at the top of the glass tube (*b*). If no vapor is introduced, only the inner and outer cones are seen. When a vapor, Cu excepted, is allowed to enter through the inner cone, a long flame burns just above the top of that cone. This is probably the inner mantel of the Bunsen flame. In most cases there is coloration of the outer cone when vapor is introduced.

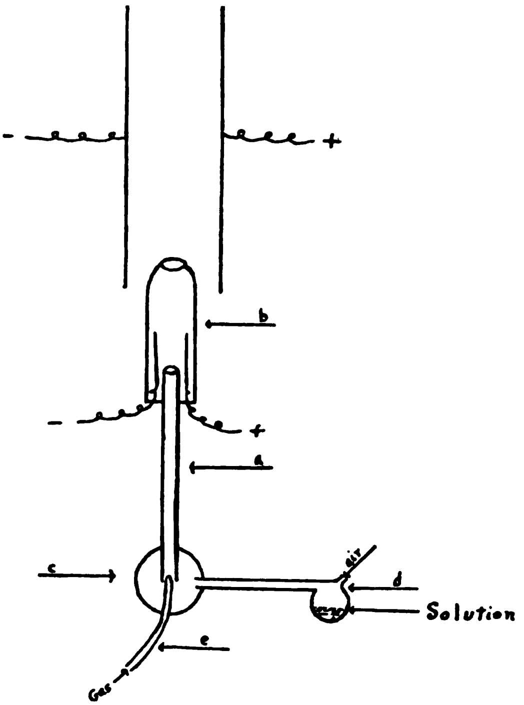

Fig. 1.

The electrical examination of the outer cone was made by placing it between two metal plates, 30 cm. square, charged by a friction machine to potentials that ranged as high as 50,000 volts. In the investigation of the inner cone and the flames due to the vapors, two metal electrodes, 1 cm. wide, 6.5 cm. long, were placed within the glass tube on opposite sides of the inner cone, and charged by wires that passed through the plug at the bottom of the glass tube. The corners of these electrodes were rounded off so as to prevent leakage. They were so placed as to be free from contact, either with the tube or with the flame due to the vapor. Even with great precaution the strength of the field between these electrodes is small, since the conditions within the glass tube are such as favor electrical conduction. Another method of studying the electrical character of the lower cone and the vapor-flame consists in establishing a field between the tube (*a*) and the tube (*b*). This was done by placing one pole of the machine in connection with the tube (*b*), while the other was attached to the brass tube. Tests were made also in which both sets of plates were charged at the same time, to see if the electrical action of the field on the flame at the inner cone produced any change in the character of the outer cone. The spectroscopic investigation was made with a large single prism spectrometer.

(*B*) RESULTS OF THE ELECTRICAL AND SPECTROSCOPIC EXAMINATION.

1. *Electrical Character of the Outer Cone.*

The outer cone of the divided Bunsen flame is a hollow cone-shaped bluish flame, in its simple state. The electrical character of this simple cone was examined by observing the effect produced by an electrical field, due to the two large oppositely charged metal plates. The position of these plates, relative to the flame and to each other, was varied. At times the lower edges of the plates were about 5 cm. below the base of the outer cone; again the plates were raised and their lower edges were about 3 cm. above the base of the cone. One plate was placed close to the flame, the other was about 8 cm. distant. Then both plates were placed close to the flame, or withdrawn equally to a distance. Under none of these conditions could any electrical wandering be observed. Sometimes the cone seemed to be distorted slightly, but this may have been due to air currents.

Then the outer cone was studied in the electrical field described above for each of the following salts, Li_2CO_3, LiCl, NaCl, KCl, RbCl, CsCl, $CaCl_2$, $MgCl_2$, $SrCl_2$, $CdCl_2$ and $BaCl_2$, introduced into the cone by the Beckmann method. The results of the examination with salts of the first Mendeléeff group will be given first.

First Group. — The Li salts were introduced in large quantities in order to color the outer cone. As soon as the cone was strongly colored, the electrical examination was made. The field seemed to produce no effect upon the cone thus colored. At times a slight distortion might be noticed, but no definite wandering could be detected.

NaCl was introduced in a sufficiently large quantity to color the outer cone. At first it was thought that there was no electrical influence, but more careful observation showed a slight wandering towards the negative electrode.

Then KCl was used. The wandering of the outer cone towards the negative plate was more marked than in the case of Na. There was also a very slight bulging of the flame towards the positive plate. RbCl gave an outer cone that was drawn towards the negative plate a little more strongly than the KCl cone. When CsCl was used the outer cone was very positive, being inclined to the negative electrode at an angle of about 45°. In the case of this salt a very slight brush-like projection extending towards the positive electrode was observed.

Second Group. — The $MgCl_2$ and $CaCl_2$ outer cones show a very strong wandering towards the negative plate. For these two salts the wandering is about equal. In $SrCl_2$ the effect of the field is small; in $CdCl_2$ it increases; in $BaCl_2$ the wandering is very strong towards the negative plate, and at the same time the substance coloring the cone is drawn to the side nearest the negative plate. This effect was so strong that, frequently on the side nearest the positive plate, the cone was almost free from the Ba-salt color.

Salt Introduced into the Outer Cone Directly. — An examination of the electrical character of the outer cone was made also when a bead of salt was introduced into the outer cone directly. Several of the salts named above were tried, but the most satisfactory and definite coloration was obtained from NaCl. When a bead of this salt was placed in the cone on the side nearest the negative electrode, the Na color was drawn towards this side, and besides there was a slight bulging of the cone towards the negative plate. This bulging was seen first near the tip of the cone, but as the field was strengthened, it was lowered and appeared as a sharp protrusion at the side of the cone. When the bead was placed on the side nearest the positive electrode, the color was carried across the top of the flame towards the negative plate and at the same time the tip of the cone was bent towards this plate.

2. *The Electrical Character of the Flame Above the Inner or So-called Carbon Cone.*

When a metal vapor was introduced into the lower cone by the Beckmann method, a flame about 10 cm. long appeared above the cone. In the examination of the electrical character of this flame, the following salts were used: NaCl, Li_2Co_3. CsCl, RbCl, KCl, $SrCl_2$ and $BaCl_2$. The first study was made by means of the two small metal electrodes within the glass tube. In all cases the wandering of this flame was strongly towards the negative plate. Further investigation was made to determine whether the wandering varied for the different salts, as was the case in the outer cone. No such variation was found. The flame bent towards the negative electrode at about the same angle which was as large as 45°, for a potential of 5,000 volts, whether Li_2Co_3 or $BaCl_2$ was used.

3. *Electrical Character of the Inner or Carbon Cone.*

When no salt was introduced, the carbon cone alone was seen at the top of the brass tube. Its electrical character was examined by means of the two small electrodes within the glass tube. When a field was established between these plates, the carbon cone was torn off the brass tube on the side towards the negative plate. It was flattened out and extended towards the negative electrode. This showed that it was strongly positive.

4. *Electrical Character of the Cones Combined.*

When the two cones were burning at the top of the glass tube, that is, when the flame is an ordinary Bunsen flame, the wandering of this flame was towards the negative plate. It has been remarked, that the outer cone in the case of a Li-salt, showed no wandering. However, if the inner cone and the Li-flame above it were allowed to rise in the glass tube so that the top of the Li-flame touched the outer cone, immediately the outer cone began to bend towards the negative plate. This wandering continued as long as the Li-flame and the outer cone were in contact, and sometimes it has been observed that even after the Li-flame had been separated from the outer cone, the bending continued for a few seconds. It would appear that something positive was given to the outer cone from the Li-flame.

5. *Two Sets of Electrodes Combined.*

When the large outer electrodes and the two small plates within the glass tube were connected, so that one inner and one outer plate were charged positively, and the two others, negatively, the outer cone and the flame at the top of the carbon cone wandered towards the negative plates. If the inner electrode was charged positively, and an outer negatively, the salt-flame inside of the glass tube was driven away from the positive electrode, and the outer cone was drawn towards the negatively charged plate. The salts used in these investigations were $SrCl_2$ and $BaCl_2$.

6. *Phenomenon of Color Disappearance.*

This investigation was carried out by the method described in Section A. When the brass tube was charged positively and the glass tube negatively, an interesting phenomenon occurred. The carbon cone was raised above the top of the brass tube from .5 to 2 cm., this distance depending on gas pressure and air supply. The cone did not retain its original shape, but was flattened out and its edges tended upwards. At the same time the salt flame, that burned at the top of the cone before, disappeared, and only traces of color could be seen above the flattened carbon cone. This color was strongest at the cone's edges. Next the air and gas supply was so regulated that the carbon cone tended to "strike back" into the brass tube. As soon as the carbon cone was drawn so close to the top of the brass tube that any contact was made between the two, immediately a stream of color appeared above the point of contact.

When the brass tube was charged negatively and the glass tube positively, the cone was drawn down over the brass tube like a cap. The colored flame above it did not disappear as in the preceding experiment. The intensity of its light seemed to be diminished somewhat, but this may have been due to the fact that the flame was spread out and the light was not so concentrated. In these two experiments the action of the carbon cone was the same whether a salt was introduced or not.

7. *Spectroscopic Examination.*

In the spectroscopic examination of the outer cone and the flame at the inner cone for the salts NaCl, LiCl, KCl, $SrCl_2$ and $BaCl_2$, the principal series lines could be found in both the outer cone and the flame above the inner cone. Two lines of the first subordinate series appeared in the inner flame of NaCl. The carbon cone gave the band-spectrum of carbon.

SUMMARY.

The results for the flame at the top of the inner cone are: (1) that when placed in the electrical field between the two small charged inner plates, it *always* wanders towards the negative plate, and that its inclination is about the same, for the same strength of field, irrespective of the salt used; (2) when studied in a field set up between an inner electrode positively charged, and an outer electrode negatively charged, it is repelled by the positive plate; (3) as soon as this flame comes in contact with a neutral outer cone, wandering towards the negative plate occurs in this outer cone. From these facts we may conclude that this flame is always electrically positive.

The results for the inner or carbon cone are: (1) it is drawn very strongly to the inner negative plate; (2) it is repelled by the brass tube when this tube is charged positively, and it is attracted to this tube when negatively charged. Hence the inner or carbon cone is always positive.

The results for the outer cone are: (1) for the five elements of the first Mendeléeff group, that were studied, the electrical wandering, when present, is towards the negative plate, and increases with the atomic weight of the metal. Li shows no wandering; the wandering begins with Na, increases for K and Rb and reaches a maximum for Cs; (2) for the elements studied of the second Mendeleéeff group, there is always electrical wandering towards the negative plate, but there is no relation between atomic weight and electrical wandering. $MgCl_2$ and $CaCl_2$ show a very strong wandering towards the negative plate; for $SrCl_2$ the wandering is small, for $CdCl_2$ it is larger than it is for $SrCl_2$, but smaller than for $MgCl_2$ and $CaCl_2$, for $BaCl_2$ the wandering is greatest.

The results for the spectroscopic examination are: (1) the principal series lines come from the flame above the carbon cone, which flame is always strongly positive, whatever salt is used. They come also from the outer cone, which is neutral, slightly positive or strongly positive according to the salt used; (2) the first subordinate series lines come only from the positive salt-flame, which is just above the inner or carbon cone for all the salts; (3) the so-called band-spectrum of carbon arises from the inner cone, which is always strongly positive.

(*C*) DISCUSSION OF RESULTS.

The primary object of the investigation described in this paper was to study the electrical character of the source of the light radiation. As the examination advanced, it became more and more evident, that although we may study the character of the source of the light radiation experimentally, yet in the interpretation of results, account must be taken of processes that are possibly closely connected with the electrical phenomenon. A complete analysis and interpretation of the results recorded in this paper would require a fuller knowledge than we have at present, of the chemical and thermal processes involved in the divided Bunsen flame. In the consideration of the chemical or other processes that occur in this flame, where it has been necessary to take account of such processes, the opinions and theories of other investigators concerning them have been used.

The electrical wandering of the outer cone for different elements of the first Mendeleéeff group is progressive like the atomic weights and electrical conductivity, and bears some relation to these properties. In the following table atomic weight, electrical conductivity and electrical wandering are given. The bending of the cone from the vertical is given in degrees. The electrical conductivity for these different metal vapors is the conductivity of a flame into which these different vapors have been introduced.

	Li.	Na.	K.	Rb.	Cs.
Atomic weight	7	23	39	85	132
Electrical wandering	0°	9°	20°	22°	45°
Electrical conductivity	2.5	8.5	64	82	116

In the elements studied for the second Mendeléeff group, elements of high and low atomic weights give the larger wanderings.

	Mg.	Ca.	Sr.	Cd.	Ba.
Atomic weight	24	40	87	112	137
Electrical wandering	36°	36°	18°	27°	45°
Electrical conductivity			Not known.		

In using the Beckmann method, metal vapor enters the inner cone. To suggest a possible explanation for the variation of electrical wandering in the outer cone, we must inquire into the changes, chemical and electrical, which the metal vapor undergoes after it enters the inner cone. What electrical character has it when it first enters the inner cone? This question, it seems, is answered by the phenomena of the disappearance of the salt-flame when the glass tube is negatively charged and the inner cone driven up from the top of the positively charged brass tube. An explanation of this occurrence is that the metal vapor is strongly positive, and that when it reaches the top of the brass tube, it is drawn towards the negatively charged brass tube, and thus prevented from entering the inner cone. This explanation is supported by the fact that whenever coloring is seen in the inner cone it appears at the extended edges, as if some metal vapor passing beneath the flattened cone at times came in contact with those edges. Again, as soon as the inner cone came in contact with the brass tube there was a sudden appearance of color as if the vapor was continually passing over the upper edges of the brass tube.

According to Fredenhagen, an oxidation process occurs in the flame above the inner cone, the oxidation continues in the outer cone, whenever the color of that cone corresponds to the elements used. While this vapor is undergoing the oxidation process in the flame above the inner cone it is still very positive and equally so for all the salts studied in both groups, as is evident from the equal wanderings of all salt-flames. But in the outer cone it has changed, being neutral, slightly or strongly positive according to the element introduced into the inner cone. During this oxidation process, therefore, Li becomes electrically neutral, Na, K and Rb become less strongly positive, while Cs apparently remains as strongly positive as before.

It might be suggested that while the metal is uniting with the oxygen it is gradually losing its positive character, and that when it has been brought to the state of a complete oxide it has become electrically neutral. In the last stages of this oxidation process it may be possible that the neutral effect has been brought about before the oxidation process has been completed. This would explain the neutral character of Li in the outer cone. In Cs or Ba the oxidation process may take place much more slowly, and the positive charge of the metal vapor would not have been neutralized when the vapor of the elements would reach the outer cone.

An explanation of the neutralization process may be offered on the ground that there are free negative electrons present in the two cones and the salt-flame. These electrons would unite with the positive metal ions that enter through the brass tube. These electrons may be present in much larger numbers in the flame and cone within the glass tube than in the outer cone.

Such a supposition would explain the apparent discrepancy between Lenard's results, that the outer edge of the Bunsen flame showed no wandering when a NaCl bead is placed in the edge, and the wandering effect observed when a bead of NaCl is introduced directly into the outer cone. In Lenard's experiment with the ordinary Bunsen flame, the cones being so closely united, the free negative electrons in the inner part of the flame would pass to the outer edge more readily than when the cones are separated as in the divided Bunsen flame. Therefore the outer cone in the ordinary Bunsen flame would contain a larger number of free electrons than the outer cone of a divided Bunsen flame, and we would expect the neutralization of a vapor to take place more quickly in the former than in the latter.

The question arises, how can the relation between electrical wandering, atomic weight and electrical conductivity be explained? A possible explanation could be offered if it be supposed that the free negative electrons are present in the two cones as stated above, and that these electrons are connected in some way with the conduction of electricity by flames. These assumptions together with the suggestion made to explain the neutralization of a metal vapor while passing through the cones may give some answer.

In the first Mendeléeff group the electrical wandering increases with atomic weight. This means that the greater the atomic weight of an element the longer a metal remains positive, and this occurs either because the electrical force acting between electrons and positive ions decreases as the atomic weights of the metals increase, or because the heavier the element, the larger the number of electrons necessary to neutralize it. This would give an explanation of the observed relation between atomic weight and electrical wandering.

Now, as has been supposed, the more neutral the outer cone, the less the number of free negative electrons present in the cone. The presence of a large number of negative electrons may give rise to that bulging or brush-like effect which has been observed in certain outer cones. But if the conduction of electricity depends upon these electrons then the greater their number in a cone the better conductor the cone will be, therefore, the more positive the cone, *i. e.*, the greater number of electrons in it, the better a conductor it will be. Then, the more positive the outer cone, *i. e.*, the greater the atomic weight of the metal from which the outer cone comes, the better it conducts. These considerations would give an explanation of the observed relation between electrical conductivity and electrical wandering.

The most important result of this investigation is the demonstration of the presence of principal series lines in sources whose electrical character presents qualitative differences. This fact is brought out most clearly in the case of Li. The Li outer cone is electrically neutral and principal series lines are obtained from it; the Li flame above the inner cone is always positive and principal series lines are obtained from it also. Therefore the principal series lines of Li arise from a positive or neutral source.

For Na, K and Rb, the outer cone contains some positive matter. This means that for these metals, either the positive metal ions have not been fully neutralized as yet, or that all the positive metal ions of an element are not neutralized with the same readiness and therefore in this outer cone we may have positive metal ions and neutral particles as well. The latter explanation, namely, that there are positive and neutral light-giving particles in the outer cone seems to be the more probable in view of the fact that only part of the cone appears to be affected by the electrical field, whereas part is not influenced by the field and therefore is neutral. When the outer cone is strongly positive, as with Cs for example, there are few if any neutral particles in this cone, and the light-emission of the outer cone comes in great part, if not wholly, from a positive source.

The value of this result, namely, that principal series lines come from a neutral as well as from a positive source, is that it helps to show that the results of Lenard and Stark in regard to the electrical character of the source of principal series lines are not really in disagreement; that the different conditions under which they worked gave them really only apparently different results as to the electrical character of the source. In the investigation described above conditions are such that the results of both Stark and Lenard are obtained. The conditions of investigation might be such that the results of Stark, Lenard and Thomson could be obtained simultaneously, *i, e.*, we could have principal series lines arising from positive, negative and neutral sources.

The most obvious inference is that, at least for principal series lines, the source need not be always positive or always neutral or always negative but that the source may be positive or negative or neutral, depending upon the conditions under which emission is brought about. The electrical character of the source of the principal series lines seems to be an accidental rather than an essential condition.

The investigation shows further that the subordinate series lines have a positive source as found by Stark and Lenard.

The band-spectrum, so far as studied in this investigation, *i. e.*, for carbon, arises from the inner or carbon cone, *i. e.*, from a positive source. Thomson found the band-spectrum for carbon arising from a negative source. Stark found the source for the band-spectrum of hydrogen and other elements to be neutral, and concluded that the source of the band-spectrum is neutral. Here as in the case of principal series lines, the band-spectrum seems to be due to sources that may be positive, negative or neutral in electrical character. Here also we may make the same inference as for principal series lines, namely, that the electrical character of the source of the band-spectrum is accidental and not a necessary condition.

I take this opportunity of acknowledging my obligation to Drs. Griffin and Shea for continued suggestion and assistance throughout this investigation, a considerable portion of which has been conducted in collaboration with the latter.

Physical Laboratory,
Catholic University of America,
May 31, 1909.

THE NOZZLE EXPANSION OF AIR AT HIGH PRESSURE.[1]

BY W. P. BRADLEY AND C. F. HALE.

INTRODUCTION.

THE problem of reducing the permanent gases to the condition of static liquids by a continuous process and on a large scale was solved in 1895 by Linde and by Hampson. Their liquefiers differ in construction, but in each of them the gas is cooled to the liquefaction point by expansion through a throttling valve or nozzle.

Two views are held at the present time as to the cause of the absorption of heat during this method of expansion. Linde[2] explains it by the Joule-Kelvin effect, and his view appears to be accepted by the majority of writers on this subject. According to this view the heat is absorbed in overcoming molecular attractions which exist in the gas at the higher pressure. The formula commonly given is

$$d = 0.276\,(p_2 - p_1)\left(\frac{273}{T}\right)^2,$$

derived from the results of Joule and Kelvin,[3] in which d is the cooling effect in degrees, p_2 and p_1 are the initial and final pressures in atmospheres, and T is the initial temperature on the absolute scale.

This view has been vigorously attacked by Pictet,[4] who insists that the heat absorption is to be accounted for by the *external* work of overcoming the pressure which prevails in front of the valve. In the Hampson liquefier and in the older single-circuit liquefier of Linde, this work consists merely in pushing back the external atmosphere. Pictet computes the cooling effect by means of the formula[1]

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[1] For generous support in the prosecution of this inquiry we are indebted to two grants from the Hodgkins Fund of the Smithsonian Institution.

[2] Wiedemann's Annalen, *57*, 328, 1896. Also abstract of paper by Schrötter, Sci. Am. Supplement, December 21, 1895.

[3] Philosophical Transactions, *152*, 579, 1862.

[4] Zeitschrift für comprimirte und flüssige Gase, 7,1, ffg., 153, ffg., 1903; *8*, 8, ffg., 1904; *9*, 51 passim to *10*, 110, 1905–1907.

$$d = \frac{T\left(1 - \frac{1}{p}\right)10.33}{273 \times 1.293 \times c \times J + 10.33},$$

where d and T have the same significance as before; p is the initial pressure in atmospheres; c is the specific heat of air at constant pressure; and J is the mechanical equivalent of heat.

The essential differences between these two formulas are obvious. If the view of Linde is correct, the cooling effect is a linear function of the difference in pressure on the two sides of the valve. Thus, for air at zero, a difference of 100 atm. corresponds to a fall of 27.6 degrees; a difference of 200 atm. to a fall of 55.2 degrees. Where the initial pressure is high and the final pressure is one atmosphere, the latter indeed may be ignored, and the cooling effect may be considered proportional to the initial pressure.

According to Pictet, on the other hand, air at zero will fall 60.7 degrees when expanding from 101 atm. to one atmosphere, as against 61 degrees when the initial pressure is 201 atm. Pictet indeed particularly emphasizes the view that pressures higher than 25–30 atm. neither can nor do have any considerable influence in the way of increasing the cooling effect unless the liquefier is grossly deficient in interchanger surface.[2] It is clear of course that if the expanded air passes over the coils of the interchanger without reaching the temperature of the inflowing air of high pressure, the defect may be remedied more or less by the use of a higher initial pressure, whereby the same mass of air will flow more slowly down the tubing to the expansion valve.

The divergence of the two views is equally noteworthy as regards the effect of initial temperature. According to Linde, the cooling effect at the valve *increases* rapidly as the initial temperature falls. Thus, for a fall of pressure of 200 atm. at zero it is 55.2 degrees. At − 82° it is 112.2 degrees, or enough to bring the air to the liquefaction point at one atmosphere. According to Pictet's formula it *decreases* for lower initial temperature. Thus, it is 61 degrees if the initial temperature is zero, but only 42.7 degrees if the initial temperature is — 82°. The final temperature in the latter case is — 124.7°, which is 70 degrees distant from the liquefaction point at one atmosphere.

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[1] In the original formula the cooling effect is represented by θ instead of d.

[2] *Loc. cit.*, 7, pp. 40–41.

It may be remarked in passing that according to Pictet's view it is not easy to account for the production of liquid air at all when a liquefier is fed with air under a pressure above the critical pressure. According to his value of d, air at 55 atmospheres must be cooled to — 172.3° before the expanded air can reach the liquefaction point at 194.5°. If the pressure is 200 atm. the air must still be cooled to — 172°. In either case, as the critical pressure is 51–53 atm. (*loc. cit.*, p. 19), the air would be completely liquefied before it reached the expansion valve (p. 17). As liquid while passing the valve can absorb no heat (p. 22), the functioning of the liquefier would cease (p. 23) until the temperature of the air just before expansion should rise above the critical temperature (= — 141°, p. 8). Then the heat absorption would begin again while the gas passed the valve, but would cease at once as soon as the temperature fell again below the critical temperature. Apparently an interchanger could functionate only so long as the initial temperature remained above — 141°. But if — 141° is the lowest effective initial temperature, the lowest obtainable final temperature according to Pictet is — 170.5° at 201 atm., or — 170.1° at 55 atm. To produce liquid at one atmosphere, about 25 degrees more are required, and these could never be realized so long as the initial pressure was higher than the critical pressure, *i. e.*, as long as further cooling would result in complete liquefaction inside the pipes.

Although it has been shown, contrary to the view of Pictet, that increase of pressure has a profound influence on the yield of a liquefier, an influence which cannot be accounted for on the basis of inadequate interchanger surface;[1] and that the air at 200 atm. has *not* reached its critical temperature before expansion,[2] but that its temperature is certainly not lower than — 100° when liquid air is collecting in the reservoir at atmospheric pressure, the general subject seemed of sufficient importance to call for a more detailed experimental examination.

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[1] Bradley and Rowe, PHYS. REV., *19*, 339, 1904.

[2] Bradley and Hale, PHYS. REV., *19*, 391, 1904. *Cf.* also Schröter, *loc. cit.*

EXPERIMENTAL PART — THE APPARATUS.

Measurement of Pressures. — Pressures were measured by a metallic gauge with a seven and one half inch dial. This was graduated to 4,000 lbs. in divisions of ten pounds. When the lag of the pointer was corrected by tapping upon some effective spot of the casing, it was possible to read to single pounds. The gauge was calibrated with care by the use of a frictionless hydrostatic balance with lever arm, pan and weights.

Measurement of Temperatures. — Four platinum resistance thermometers were used, one in the high pressure circuit just above the valve, and three in the expanded air at different distances from the valve, in a manner which will be set forth below in detail. They were made of pure insulated platinum wire, 0.0798 mm. in diameter and 35–40 cm. in length. The wire was wound upon the end of a heavy copper lead, which in addition to its two-ply cotton insulation was wrapped with adhesive insulating tape. The latter helped to keep the turns of the platinum wire in place. One end of the platinum wire was soldered to the tip of the copper lead upon which it was wound. The other end was soldered to a similar copper wire which served as the other lead.

Resistances were measured by means of a Wheatstone bridge and a D'Arsonval galvanometer. The maximum sensitiveness was about .01 ohm, corresponding to 0.3°. This was quite sufficient, inasmuch as the temperatures to be read could scarcely be held constant to less than that amount.

The thermometers were carefully calibrated in pure cracked ice, in a carbon dioxide-ether mixture, and in pure liquid oxygen. The oxygen was prepared from sodium peroxide and water, and was purified by passing it through a saturated potassium hydroxide solution, through conc. sulphuric acid, previously boiled to expel air, and through phosphorus pentoxide. Great care was taken to remove traces of air from the apparatus. The gas was condensed by means of liquid air boiling under atmospheric pressure, and was perfectly clear and limpid. When enough liquid oxygen had accumulated to cover the thermometer, the liquid air bath was lowered and the oxygen was allowed to distil away under atmospheric pressure. To avoid superheating, pure oxygen gas was allowed to bubble through the liquid from a capillary tube. Allowing for the resistance of the copper leads and for the barometer readings, and assuming that the normal boiling point of oxygen is — 182.8°,[1] and that of the carbon dioxide-ether mixture is — 78.34°,[2] the curve of resistance was practically rectilinear. As the error involved in assuming the curve to be rectilinear from zero to — 78.34°, and from — 78.34° to — 182.8° respectively was entirely negligible, this course was adopted.

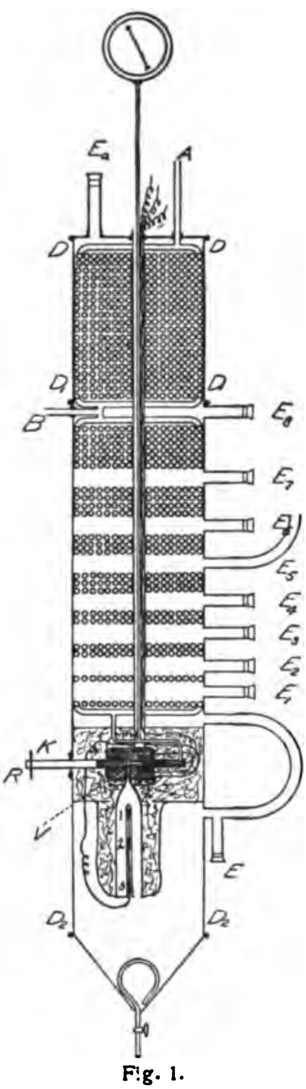

Fig. 1.

The Interchanger.— Previous experience had demonstrated that an interchanger, such as is found in all air liquefiers, would be the most convenient method of securing constant low temperatures over a wide range. The plan was the simple one of constructing such an interchanger in sections, suitably spaced from one another vertically, and provided with a corresponding number of exhaust pipes for the expanded air. The arrangement is illustrated in Fig. 1.

The interchanger consisted of two tubes of equal length, wound in parallel so as to save friction. The tubing was of seamless-drawn copper, of 4.8 mm. internal diameter and 0.8 mm. wall. The discs when wound were about 11 cm. in diameter. A

[1] Travers, Sentner and Jacquerod, Zeitschrift für physikalische Chemie, *45*, 416, 1903.

[2] Holburn, Annalen der Physik (4), *6*, 245, 1901.

space of 1.5 mm. was left between adjacent turns of tubing in each disc, and a similar space between adjacent discs in each section. There were nine sections in all, containing respectively 1, 1, 2, 2, 3, 3, 4, 6 and 15 discs of tubing, reckoned from the valve upward. The space between adjacent sections was about 3 cm.

The interchanger was wrapped in three layers of flannel, and was provided with a very snugly fitting case of tin, the lower conical end of which served as a reservoir for liquid air when the temperature was low enough to produce it. At the side of the casing, at a level corresponding to the spaces between the sections of the interchanger, eight wooden outflow tubes, E_1–E_8, were attached, the flannel wrapping of the interchanger being suitably cut away at the inner end of each. In the cap at the top was another outflow tube E_9 corresponding to the usual one in an air liquefier.

For most of the work, except that at low pressures, the thick upper section of the interchanger was superfluous, and accordingly provision was made for cutting it out from the air circuit when desired, and for the introduction of the compressed air at *B*, between it and the next lower section. When *B* served as the inlet, *A* was capped, and vice versa.

The casing was made in two sections, joined at D_1D_1 by bolted flanges with rubber packing. By unbolting at D_1D_1 and withdrawing the valve stem *R*, the interchanger and valve could be lifted out of the lower section of the case, thus greatly facilitating adjustments or necessary repairs. In a similar way the conical base of the casing could be removed at the joint D_2D_2, as well as the cap at *DD*. The casing was heavily wrapped with hair felt for the sake of insulation from the heat of the room.

When the air entered at *A*, for example, it passed through the entire length of the tubing of the interchanger to the valve *V*. After expansion through the valve it passed out of the case, either at *E*, without any contact with the interchanger, or at some one of the other exhausts, E_1–E_9, after passing over one or more sections of the interchanger. Except for a circumstance which will be explained later, only one exhaust was opened at a time.

In passing downward through the interchanger, parallel-wound though the latter was, the air met with a certain amount of frictional resistance. For this reason the pressure gauge was connected with the system at a point as near the valve as possible. This connection was made through the core of the interchanger, as may be seen from the figure.

The Valve. — The greatest care was observed in the construction of the valve, with a view to preventing interchange of heat between the air before, and the air after, expansion. A valve constructed of metal, as is usual in air liquefiers, must itself act in some degree as an interchanger, owing to the conducting power of its walls. In order to avoid this source of error as far as possible, the valve walls and seat were made of vulcanite. The details are shown in Fig. 2.

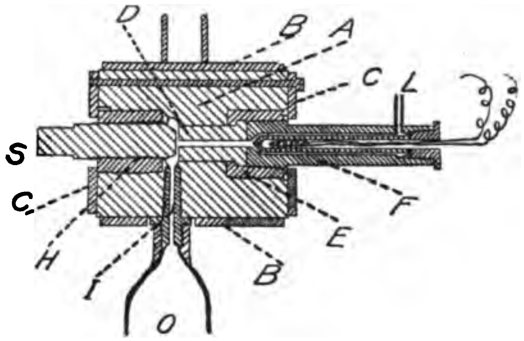

Fig. 2.

A is a vulcanite cylinder, 5 cm. long and 4 cm. in diameter. In order to reinforce it against bursting, it is surrounded with a cylindrical brass sleeve *B*, which is cut horizontally at a point not shown in the figure, and is provided with flanges in such a way that it can be bolted around the vulcanite as tightly as may be necessary. Incidentally, it is this brass sleeve which holds the valve in place with reference to the interchanger, for it is attached to the lower end of the tubular core of the latter. For longitudinal reinforcement, circular brass plates *CC* are provided at the ends of the vulcanite, and these are held firmly together and against the ends of the vulcanite by bolts, one of which is shown. The plates are not in metallic contact with the sleeve.

The valve seat *D* is a separate piece of vulcanite, fitting snugly into the core of the main block. It is carried by the brass sleeve

E, which backs against one of the reinforcing plates for greater strength, and is held firmly in place in the sleeve by the hollow brass screw-plug *F*, which will be described presently.

The valve stem *S* is of brass. In preliminary experiments we used one of vulcanite for the sake of better insulation, but it proved very unsatisfactory. The pressure upon the stem, while not excessive, is considerable, and the threads are apt to wear. If the stem fits snugly into its sleeve, it is apt to stick fast at low temperatures, with the attendant danger of breaking under torsional strain. If it turns more freely, air is liable to leak by it into the space around the valve box. Probably the danger from conduction in a metallic stem is very slight, for the passage of heat into it from the parts in contact with the unexpanded air is a very roundabout one indeed. For the sake of greater safety, the stem plays in a brass sleeve *H*, which backs against the end plate, and is pinned to prevent its turning with the stem. The continuation of the stem (*R*, in Fig. 1) is of wood, and passes through a soft rubber disc in the interchanger case at *K*. A hollow screw-plug *I* conducts the expanded air from the valve cavity to the outflow tube *O* which carries three of the thermometers.

Before the valve was used, it was thoroughly tested for strength and tightness. At the conclusion of the experiments it was in as perfect condition as at the beginning.

The Arrangement of the Thermometers. — The brass screw-plug *F* (Fig. 2) consists of three parts — an outer cylindrical shell, a thin-walled thermometer pocket, with a flange at the open end, and a binding nut at the outer end to hold these together in place. On the inner side of the flange of the thermometer pocket was a fiber washer to make the joint tight. Air from the interchanger enters at *L* and passes over the entire length of the thermometer pocket into the valve seat *D*. The internal diameter of *L* and of *D* is 2.18 mm. and the annular space between the thermometer pocket and its case was so calculated that the area of its cross-section was the same as that of *L* and *D*.

The thermometer for measuring the initial temperature was placed in the inner end of the pocket, and was cast firmly in place by pouring in melted paraffin. The lead wires passed out through the binding nut and up through the core of the interchanger to the bridge and galvanometer, as shown in Fig. 1.

The temperature after expansion was measured by three thermometers similarly enclosed in a single pocket, each separated from its next neighbor by a space of 4 cm. The general arrangement of these also can be seen in Fig. 1. The pocket is provided at its lower end with three radiating slitted straps, as illustrated in Fig. 3, by means of which it can be bolted firmly to the flanges of outflow tubes of different diameters. In the figure, *P* is the mouth of the pocket, *SSS* are the straps, *F* is the flange at the base of the outflow tube, and *E* is the open space through which the exhaust escapes into the casing of the interchanger.

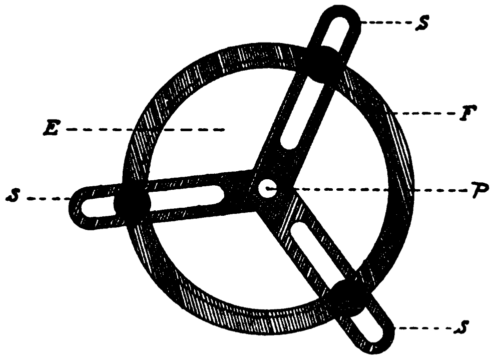

Fig. 3.

The reason for the use of three thermometers at this point was the following. Joule and Kelvin[1] found that when air was allowed to expand through a single orifice, its temperature was lower close to the orifice than at a distance of a quarter of an inch. Our own preliminary experiments confirmed this observation. In fact when a mercury thermometer was thrust up into the outflow tube toward the small orifice at the top, it registered lowest when the bulb was in the position corresponding to that which is the normal one for the ball in a ball-nozzle — a position which is very close to the orifice, and in which the thermometer bulb is held with some force in a condition of equilibrium by the action of the air current. If the bulb was [illegible] the [illegible] the temperature [illegible] towards the walls of the [illegible] the [illegible] the temperature rose, but [illegible] inches from the [illegible] the temperature [illegible] was [illegible] constant. [illegible] Fig. [illegible] therefore [illegible] the [illegible] temperature [illegible] but also [illegible] the [illegible] temperature was reached.

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[1] Philosophical Magazine (4), *4*, 481, 1852. *Cf.* Travers, "Study of Gases," pp. 189–90.

Insulation of the Valve.—[illegible] was [illegible] the insulation of the valve [illegible] It is [illegible] part of the [illegible] the valve [illegible] do [illegible] the [illegible] temperature were [illegible] affected [illegible] the valve and its [illegible] were [illegible] with a [illegible] hair felt, which was [illegible] adhesive tape [illegible] wound [illegible] small differences in pressure. Then other layers of felt and adhesive tape were applied, until a drum-shaped [illegible] was built up of the same diameter as the casing of the interchanger. This drum was wrapped with flannel like the interchanger, and it fitted the casing so tightly that air currents were prevented from passing by it. [illegible] it was now impossible inside the casing [illegible] the apparatus [illegible] the valve [illegible] the interchanger [illegible] the ordinary way, a well-insulated [illegible] was attached [illegible] as shown at the right in Fig. [illegible]

The Air Supply.—Air was delivered to the interchanger from a four-stage Norwalk compressor at a maximum rate of about [illegible] cubic feet per minute. [illegible] any deposition of solid matter in the valve, or in the tubes immediately adjacent, would alter the available diameter of the respective passages and so [illegible] make more difficult the task of keeping the valve in proper adjustment, [illegible] also, it was feared, occasion abnormalities in the cooling effect at the valve because of changes in the relative velocities of the air before and after expansion, the air was purified very carefully by passing it through strong steel cylinders containing sticks of solid potassium hydroxide and well screened granular calcium chloride respectively. No difficulty was experienced in the [illegible] from the formation of solid, except when the initial or final temperature was in the vicinity of zero or — 78°. The resulting annoyance even then was trifling when a normal amount of air passed the valve, except at the beginning of a run. When less air than the normal quantity was used, for a purpose to be explained later, it was often necessary to avoid these points entirely on the temperature curves. It was clear that the purification was not an absolute one.

It was never difficult to tell when the conditions at the valve were abnormal in this regard. The fact was heralded by slight but spasmodic motions of the gauge pointer a considerable time before any change could be detected at the galvanometer.

Experience has shown that the purification of the air supply is very greatly simplified if the exhaust from the expansion valve is delivered again into the intake of the compressor, instead of using steadily new air from outside as is often done. Accordingly the exhaust was taken by means of a flexible rubber hose from the exhaust pipes E–E_9 to a forty gallon reservoir, which communicated directly with the intake of the compressor and also with the external air. In this manner the exhaust was not only saved but was maintained at atmospheric pressure. In the event of perfect functioning of the compressor and of perfect adjustment of the expansion valve, the air circuit was practically, though not actually, a closed one,—unless indeed air were being withdrawn from the circuit by liquefaction.

Method of Operation.— At the beginning of a run warm air was blown through the whole interchanger, at comparatively low pressure and with the valve wide open, in order to remove any traces of moisture left from the preceding run. During this period, the exhaust was allowed to escape from E, so as not to cool the interchanger. The valve was then shut, until the pressure reached the desired amount, when it was opened again sufficiently to deliver as nearly as possible the exact amount of air which was being pumped by the compressor. The exhaust meanwhile was delivered through that one of the wooden tubes E_1–E_9 which experience had shown would carry the valve temperature down to the desired region.

During the cooling of the interchanger and the consequent contraction of the air within it, it was necessary to adjust the valve frequently in order to maintain constant pressure. On the other hand, when thermal equilibrium was reached, the valve could often remain at a single adjustment for periods of twenty minutes or half an hour without sensible change in the pressure.

The time required for the establishment of thermal equilibrium varied of course with the proportion of the interchanger which was thrown into the circuit. Without the interchanger, fifteen minutes would usually suffice. When the entire interchanger was in action an hour might be required. For this reason it was found of great advantage in the final series of experiments, to precool the air supply, when necessary, by means of ammonia refrigeration. To this end the air from the purifiers passed through one half of a double copper coil, while liquid ammonia evaporated at constant pressure through the parallel half, which was soldered coaxially to the first. In this way, by the use of a throttling valve in the ammonia circuit, the temperature of the air supply could be shifted in a minute or two to any desired point, from about — 30° to the temperature of the room. In addition to the saving of time thus effected through the use of a minimum of interchanger surface, the precooling of the air supply in this way facilitates the attainment at the valve of any specific temperatures which may be desired, instead of a certain number of temperatures spaced from one another in correspondence with the respective cooling effects of the individual sections of the interchanger.

In order to economize time still further, especially towards the end of a period of thermal adjustment, when the process is always slowest, the practice was to choose that one of the outflow tubes E_1–E_9 which would give a final temperature somewhat too low; then, when the desired temperature was reached, to check further progress downward by switching out a small portion of the exhaust at E, so as to remove it entirely from action on the interchanger. By manipulating a suitable throttling valve at this point, the proper proportion could be easily hit upon experimentally. Of course this portion of the air was led back into the compressor like the rest.

A sample page from the original note-book will make these matters clear. It should be noted that temperature readings were usually not recorded at all until equilibrium was near at hand.

TABLE I.

Record of Observations on March 7, 1907.

Time.	Resistance —Inflow.	Resistance—Outflow.			No. Sections Interchanger.	Pressure (Gauge).
		No. 1.	No. 2.	No. 3.		
Hr. Min.	Ohms.	Ohms.	Ohms.	Ohms.		Lbs.
3 30	7.70	4.79	4.79	5.15	1	3,000
35	7.64		4.73	5.07		
40	7.61		4.70	5.06		
45	7.58		4.67	5.02		
48	7.58	4.86	4.66	5.01		
50	7.58	4.86	4.66	5.01		
4 20	6.68		3.40	3.66	3	3,000
25	6.67	3.51	3.38	3.63		
30	6.66	3.50	3.35	3.61		
33	6.66	3.50	3.35	3.61		
4 55	5.70	1.86	1.78	1.95	6	3,000
5 00	5.70	1.86	1.78	1.95		
5 10	5.40		1.75	1.87	8	3,000
15	5.37	1.83	1.74	1.87		
20	5.37	1.83	1.74	1.87		
		Throttle opened a little.				
25	5.43		1.74	1.87		
28	5.43	1.83	1.74	1.87		
		Throttle opened more.				
35	5.57	1.83	1.74	1.87		
40	5.57	1.83	1.74	1.87		

The foregoing table represents the way in which the results of different initial temperatures were obtained while the pressure remained constant. It appeared later that still less time was consumed, on the whole, when a series of readings was based on a constant amount of interchanger surface, with varying pressure. Table II. illustrates this method.

The results obtained by these two methods were entirely concordant.

RESULTS.

Table III. contains the results of our final series of experiments. This series was made after the lapse of more than a year and a half spent in perfecting the apparatus and in acquiring facility and precision in its handling. The series itself required the greater part of two months for its completion. Typical readings were checked repeatedly at the beginning of the runs on different days.

TABLE II.

Record of Observations on March 19, 1907.

Time.	Resistance —Inflow.	Resistance—Outflow.			No. Sections Interchanger.	Pressure (Gauge).
		No. 1.	No. 2.	No. 3.		
Hr. Min.	Ohms.	Ohms.	Ohms.	Ohms.		Lbs.
2 55	7.66	5.73	5.48	5.90	4	1,500
3 00	7.60	5.67	5.41	5.82		
3 03	7.55	5.61	5.36	5.76		
06	7.55	5.61	5.36	5.76		
3 30	7.25	4.92	4.69	5.05	4	2,000
35	7.24	4.88	4.66	5.03		
40	7.22	4.88	4.65	5.00		
45	7.22	4.88	4.65	4.98		
48	7.22	4.87	4.63	4.98		
4 00	7.09	4.38	4.17	4.50	4	2,500
05	7.06	4.38	4.17	4.48		
10	7.02	4.34	4.13	4.42		
12	7.02	4.32	4.12	4.43		
15	7.02	4.34	4.13	4.43		
16	7.02	4.34	4.13	4.43		

Five different pressures were employed, viz., 1,000, 1,500, 2,000 2,500 and 3,000 lbs. gauge. There was a certain amount of back pressure caused by the friction of the exhaust as it passed through the coils of the interchanger, and through the pipes which conveyed it to the reservoir where atmospheric pressure prevailed, as described above. It amounted to less than 0.05 per cent. of the total pressure difference. This is entirely negligible in comparison with the unavoidable errors in reading the gauge and the thermometer resistances, and was left out of account.

The initial temperature varied from + 20° to — 120°.

It will be observed that the three thermometers always differ from one another for a given initial temperature, except at the temperature of liquefaction. The thermometer nearest the valve, no. 1, reads lowest, and that which is farthest away, no. 3, reads highest; and in general the difference between nos. 1 and 2 is greater than between nos. 2 and 3. These differences are not due to defective insulation around the outflow tube which contains the thermometer pocket, for they become *greater* when the general temperature of the blast is higher. Defective insulation would produce exactly the

Table III.

Final Results.

Pressure.	Thermometer Resistances (Corrected).				Temperatures.			
	Inflow.	No. 1.	No. 2.	No. 3.	Inflow.	No. 1.	No. 2.	No. 3.
Lbs.	Ohms.	Ohms.	Ohms.	Ohms.	Deg.	Deg.	Deg.	Deg.
1,000	9.12	7.46	7.21	7.79	+ 9.8	− 11.0	− 6.9	− 6.0
	8.19	6.54	6.30	6.80	− 17.1	− 40.8	− 37.9	− 37.4
	7.58	5.92	5.69	6.16	− 34.7	− 60.8	− 58.8	− 57.7
	6.75	—	4.85	5.25	− 58.7	—	− 87.4	− 86.4
	6.57	4.87	4.68	5.05	− 63.9	− 94.5	− 93.0	− 92.6
	6.10	4.27	4.09	4.43	− 77.4	−113.6	−112.7	−111.8
	6.04	4.19	4.02	4.34	− 79.1	−116.7	−115.0	−114.6
	5.59	3.61	3.46	3.75	− 91.8	−134.3	−133.7	−132.9
	4.92	2.53	2.43	2.63	−110.6	−168.8	−168.1	−167.6
	4.59	1.75	1.67	1.89	−119.9	−193.6	−193.5	−190.5
	4.53	1.74	1.65	1.78	−121.6	−193.9	−194.1	−193.9
1,500	9.31	7.43	7.19	7.77	+ 15.3	− 11.9	− 7.5	− 6.7
	8.61	6.72	6.48	7.00	− 4.9	− 34.9	− 31.8	− 31.1
	8.20	6.27	6.05	6.53	− 16.8	− 49.6	− 46.5	− 46.0
	7.54	5.58	5.35	5.78	− 35.8	− 71.8	− 70.4	− 69.7
	6.71	4.56	4.41	4.75	− 59.8	−104.4	−102.0	−101.9
	5.84	3.32	3.19	3.44	− 84.8	−143.8	−142.7	−142.5
	5.34	2.42	2.38	2.58	− 98.8	−172.4	−169.8	−169.1
	5.21	2.14	2.11	2.29	−102.4	−181.2	−178.8	−178.1
	5.06	1.75	1.78	1.93	−106.7	−193.6	−189.8	−189.3
	5.05	1.74	1.65	1.82	−107.0	−193.9	−194.1	−192.7
	5.03	1.74	1.64	1.78	−107.5	−193.9	−194.5	−193.9
	4.97	1.73	1.64	1.77	−109.2	−194.2	−194.5	−194.2
	4.89	1.72	1.64	1.77	−111.5	−194.5	−194.5	−194.2
2,000	9.46	7.42	7.15	7.74	+ 19.7	− 12.3	− 8.9	− 7.6
	8.62	6.47	6.25	6.76	− 4.6	− 43.0	− 39.7	− 38.7
	7.34	5.00	4.81	5.20	− 41.6	− 90.4	− 88.7	− 87.9
	6.84	—	4.16	4.50	− 56.0	—	−110.4	−109.6
	6.16	3.35	3.21	3.46	− 75.7	−142.8	−142.1	−141.9
	5.79	2.75	2.62	2.83	− 86.2	−161.8	−161.7	−161.4
	5.36	1.76	1.81	1.98	− 98.2	−193.3	−188.8	−187.8
	5.29	1.73	1.67	1.81	−100.2	−194.2	−193.5	−193.1
	5.18	1.72	1.64	1.76	−103.0	−194.6	−194.5	−194.6
	5.10	1.72	1.64	1.76	−105.6	−194.6	−194.5	−194.6
2,500	9.38	7.16	6.89	7.45	+ 17.3	− 20.5	− 17.8	− 16.8
	8.61	6.29	6.05	6.53	− 4.9	− 48.9	− 46.5	− 46.0
	7.75	5.29	5.07	5.48	− 29.8	− 81.2	− 80.0	− 79.2
	6.80	4.02	3.85	4.17	− 57.2	−121.5	−120.7	−119.9
	5.76	2.34	2.23	2.42	− 87.0	−174.9	−174.8	−174.1
	5.49	1.74	1.66	1.87	− 94.6	−193.9	−193.8	−191.2
	5.43	1.73	1.65	1.78	− 96.3	−194.2	−194.2	−194.0

TABLE III.—*Continued.*

Final Results.

Pressure.	Thermometer Resistances (Corrected).				Temperatures.			
	Inflow.	No. 1.	No. 2.	No. 3.	Inflow.	No. 1.	No. 2.	No. 3.
Lbs.	Ohms.	Ohms.	Ohms.	Ohms.	Deg.	Deg.	Deg.	Deg.
2,500	5.35	1.72	1.64	1.76	− 98.6	−194.6	−194.5	−194.6
	5.21	1.71	1.63	1.75	−102.4	−194.9	−194.8	−194.9
	5.10	1.71	1.63	1.75	−105.6	−194.9	−194.8	−194.9
3,000	9.49	7.14	6.90	7.45	+ 20.5	− 21.1	− 17.4	− 16.8
	8.63	6.13	5.89	6.36	− 4.3	− 54.0	− 52.0	− 51.4
	7.94	5.33	5.10	5.50	− 24.3	− 79.9	− 79.0	− 78.6
	7.48	4.76	4.56	4.91	− 37.6	− 98.0	− 97.0	− 96.9
	6.55	3.40	3.25	3.51	− 64.4	−141.2	−140.7	−140.3
	5.60	1.76	1.68	1.85	− 91.5	−193.3	−193.1	−191.8
	5.52	1.74	1.65	1.77	− 93.8	−193.9	−194.2	−194.3
	5.47	1.73	1.65	1.77	− 95.2	−194.2	−194.2	−194.3
	5.27	1.73	1.64	1.77	−100.9	−194.2	−194.5	−194.3

opposite effect. The differences should become less at higher temperatures and disappear altogether when the blast has the temperature of the room. They are due on the contrary to the high velocity at which the blast enters the outflow tube through the orifice at the top, and the reading of no. 3 is to be considered the true final temperature, as has been already explained.

When the blast at the mouth of the outflow tube contains liquid air, there is of course no longer any possibility of local differences of temperature within the tube. Doubtless the ratio of liquefied to unliquefied air is greater at the top than at the bottom, on account of the differences of velocity, but the temperature is the same.

If the initial temperature is a suitable one, liquid may exist indeed only at the upper part of the tube. Instances of this are found in the tenth reading for 1,000 lbs., the tenth and eleventh for 1,500 lbs., the seventh for 2,000 lbs., etc., where no. 1 or nos. 1 and 2 record the temperature of liquid air, while no. 3 is higher.

Fig. 4 displays the nature of the curves which are obtained by plotting initial temperatures as abscissas and final temperatures as ordinates.

The curves exhibit certain unmistakable facts.

It is clear that the final temperature falls more rapidly than the initial temperature, instead of less rapidly, as Pictet's formula re-

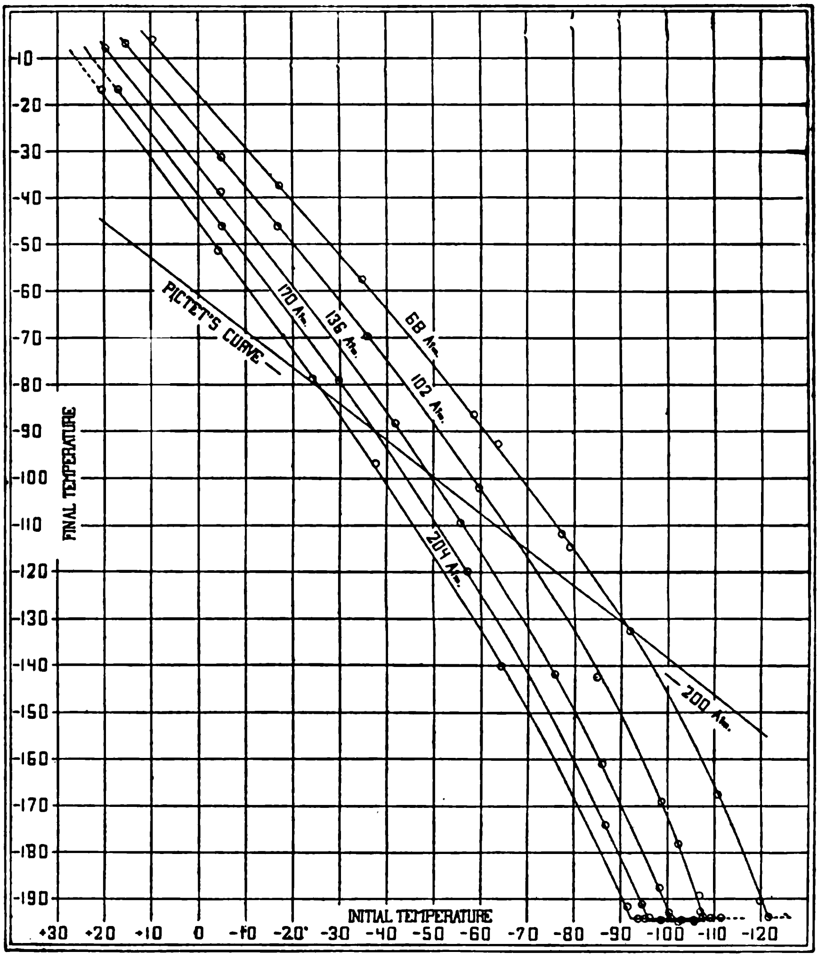

Fig. 4.

quires. Qualitatively, this result is in accord with the view of Linde and Schrötter. For the sake of better comparison, Pictet's curve for 200 atm. is plotted with the rest. For all pressures between 100 atm. and 200 atm., his curves are practically identical, as has been said.

Again, it is clear that at none of these pressures does the air on the high pressure side of the valve need to descend to the critical temperature in order to produce liquid air. At 1,000 lbs. pressure, the initial temperature descends only to — 121°; at 1,500 lbs., to

— 107.5° ; at 2,000 lbs., to — 101° ; at 2,500 lbs., to — 96.5° ; and at 3,000 lbs., to only — 93°. Indeed at 3,000 lbs. liquefaction is attained at the valve from an initial temperature which is forty degrees higher than the critical temperature, and by a cooling effect of approximately one hundred degrees, where the formula of Pictet allows only forty. It is certain therefore that the use of high pressure does not tend to promote liquefaction in the high pressure circuit, as Pictet fears, but just the reverse. The further question whether liquefaction in the high pressure circuit would be deleterious if it did occur, has already been answered in the negative.[1]

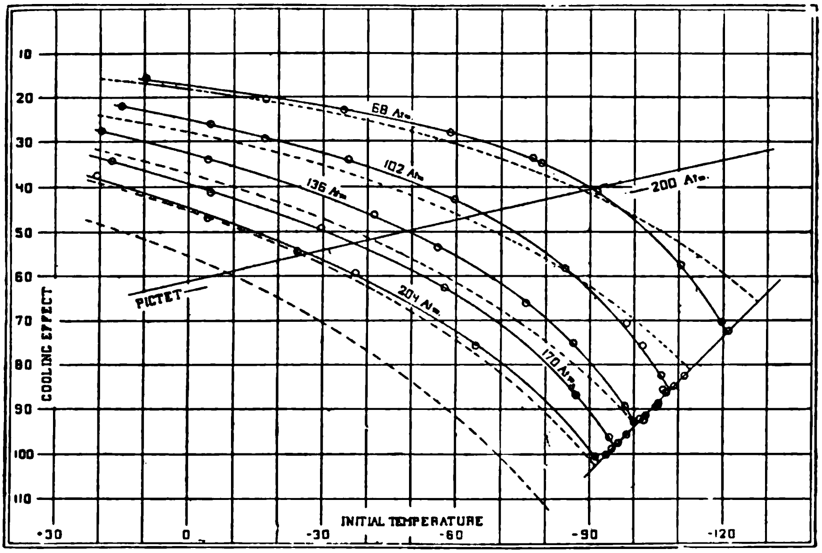

Fig. 5.

Thus the view of Linde is qualitatively vindicated. In order to exhibit the quantitative relations between our results and those called for by the views of Linde and Pictet, it will be more convenient to plot initial temperatures against the cooling effect. This is done in Fig. 5, with initial temperatures and cooling effect as abscissas and ordinates respectively.

In the figure the results obtained by us are represented by the circular plots and the continuous line curves are drawn through them as smoothly as possible. These curves terminate at the right in a rectilinear representing liquefaction. The dotted curves are calculated from the formula of Joule-Kelvin.

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[1] Bradley and Fenwick, Journal of Physical Chemistry, *10*, 275–289, 1906. Cottrell, ibid., 264–274.

The curve for Pictet's formula is seen still more clearly than before to resemble the experimental ones in no respect whatsoever. Indeed, it is difficult to see how that formula could have been based upon experimental data at all.

On the other hand, the Joule-Kelvin curves show a pronounced general similarity to our own, though they differ markedly in certain details. Thus, every one of them, at least for the greater portion of its length, lies lower than the corresponding experimental ones, and the difference between corresponding curves becomes rapidly greater as the pressure increases. At 68 atm., the difference is about one degree for an initial temperature of — 20°, while at 204 atm. it is ten. At 68 atm., it is two and one half degrees for an initial temperature of — 80, while at 204 atm., it is twenty-three.

On the contrary, for low pressures and *low initial or final temperatures*, the cooling effect according to the Joule-Kelvin formula is considerably *less* than that which we have observed. At the point of liquefaction it is twelve and one half degress less for 68 atm., and nine degrees less for 102 atm. For 136 atm., the two curves are practically coincident at the point of liquefaction.

It is evident therefore that the situation is much more complicated at high pressures and at low temperatures than is represented in the Joule-Kelvin formula, which was derived from experimental results at moderate temperatures and pressures.

Before entering upon a discussion of this question, certain other matters must be considered.

It has already been mentioned that expanded air, close to the valve, is colder than that farther away, even under satisfactory conditions of insulation. This phenomenon is explained as due to the greater kinetic energy of the air so long as it retains any portion of the abnormal velocity which it acquires in passing through the minute aperture of the valve. Thus Travers:[1] "The increase in the kinetic energy of the issuing gas necessitates an absorption of heat, so that there would be a fall of temperature — implying a conversion of molecular kinetic energy into translational energy of the mass of the gas. This fall of temperature, which may be termed *velocity cooling*, would disappear at a short distance from the plug, the velocity of the stream of gas being reduced by friction within the walls of the tube or with the gas in the vessel into which it flows, resulting in the formation of eddies ; the kinetic energy of the stream of gas would thus be reduced to its original value. The effective cooling of the gas is the result of internal work only, though the velocity cooling which takes place close to the plug or valve, through which the gas is escaping, may result in a fall of temperature greater than could be accounted for by the Joule-Thomson effect ; the velocity cooling would, however, be confined to a very limited region."

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[1] Study of Gases, p. 190.

In order experimentally to eliminate velocity cooling, so as to study the Joule-Kelvin effect alone, it would be necessary to provide such mechanical conditions as should give the air after expansion the same velocity which it had before reaching the valve. This is a difficult, if not wholly impossible task. If the question was simply one of relative initial and final pressures, equal velocity before and after expansion could easily be secured by giving the high pressure pipe and the outflow tube sectional areas in the clear which should be to each other inversely as the pressures — that is, in practice, by using different outflow tubes having different sectional areas for each change in the pressure ratio. But complication enters from the fact that the Joule-Kelvin cooling effect increases inversely as the square of the initial temperature, so that for every initial temperature there is a different volume ratio between the compressed and the expanded air, and therefore a change in the velocity ratio also. Equal velocities before and behind the valve could only be secured in practice by changing the outflow tube, not only for every change in the pressure ratio, but for every change in the temperature ratio as well.

However, it was hoped that the effect of velocity cooling might be eliminated by calculation, provided the total cooling effect was determined through the use of a set of outflow tubes of sufficiently diverse diameter. Our original plan contemplated the determination of a complete set of readings like that of Table III. for each one of [illegible] in the set, and we were not a little surprised at the discovery that such a procedure was quite unnecessary, as will now be shown.

TABLE IV.

[illegible]	Temperatures. [illegible]				Final Temp. Tube E.	Differences.	Average Difference.
	Inflow.	No. 1.	No. 2.	No. 3.			
[illegible]	Deg.	Deg.	Deg.	Deg.	Deg.	Deg.	Deg.
[illegible]	[illegible]	− 2.3	− 2.2	− 2.8	− 2.5	−0.3	
	− [illegible]	− 27.2	− 27.1	− 26.9	− 27.0	−0.1	
	− [illegible]	− [illegible]	− 77.3	− 77.4	− 77.7	+0.3	+0.14
	− [illegible]	−104.4	−104.1	−104.3	−104.6	−0.3	
	[illegible]	−149.2	−149.1	−149.1	−149.3	+0.3	
[illegible]	− [illegible]	− 14.2	− 14.3	− 14.5	− 14.6	−0.1	
	− [illegible]	− [illegible]	− 35.5	− 36.2	− 35.7	−0.4	
	− [illegible]	− 74.2	− 73.5	− 73.6	− 73.4	−0.2	
	− [illegible]	[illegible]	[illegible]	−111.8	−111.4	−0.4	−0.30
	− [illegible]	−138.1	−138.7	−138.4	−137.0	−1.4	
	− [illegible]	[illegible]	−165.5	−165.1	−165.0	−0.1	
	[illegible]	[illegible]	−183.2	−183.1	−183.5	+0.4	
[illegible]	− [illegible]	− 19.3	− 19.4	− 19.4	− 19.0	−0.4	
	− [illegible]	− 62.3	− 62.6	− 62.5	− 62.0	−0.5	
	− [illegible]	− 97.7	− 98.0	− 97.8	− 97.1	−0.7	−0.70
	− [illegible]	−139.3	−139.0	−139.0	−138.6	−0.4	
	− [illegible]	−177.4	−176.5	−176.3	−174.8	−1.5	
[illegible]	− [illegible]	− 25.3	− 23.9	− 24.1	− 23.7	−0.4	
	− [illegible]	− 74.3	− 74.1	− 74.2	− 74.0	−0.2	
	− [illegible]	−114.5	−114.7	−114.9	−114.6	−0.3	−0.50
	− [illegible]	−153.3	−152.3	−152.1	−149.3	−2.8	
	− [illegible]	−162.1	−161.7	−161.7	−159.8	−1.9	
	− [illegible]	−169.8	−169.4	−169.1	−168.8	−0.3	
[illegible]	− [illegible]	− 29.1	− 27.7	− 27.9	− 27.7	−0.2	
	[illegible]	− 65.0	− 64.3	− 65.0	− 65.5	+0.5	+0.08
	[illegible]	−121.2	−120.7	−120.5	−120.3	−0.2	
	− [illegible]	−175.2	−174.1	−173.5	−173.7	+0.2	

Table V gives some sample temperature readings during the use of the smallest outflow tube. This tube had an available sectional area between the inner wall and the outer wall of the thermometer tube of [illegible] sq. mm. As before, the records of all three of the thermometers were taken. For comparison's sake, interpolated values from Table III. are given for thermometer no. 3. Table III. was obtained by the use of an outflow tube (F) of 669.5 sq. mm. [illegible] used.

Thus the use of the smallest outflow tube, though it increases the velocity of the expanded air seventeen fold, increases the apparent cooling effect by only four tenths of a degree.

It may be noted parenthetically that the difference of the readings of themometers no. 1 and no. 2 for high initial temperature is very much less in Table IV., during the use of the smallest outflow tube, than in Table III. The reason is a simple one. When the thermometer pocket was attached to this tube in the manner already described, it was found that its flange and straps practically closed the mouth of the latter. In order to provide room for the escape of the expanded air, suitable washers were therefore used between the flange of the outflow tube and the straps of the thermometer pocket. But this brought the upper end of the pocket correspondingly farther away from the region of excessive velocity, and abnormal cooling effect, in the tip of the outflow tube.

TABLE V.

Pressure.	Temperatures. Tube *C*—1,552 sq. mm.				Final Temp. Tube *B*.	Differences.	Average Difference.
	Inflow.	No. 1.	No. 2.	No. 3.			
Lbs.	Deg.	Deg.	Deg.	Deg.	Deg.	Deg.	Deg.
1,000	− 39.6	− 66.3	− 63.9	− 63.1	− 63.5	+0.4	+0.1
	−113.7	−177.1	−175.8	−175.6	−175.4	−0.2	
1,500	− 38.7	− 76.3	− 74.1	− 73.9	− 73.7	−0.2	+0.1
	−105.8	−191.7	−188.4	−187.4	−187.8	+0.4	
2,000	− 37.6	− 83.4	− 83.1	− 82.3	− 82.3	0	−0.35
	− 89.3	−169.5	−168.8	−168.5	−167.8	−0.7	
2,500	− 36.1	− 90.4	− 89.3	− 88.5	− 88.7	+0.2	−0.6
	− 81.4	−165.6	−164.8	−164.5	−163.0	−1.5	
3,000	− 83.4		−175.1	−174.1	−174.7	+0.6	+0.6

The next table contains readings taken during the use of the largest outflow tube of the set. This tube had a net sectional area of 1,552 sq. mm., giving a velocity less than one fortieth that of the smallest tube, and about one half that of tube *B* by which Table III. was obtained. As only small differences were now expected, very few readings were made.

The result is an apparent average *increase* in the cooling effect of one tenth of a degree. With the widest outflow tube the cooling the

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table. However, the differences are not great enough numerically to introduce any essential change in the conclusions derived from our main results. They are scarcely greater than the experimental errors.

It is well known that for various reasons the output of a compressor changes somewhat from day to day, and even from hour to hour during a single run. When these changes occurred during a single run, they proved very annoying, for they not only necessitated such a manipulation of the valve as would keep the initial pressure constant, but often occasioned considerable delay in the temperature readings, until a new equilibrium could become established in the interchanger. Still, when once the equilibrium had become established, we could never observe that the cooling effect had become abnormal.

However, it seemed desirable to test experimentally the effect of considerable variations in the amount of air forced through the valve. This was done in two ways: — during the preliminary stage of the work, by changing the speed of the compressor; later, in the much more convenient way of throttling the intake of the same. By the latter method especially the amount of air delivered was easily changed within a few minutes from the normal rate of twenty cubic feet to about ten per minute, and then back again. Although considerable time was spent in the attempt, we could never detect any essential change in the results. Such changes had one very noticeable effect, which threw a side light upon the efficiency of the hair felt insulation about the interchanger. For example, in one sample experiment, taken almost at random, the full amount of air was being pumped, and was precooled as already explained, by ammonia. The pressure was 2,000 lbs., and very little of the interchanger was exposed to the exhaust. The initial temperature was — 39.3°, and the final temperature was perfectly steady at — 85.4°. At once the compressor output was cut down one half, and the expansion valve adjusted to correspond. The air entering the interchanger now took twice as long as before to reach the valve and so was exposed twice as long to the inflow of heat through the insulation. The result was a gradual rise of the initial temperature to — 30.6°, and of the final temperature to — 72.9°. Then the intake of the compressor was thrown open and the full amount of air was again delivered. The initial temperature fell steadily to − 38.7°, and the final temperature to − 84.8°. All three of these final temperatures are normal for the corresponding initial temperatures, as one may see by interpolating for the latter in Figs. 4 or 5.

As the valve was originally constructed, the face of the stem, as well as that of the valve seat, stood at right angles to the direction of the flow of the compressed air. Thus, at the moment of expansion, the air turned abruptly at right angles in order to reach the little cavity around the stem. Although it was not clear how this arrangement could affect the results, the valve was changed for the sake of testing the matter. The stem was made conical in the positive sense, and the seat, in the negative. Still, the cooling effect was the same, within the limits of error.

The same held true after a further modification, whereby the sectional area of the narrow passage leading downward from the valve cavity to the top of the outflow tube was increased sixteen fold.

In short, it is apparent that within the limits of error, the only factors which determine the cooling effect when air expands from a high pressure to one atmosphere are the pressure difference and the initial temperature. This brings us back to a discussion of Table III. and the curves of Fig. 5.

At first, the approximate parallelism between these curves and those calculated from the Joule-Kelvin formula led to the hope that, by a simple change in the constants, that formula might continue to serve. In this we were disappointed. It is true that by substituting for the expression $0.276\ (p_2 - p_1)$, $0.276\ (p_2 - p_1) - 0.017\ (p_2 - p_1)^2$, curves are obtained which are practically coincident with the experimental ones over about three fourths of their length. The following table shows this. The "observed" values are read from the smoothed curves of Fig. 5.

It will be noticed that the differences are within the limits of error down to an initial temperature of − 80°, for 68 atm. and for 170 atm.; to − 60°, for 102 atm.; and to − 50°, for 136 atm. Beyond these limits they become rapidly larger, and with the minus sign. That is, the modified Joule-Kelvin curves diverge from, and lie above those which are derived from observation, and represent thereafter a lesser cooling effect.

TABLE VII.

Modified Joule-Kelvin Formula.

Initial Temperature.	"p_2-p_1"								
	68.03 Atm.			102.04 Atm.			136.05 Atm.		
	"*d*" Calc.	"*d*" Obs.	Diff.	"*d*" Calc.	"*d*" Obs.	Diff.	"*d*" Calc.	"*d*" Obs.	Diff.
0	17.6	17.1	+ .5	25.6	25.0	+ .6	32.9	32.6	+ .3
− 10	19.0	18.7	+ .3	27.6	27.4	+ .2	35.5	35.6	− .1
− 20	20.5	20.3	+ .2	29.8	30.0	− .2	38.3	38.7	− .4
− 30	22.2	21.9	+ .3	32.3	32.7	− .4	41.6	42.1	− .5
− 40	24.2	23.8	+ .4	35.1	35.7	− .6	45.2	46.0	− .8
− 50	26.4	25.8	+ .6	38.3	39.0	− .7	49.4	50.4	− 1.0
− 60	29.0	28.2	+ .8	42.0	43.0	− 1.0	54.1	55.5	− 1.4
− 70	31.9	31.6	+ .3	46.3	48.3	− 2.0	59.6	61.8	− 2.2
− 80	35.3	35.4	− .1	51.2	54.9	− 3.7	65.9	69.5	− 3.6
− 90	39.2	40.2	−1.0	56.9	63.4	− 6.5	73.3	79.5	− 6.2
−100	43.9	47.4	−3.5	63.7	74.3	−10.6	82.0	92.8	−10.8
−110	49.4	57.2	−7.8						
	170.07 Atm.			204.08 Atm.					
0	39.7	39.4	+ .3	45.9	44.6	+1.3			
− 10	42.8	42.6	+ .2	49.5	48.2	+1.3			
− 20	46.2	46.0	+ .2	53.5	52.1	+1.4			
− 30	50.1	49.7	+ .4	58.0	56.4	+1.6			
− 40	54.5	54.0	+ .5	63.0	61.1	+1.9			
− 50	59.5	58.7	+ .8	68.8	66.4	+2.4			
− 60	65.2	64.2	+1.0	75.4	72.5	+2.9			
− 70	71.8	71.0	+ .8	83.0	79.5	+3.5			
− 80	79.4	79.6	− .2	91.9	88.2	+3.7			
− 90	88.3	91.6	−3.3	102.2	99.2	+3.0			

The differences for 204 atm. are all rather large, and they become larger as the initial temperature falls. Moreover, they all have the plus sign, meaning that the calculated cooling effect throughout is greater than it should be.

It became quickly evident therefore that in all probability time would be saved in the end by deriving a new formula directly from the experimental results.

Indeed, two such formulas will be given. They will be called formulas *A* and *B* respectively. Each of them fits the facts within the limits of error. Unfortunately, both of them are rather complicated.

For the suggestion which led to Formula *A*, we are indebted to

Mr. T. E. McKinney, Ph.D., who was at the time associate professor of mathematics in this University. It was suggested, namely, that the general formula

$$d = a_0 + \frac{a_1}{T - a_2}$$

could doubtless be made to represent the cooling effect observed under each one of the five pressure differences, by assigning five sets of values to a_0, a_1 and a_2 (T being the initial temperature on the absolute scale) and that then, without too great complication, the five values for each of these constants might prove capable of interpretation as functions of the pressure difference, P. So far at least as a_0 and a_1 are concerned, the expectation proved well founded. The case of a_2 was more complicated. Formula *A* follows:

$$d = 23.9 - \frac{7{,}801.5}{379.65 - P} + \frac{\dfrac{6{,}833{,}400}{449.66 - P} - 14{,}905}{T - 84.8 - .54516P + 9.4648 \times 10^{-4}P^2 + 2.3973 \times 10^{-6}P^3 - 7.0977 \times 10^{-8}P^4}.$$

Table VIII. gives a comparison of the observed cooling effect with that calculated by Formula *A*.

Fig. 6 illustrates the cooling effect as calculated by means of Formula *A*. As before, the circular plots represent the experimental results. As will be seen, the curves fit rather better for initial temperatures below −40°, or thereabouts, than for temperatures above that point.

For the derivation of *Formula B*, we are under obligations to Mr. B. H. Camp, M.A., of the mathematical department. P, as before, is the pressure difference in atmospheres, and d is the cooling effect, but t is the initial temperature in degrees of the Centigrade scale.

$$\begin{aligned} d = 0.2754P - 2.744 \times 10^{-4}P^2 - (2.617 \times 10^{-3}P - 4.991 \times 10^{-6}P^2)t \\ - (4.175 \times 10^{-7}P - 3.86 \times 10^{-5})t^2 - (4.175 \times 10^{-9}P \\ + \frac{(163.3 - P)^2}{4.649 \times 10^{10}} - 7.341 \times 10^{-7})t^4. \end{aligned}$$

TABLE VIII.

Formula A.

P (Atm.)	Cooling Effect (d).			Av. $d_c - d_0$
	d_c	d_0	$d_c - d_0$	
68.03	16.2	15.8	+ .4	−.2
	19.4	20.3	− .9	
	22.3	23.0	− .7	
	27.7	27.7	0	
	29.2	28.7	+ .5	
	34.1	34.4	− .3	
	34.8	35.5	− .7	
	41.3	41.1	+ .2	
	56.8	57.0	− .2	
	69.6	70.6	−1.0	
	72.5	72.3	+ .2	
102.04	22.7	22.0	+ .7	+.1
	26.2	26.2	0	
	28.7	29.2	− .5	
	33.7	33.9	− .2	
	42.7	42.1	+ .6	
	58.1	57.7	+ .4	
	72.2	70.3	+1.9	
	77.0	75.7	+1.3	
	83.0	82.6	+ .4	
	83.8	85.7	−1.9	
	84.7	86.4	−1.7	
136.05	28.4	27.3	+1.1	+.1
	33.8	34.1	− .3	
	46.1	46.3	− .2	
	53.0	53.6	− .6	
	66.1	66.2	− .1	
	75.6	75.2	+ .4	
	89.9	89.6	+ .3	
	92.8	92.9	− .1	
170.07	34.4	34.1	+ .3	−.2
	40.4	41.1	− .7	
	49.2	49.4	− .2	
	62.9	62.7	+ .2	
	86.9	87.1	− .2	
	95.5	96.6	−1.1	
	97.7	97.7	0	
204.08	38.6	37.3	+1.3	+.3
	46.2	47.1	− .9	
	53.9	54.3	− .4	
	60.1	59.3	+ .8	
	76.3	75.9	+ .4	
	101.1	100.3	+ .8	

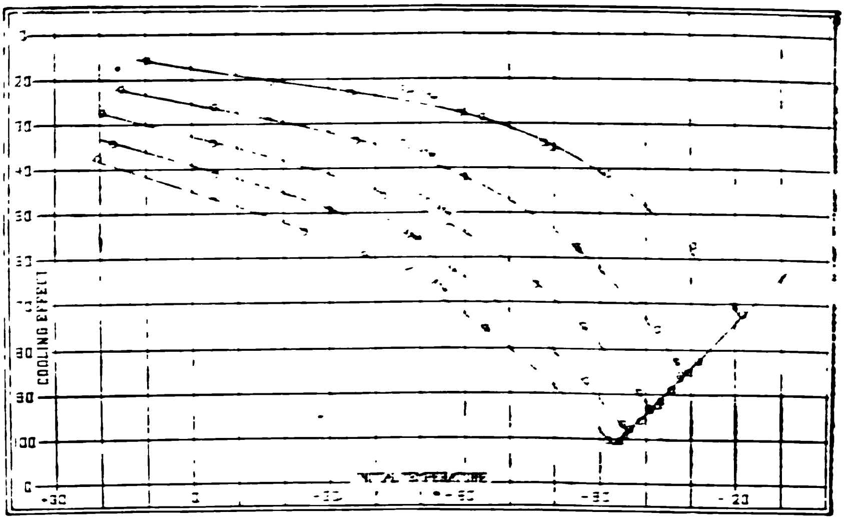

Fig. 6.

The cooling effect as calculated by Formula *B* is compared with the observed cooling effect in Table IX. and in Fig. 7.

As Formula *A* accords a little better with the observed cooling effect for lower initial temperatures, so Formula *B* accords rather

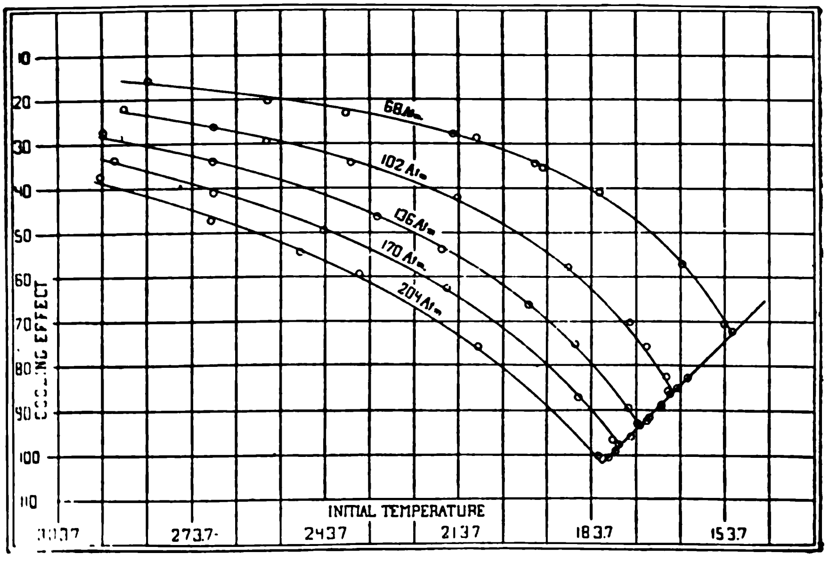

Fig. 7.

TABLE IX.

Formula B.

P (Atm.)	Cooling Effect (d).			
	d_e	d_0	d_e-d_0	Av. d_e-d_0
68.03	16.0	15.8	+ .2	+.3
	20.1	20.3	− .2	
	22.8	23.0	− .2	
	27.6	27.7	− .1	
	29.0	28.7	+ .3	
	33.9	34.4	− .5	
	34.7	35.5	− .8	
	42.0	41.1	+ .9	
	59.0	57.0	+2.0	
	71.2	70.6	+ .6	
	73.8	72.3	+1.5	
102.04	22.2	22.0	+ .2	−.2
	26.5	26.2	+ .3	
	29.1	29.2	− .1	
	33.7	33.9	− .2	
	42.1	42.1	.0	
	57.9	57.7	+ .2	
	72.2	70.3	+1.9	
	76.7	75.7	+1.0	
	82.7	82.6	+ .1	
	83.2	85.7	−2.5	
	83.9	86.4	−2.5	
136.05	27.4	27.3	+ .1	−.3
	33.9	34.1	− .2	
	45.5	46.3	− .8	
	52.2	53.6	−1.4	
	65.6	66.2	− .6	
	75.4	75.2	+ .2	
	89.8	89.6	+ .2	
	92.6	92.9	− .3	
170.07	33.9	34.1	− .2	+.3
	40.7	41.1	− .4	
	49.1	49.4	− .3	
	62.8	62.7	+ .1	
	88.1	87.1	+1.0	
	97.0	96.6	+ .4	
	99.1	97.7	+1.4	
204.08	38.1	37.3	+ .8	−.1
	46.6	47.1	− .5	
	53.7	54.3	− .6	
	59.6	59.3	+ .3	
	76.0	75.9	+ .1	
	99.9	100.3	− .4	

better with the cooling effect for the higher initial temperatures. Indeed, it was Formula *B* and its curves which first called attention to the fact that the observed cooling effect from 0° C. upward almost involves an inflexion of the curvature which prevails below that point. At that time, unfortunately, it was too late to enter upon the matter again experimentally. It is clear, however, that such an extension of the present inquiry is much to be desired.

Since the experimental part of this paper was completed, much interest has been taken regarding the nature and location of the so-called inversion point of air and other gases, that is, the temperature, or temperatures, at which the cooling effect is zero. Particular reference may be made to the experimental work of Olszewski,[1] and the calculations of Dickson,[2] based on van der Waals' equation of state.

In 1898 Witkowski showed that the inversion point must be a function of the pressure. This was confirmed by Porter[3] and by Dickson,[2] on theoretical grounds, as well as by the experimental results of Olszewski. However, the relation of the inversion point to the pressure is a direct one according to Olzewski, in harmony with the deductions of Porter, while according to Dickson it should be an inverse one.

Again, Olszewski finds the inversion temperatures to lie between 198°–259° Centigrade for pressures ranging from 40 to 160 atm., while, according to Dickson's calculations, they should lie between 607.3° and 555°. Porter's calculations are for hydrogen only.

It is to be noted, as Dickson points out, that these discrepancies may be accounted for, if it may be assumed that there is some fundamental difference between expansion through a nozzle and expansion through a porous plug.

Under the circumstances, it may not be without interest to compare Formulas *A* and *B* as given above, by setting $d = 0$ in each, for the five pressures used in this work. When this is done, Formula *A* gives the following results, in Centigrade temperatures.

68.03 atm.	102.04 atm.	136.05 atm.	170.07 atm.	204.08 atm.
2,494°	971°	679°	534°	431°

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[1] Philosophical Magazine (6), *13*, 722–4, 1907.

[2] Idem (6), *15*, 126–46, 1908.

[3] Philosophical Magazine (6), *11*, 554–68, 1906.

The method of expansion used in these experiments and in Olszewski's work is probably the same. As the values just given differ greatly in magnitude from those found by Olszewski and as the relation between the calculated inversion temperature and the pressure is an inverse one, it is clear that however closely Formula *A* may correspond to the experimental data from which it was derived, it would not be a safe one to use for high initial temperatures. Indeed, it is our conviction that the cooling effect obtained by the use of a nozzle is a very complicated matter. For almost any narrow range of data, some comparatively simple formula may doubtless be derived which will fit passably well. The hope that any simple formula will be found to represent data of wide range, either of temperature or pressure, is probably futile. This matter will be considered again at the close.

As has been said, Formula *B* corresponds rather better than Formula *A* to the experimental data at the higher initial temperatures. It might be expected therefore that it would lead to a closer approximation to the inversion temperatures. And numerically speaking, this is true for the two highest pressures. Thus for 204.08 atm., the inversion temperature would be 71.8°, and for 170.07 atm. it is 79° Centigrade. The curves for the other three pressures show no point at all where the cooling effect is zero, although that for 136.05 atm. comes within half a degree of doing so, before it turns again downward. It will be noticed moreover that the two inversion temperatures which are given for the higher pressures, bear an inverse ratio to the pressures, as in the case of Formula *A*.

In another connection[1] favorable use has already been made of the theory of the gaseous and liquid states which has been advanced and vigorously defended by J. Traube.[2] There seems good reason to believe that this theory is capable of explaining satisfactorily the phenomena connected with the Joule-Kelvin effect and the inversion temperature. Although the data at hand at present are doubtless much too meager to warrant anything more than tentative, we venture the following suggestions.

[1] Bradley, Brown and Hale, PHYS. REV., *27*, 104, 1908.

[2] Zeitschrift für anorganische Chemie, *37*, 225, 1903; *38*, 399, 1904.

A perfect gas should become warmer on passing through a throttling valve or nozzle, on account of the friction encountered. In making this statement, no regard is paid to changes in kinetic energy during the passage of the valve and immediately afterward, for, if the initial and final velocities are equal, these will compensate one another. Instead of speaking of a gas whose temperature is raised under these circumstances as "ultra perfect," as has somewhat commonly been the practice, particularly in the case of hydrogen, it should rather be considered as more or less approximate to, but still less than, "perfect."

Under any circumstances attainable in practice, a gas is to be thought of as containing in solution, or admixed, some characteristic proportion of "liquidons," a proportion which will depend directly on the pressure and inversely on the temperature. In order to convert these liquidons into true gas molecules (gasons), heat is required.

When a gas expands from a higher pressure to a lower one, by passing through a nozzle, the initial proportion of liquidons will decrease to that which is proper for the new pressure, and an absorption of heat will occur. If the heat thus called for is less than that developed by friction, the net result in the gas as a whole will still be a rise in temperature. On the other hand, when the evaporation of liquidons is copious enough to call for more heat than is produced by friction, a net cooling effect will be obtained. The total cooling effect, and therefore the net cooling effect, if any, will vary directly as the initial pressure, and inversely as the initial temperature and the final pressure.

Probably the heat of friction depends on a considerable number of factors, referable in part to the nature and condition of the gas itself and in part to the materials, condition and structure of the valve or nozzle.

The proportion of liquidons present in the gas at low pressures will not be large unless the temperature becomes greatly lowered. The real cooling effect will therefore not be large except for low initial temperatures. But in proportion as the initial temperature approaches the critical temperature the proportion of liquidons, and therefore the cooling effect, will increase more and more rapidly, until the rate of increase becomes very great indeed. This appears to be the explanation of the peculiarities of the curve for 68 atm., in Figs. 5, 6 and 7.

At comparatively high pressures, on the other hand, the proportion of liquidons will be greater at all times, and so, by consequence, will the cooling effect be. But as the cooling effect is so much greater, liquefaction will be reached while the initial temperature is much further from the critical temperature. Therefore the region of rapid increase in the cooling effect will be greatly restricted or even eliminated. The high pressure curve will terminate in the rectilinear of liquefaction before any sharp increase in its curvature is possible. This condition of things is illustrated in the curve for 204 atm., which is much lower, much shorter, and of much less pronounced curvature at the lower end than any of the others. This curve indeed is to be thought of as "corresponding" only to those portions of the other curves, which lie directly above it — its section of sharp curvature at the right having been eliminated by the earlier occurrence of liquefaction.

From this view-point the inversion temperature would be the temperature at which the cooling effect due to the evaporating liquidons exactly counterbalances the heat due to friction. Naturally this temperature would be higher for higher pressures, as Olszewski has shown, because of the greater proportion of liquidons which are then present.

SUMMARY.

1. The cooling effect produced by the nozzle expansion of air has been studied experimentally for initial temperatures ranging from + 20° to — 120°, and for pressures from 68 atm. to 204 atm.

2. The cooling effect is a direct function of the initial pressure and an inverse function of the initial temperature. This result is qualitatively in harmony with the formula commonly used to express the results of the Joule-Kelvin effect, as applied in particular to the liquefaction of air by Linde, but is diametrically opposed to the formula by which Pictet expresses his view of nozzle expansion.

3. For moderate initial temperatures, the cooling effect is less than that called for by the Joule-Kelvin formula, and the disparity becomes increasingly greater as the pressure increases.

4. For low initial temperatures, the cooling effect becomes rapidly greater than that which the formula indicates, provided the pressure is not too great. At the higher pressures the experimental curves tend to become more nearly parallel to those calculated from the formula.

5. Within the limits of error the cooling effect is not modified by changes in the kinetic energy (velocity cooling) whose ratio is as great as 40 : 1; by the form given to the surfaces of the valve stem and seat; nor by considerable changes in the volume of air which is made to pass the valve.

6. Two formulas, *A* and *B*, are derived from the experimental data. The differences between the observed cooling effect and that calculated by these formulas are only infrequently greater than one degree, and average two or three tenths of a degree.

7. So far as these formulas lend themselves to the calculation of the inversion temperatures of air, the latter are inverse functions of the pressure. But the extrapolation is too extreme to be of significance.

8. Certain suggestions are made looking toward an explanation of the Joule-Kelvin effect.

It is intended to undertake at once an extension of the range of these experiments, to include much higher temperatures. The interest which attaches at present to the subject of inversion temperature, as well as certain anomalies observed in connection with the higher of the temperatures already employed, seem to make this highly desirable.

Wesleyan University,
Middletown, Conn.,
February 10, 1909.

THE EFFECT OF TEMPERATURE VARIATIONS ON THE LUMINOUS DISCHARGE IN GASES FOR LOW PRESSURES.

By R. F. Earhart.

IN 1903 Carr[1] made a series of observations on the potential difference required to cause a discharge of electricity between parallel plates, for distances varying from 1 mm. to 10 mm., and for different pressures.

It is well known that for a particular distance a pressure exists at which a minimum potential difference will produce a discharge. Either lowering or increasing the pressure will require a higher potential for the ionization of the gas. The pressure corresponding to the minimum potential is called the critical pressure. Carr found the minimum potential required for discharge to be independent of the distance between the plates. The critical pressure, however, varied with the separation of the plates. The ionization in this case is essentially ionization by collision. The writer has for some time been interested in the effect which temperature would have on the ionization of a gas when the ionization was produced by collision.

I, therefore, determined to repeat Carr's measurements with some slight modifications, subjecting the gas to various temperatures. Before attempting measurements at high temperatures it seemed desirable to carry out a series of measurements with a rather simple form of apparatus for moderate temperatures, in order to find whether consistent results were to be expected at the higher temperatures where one would reasonably expect the effects to be more pronounced. The apparatus which has been used for variations in temperature from — 78° C. to 325° is shown in Fig. 1. It consists of two circular aluminium plates 1 cm. in diameter and 5 mm. apart. These were enclosed in a glass bulb approximately 4 cm. in diameter, provided with suitable tubes for exhausting and for the introduction of fresh gas, if desired. This bulb was placed in a furnace and heated electrically. The furnace was constructed of a porous battery jar around which a layer of asbestos paper was wrapped; over this was wound No. 18 nickel wire and this surrounded with loose magnesium packing (*P*) several inches in thickness. A thin layer of magnesium paste was used to surface the packing. This furnace was contained in a cylindrical metal box, in order that it might be removed, if desired, and the whole nested in a much larger wooden box containing magnesium packing. The mercury thermometer shown in the figure was used for temperatures below 200° C.; for temperatures above this a Pt resistance thermometer was substituted. After the discharge bulb and thermometer were in place a cover of mica (*M*), in the diagram, closed the furnace, and loose magnesium packing was used to fill in the space above the chamber. For temperatures above that of the room the current in the heating coil was varied until the desired temperature was attained, and after 30 minutes of constancy readings were taken. It is only necessary to know the

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[1] Proc. Roy. Soc., LXXI., p. 314.

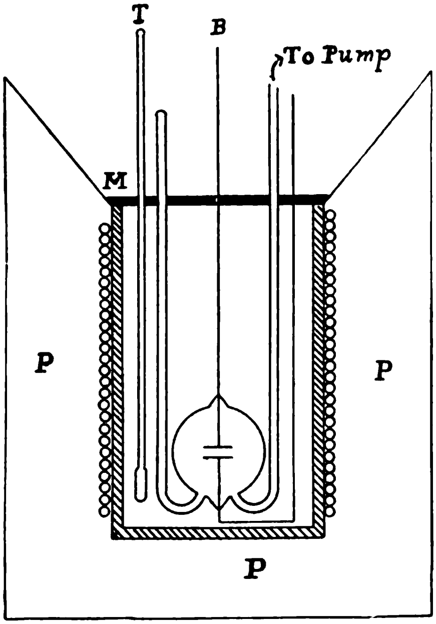

Fig. 1.

approximate temperature and no refinements were attempted. A series of measurements usually occupied three or four hours. Temperatures varying not more than 2 or 3 degrees from the mean were easily secured and considered satisfactory. Pressure measurements were made with the McLeod gauge in the usual manner.

The electrical arrangement is shown in Fig. 2. One terminal of a storage battery (*B*) capable of giving 1,200 volts was connected to earth. The other terminal was connected to one of the plates in the discharge chamber (*C*) through a liquid resistance of CdI in amyl alcohol (*R*). The second plate was also connected to earth. A Weston voltmeter (*V*) together with a suitable multiplier gave the potential difference between the plates in the discharge chamber. The potential required to produce luminosity in the gas was observed.

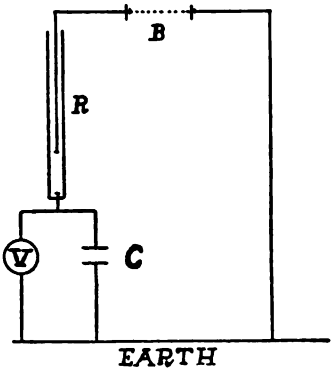

Fig. 2.

There is a discharge preceding this, but luminosity marks a sharply defined stage in the ionization of a gas. In general, near the critical pressure, an increase of a single volt will produce a change from no observable luminosity in a dark room, to one which is easily discernible in a lighted room. The behavior of the voltmeter needle during this change from no luminosity to luminosity is a most sensitive detector of the luminous discharge. As the potential is built up by decreasing the resistance in the circuit, the voltmeter needle advances until the luminous discharge occurs and then drops back 40, 50 or even 100 volts, depending on the nature and pressure of the gas. If now the potential be decreased by increasing the resistance in (*R*), the potential diminishes until the luminous discharge ceases. The voltmeter needle then jumps quickly forward. This motion depends also upon the nature and pressure of the gas. The changes in the potential, which occur as the discharge changes in character from the luminous to non-luminous condition or vice versa, are much less below than above the critical pressure. This is what one might expect on account of the difference in current density for the two cases. For a time both the luminosity and the behavior of the needle were noted in taking observations, but the connection between the luminosity and the behavior of the needle was so evident to the observer that the direct observation of the luminosity was dispensed with for the higher temperatures. This greatly facilitated the ease of observations. In the tables, which follow, both the potential required to produce luminosity and the potential required to maintain are given.

In an article appearing in the Philosophical Magazine,[1] Mr. J. H. Brown describes an experiment on the current produced after a discharge has been created, and has studied the current density produced for potentials which lie between the potential required to produce the discharge and the potential required to maintain such discharge. He calls attention to the fact that a high resistance in series with the spark gap renders the discharge stable and enables one to secure consistent readings. In so far as the readings tabulated are consistent, I think they were rendered possible by utilizing only a part of the full voltage, which my battery was capable of giving. In order to obtain results at varying temperatures comparable with one another, two factors should be considered, viz., the lag, and the fact that a discharge passing through the gas may raise the temperature of the gas. If the discharge is of long duration, the temperature within the discharge chamber will probably be considerably above that of the vessel in which the chamber is contained, and the thermometer in the outer vessel will not indicate even the approximate temperature of the gas through which the discharge takes place. The generally accepted theory of ionization by collision considers the discharge produced by the action of the field as due to the fact that the strong fields will give great velocity to the free ions existing between the electrodes, and that these ions by their impact on neutral atoms or molecules create a copious supply of fresh ions. Inasmuch as a luminous discharge presupposes the existence of an ionized gas, it seemed legitimate to produce an initial discharge for a brief interval in order to create a supply of ions and after the lapse of a minute or two during which readjustments of temperature would take place, to apply a potential, which would produce the luminous condition desired. An illustration of the conditions for producing discharge is inserted.

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[1] Phil. Mag., Vol. 12, 1906, p. 210.

The apparatus had stood at room temperature for 48 hours, with air as the dielectric, and at a pressure of 3.20 mm. The potential required to produce the initial discharge was 784 volts. The potential was built up of from zero to 784 volts in, perhaps, 5 seconds. The potential was immediately reduced to zero after the luminosity appeared. Two minutes afterward, with the same rate of increasing the potential, luminosity was produced at 502 volts. A repetition two minutes later gave the same result. This was repeated after an interval of seven minutes, when the discharge occurred at 524 volts. Two minutes later the break occurred at 502 volts. I next allowed a seven-minute interval to elapse and raised the potential to 502 volts. After 30 seconds or thereabout, the discharge came. In all cases the potential required to maintain the luminous current was 355 volts. Sets of readings similar to the foregoing were obtained for a number of pressures. The method employed in making the observations was to apply a potential which would cause a discharge, but to maintain this for the briefest possible time, after the lapse of a short interval to build up the potential slowly and to note the occurrence of the discharge, then immediately to lower the potential until the luminous discharge ceased. In this way it was possible to secure consistent readings and to check the potential readings as often as desired. The gases operated upon were air, hydrogen and CO_2. The lowest temperature was secured by using CO_2 snow. The heating chamber was used as a container for the CO_2 snow. The temperature in this case was assumed to be — 78° C. Each of the tables which follow are based upon two series of observations. In a single series of observations, usually about 12 points on a pressure potential curve were secured. Another series was taken under similar conditions and 10 or 12 additional points secured. These two series are brought together in a single table.

These are represented graphically in Figs. 3 to 8 inclusive. Fig. 3 is a family of curves representing the values obtained when air was the dielectric. The curve corresponding to 120° is omitted in the graph. The values for this curve agree with one another and bear the proper relation to the other curves of the family. It was thought, however, that a sixth curve would complicate the figure unduly. The curves are similar in form to those secured by Carr. The results are hardly comparable with his, however. In the present case the discharge might occur at the edges of the plates, whereas in Carr's experiment the discharge was confined to a uniform field. Moreover, Carr measured the potential required to produce the initial discharge. Luminosity occurs, undoubtedly, after the so-called dark discharge and marks a more or less advanced stage in the ionizing process. The results do not agree quantitatively with Paschen's Law. Paschen's Law, which applies rather to the initial discharge, states that the potential required for discharge varies with the mass of the gas separating the electrodes. According to Paschen's Law, for pressure greater than the critical pressure, decreasing the pressure will lower the potential required for discharge; for pressures less than the critical pressures diminishing the pressure increases the required potential. In other words, above the critical pressure the potential is lowered as the mass of the gas is decreased, while above the critical pressure decreasing the mass of the gas requires a higher potential.

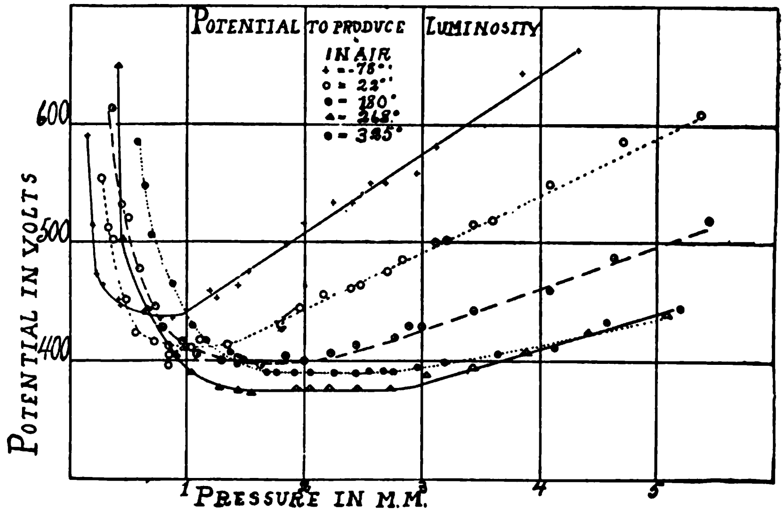

Fig. 3.

If we compare similar pressures at different temperatures, then at the higher temperature a smaller mass of gas will exist between the electrodes, and the potential required for discharge be diminished for the higher temperature, provided the pressure is greater than the critical pressure. Again, if the pressure be less than the critical value, raising the temperature will likewise decrease the amount of gas between the electrodes, and this will require higher potentials, as the temperature is increased. For the temperatures from 22° to 268° this holds in a qualitative way, and, with a rough approximation, in a quantitative way. It fails to express the relation between 268° and 325°, even in a qualitative manner. In Fig. 4, the curves which represent the potential required to maintain luminosity for 120° and 268° are omitted on account of the crowding of the lines.

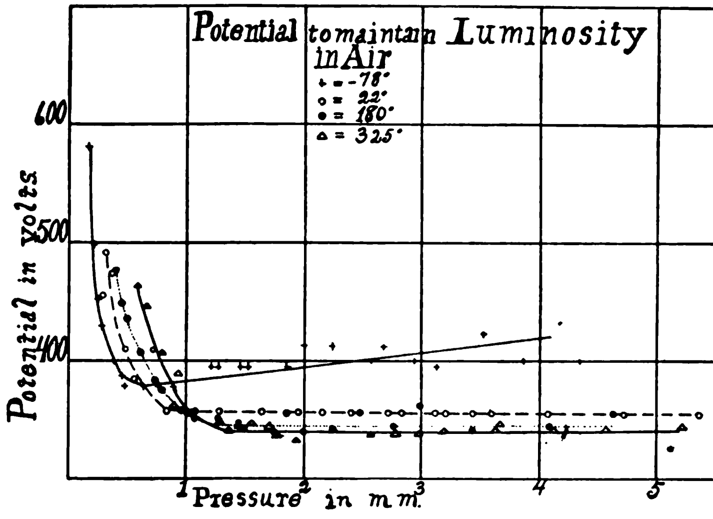

Fig. 4.

In the case of air for pressures above the critical pressure the potential required to maintain the luminosity at any temperature is approximately the same for all pressures within the limits of the experiment. Here the curves for the higher temperatures fall below those for the lower temperatures; while below the critical pressure the high temperature curve lies higher than its fellows.

For air at 22° the potential to maintain luminosity was 355 volts. This agrees with the minimum spark potential as measured by Carr, Strutt, Hobbs and others. It is also equal to the cathode fall in potential.

In Figs. 5 and 6 the results secured with hydrogen are repre-

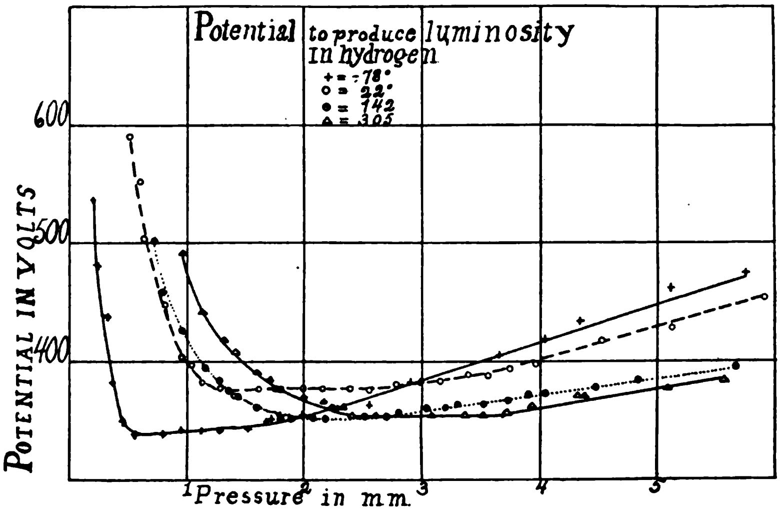

Fig. 5.

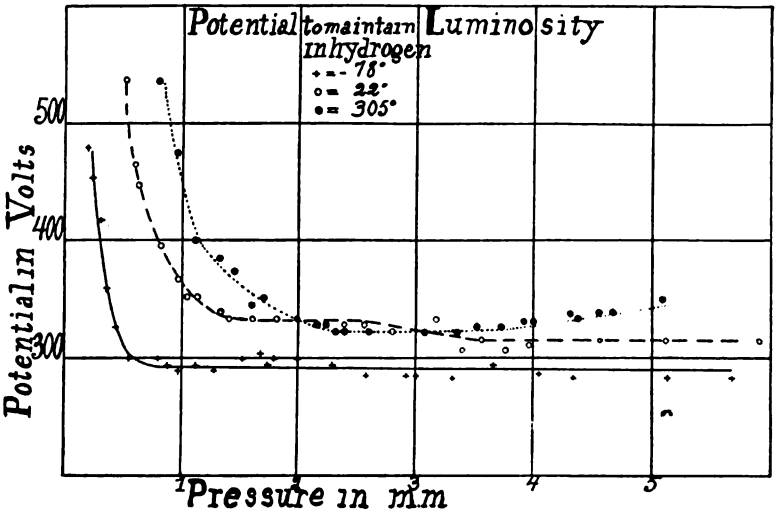

Fig. 6.

sented graphically. The hydrogen was prepared by the action of H_2SO_4 on platinized c.p. zinc. The H was passed through solutions of potassium permanganate and pyrogallic acid, and dried with phosphor pentoxide. Hydrogen of only moderate purity is secured in this manner. I am inclined to believe that the somewhat anomalous results for the potential required to maintain the luminous condition are due to impurities. For — 78° the curve is similar to the air curve and shows a value of 288 volts. The minimum spark potential for H is given by Carr as 280, and by Strutt as from 301–308 volts.

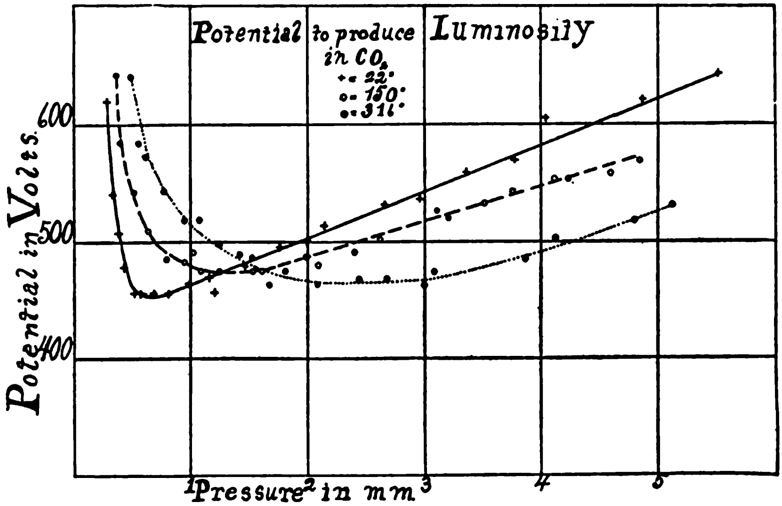

Fig. 7.

Many observers have found the phenomenon connected with discharge in hydrogen greatly altered by minute traces of oxygen. After making a series of observations with the moderately good H, I purposely admitted an exceedingly small percentage of air. The potential required to start the discharge was not perceptibly affected; the potential required to maintain discharge was raised considerably.

In case of the carbon dioxide only three temperatures were used. The CO_2 was prepared chemically, using marble and HCl in a Kipp apparatus. The results are indicated in Figs. 7 and 8. After the higher temperature measurements had been made with CO_2 the lower electrode (cathode) showed evidence of violent bombardment and high temperature. Nothing of this sort occurred with air or hydrogen. After the series of observations at 316° on CO_2, the temperature was inadvertently raised much higher, and in that condition a few discharges produced. An examination of the tube afterward showed no evidence of the tube itself being injured or affected by this higher temperature. The cathode was bent downward and away from the anode and had the appearance of a mushroom.

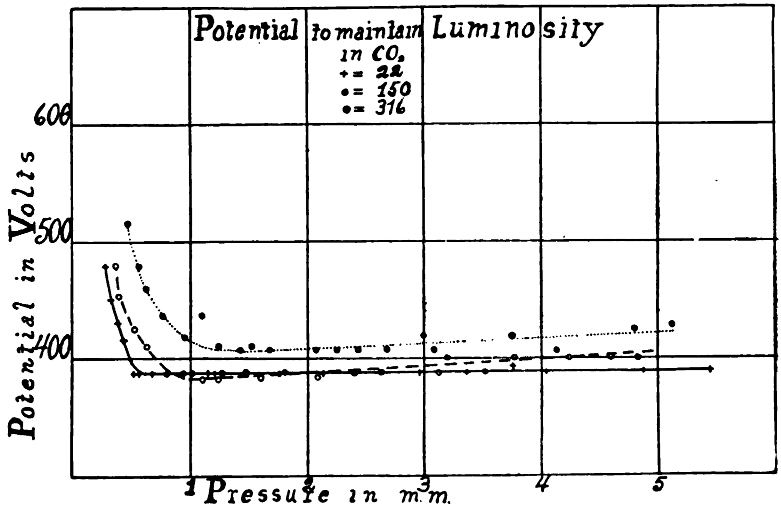

Fig. 8.

No very great accuracy can be claimed for the determination of the critical pressures, especially at the higher temperatures. Indeed, the curves show that for the higher temperatures the region at which the minimum discharge occurs applies to a considerable variation in pressure. For the lowest temperature the location of the critical pressure is determined between rather narrow limits. At 300° C. all of the gases show that a variation in pressure of 2 millimeters may occur without altering the potential required for discharge to any considerable extent. The curves show in general a flattening for this region of minimum potential as the temperature is increased. It seems quite possible that the bend in the curves representing the potential required to maintain discharge will locate the critical pressure more definitely than the discharge curve.

The relation of the curves for air indicates in a general way that there is no particular modification in the mechanism of the discharge until a temperature in the neighborhood of 300° C. is reached. Here the simple relative position of the curves undergoes a change indicating some modifying effect produced near this temperature. The effect of temperature on the potential required to produce the luminous condition is much more apparent than that of the potential required to maintain the luminous condition. In the case of air, increasing the temperature permits a lowering of the potential which will maintain the luminosity. In the case of H and CO_2 the luminous condition is produced at lower potential as the temperature increases, while the potential required to maintain the luminosity is increased. The differences in the latter case for H and CO_2 are extremely small. Since the potential required to maintain the discharge seems to be sensibly affected by traces of foreign gases and the variations are of small magnitude, it is possible that the potential required to maintain the luminosity is not affected through the temperature range used. This would indicate that in the phenomenon connected with Paschen's Law and the effects of pressure and temperature, we are dealing with the mechanism for producing ions. However, it is of equal interest to gather data concerning the conditions where this mechanism ceases to be effective. (Further comment is deferred until more data are available.)

The experiment described is preliminary to one in which it is hoped to study the electrical discharge in the region of the critical pressure and for temperature variations extending up to 800° C. or thereabouts.

Physical Laboratory,
Ohio State University.

ON THE DEFINITION OF AN IDEAL GAS.[1]

BY A. G. WEBSTER AND M. A. ROSANOFF.

THE definition of absolute temperature frequently found in treatises on thermodynamics or on physical chemistry leaves much to be desired. This important quantity is often defined by the equation $pv = RT$, where p and v represent the pressure and specific volume of an ideal gas. We are thus involved in the question, *what is an ideal gas?* To this the answer is often given, one which obeys the two gas laws, that of Boyle and Mariotte, and that of Gay-Lussac and Charles, or combining the two in an equation, $pv = RT$. We are thus led around in a circle to an absurdity. Of course the absolute temperature should not be thus defined, but should be defined from Carnot's principle, as was done by Lord Kelvin. But inasmuch as there are no ideal gases in nature, from which to make thermometers, the verification of how nearly a real gas verifies the definition of an ideal one is a matter of some difficulty. The verification of the law of Boyle is an easy criterion and if to this we add the proper second specification we get a proper definition. This is often done by saying that if the energy of a gas is independent of the specific volume, but depends only on the temperature, the gas is ideal. This may be replaced, as shown below, by the statement that the cohesion of the gas is zero. A third statement is that found, for instance, in a very careful paper by Buckingham, on the thermodynamic scale,[2] in which the correct definition is given, that "the ideal gas is one which follows Boyle's law and in which a free expansion, with no external work, would cause no change in the temperature." This criterion is tested in the Joule-Kelvin porous plug experiment, and the heating or cooling that usually takes place is called the Joule-Kelvin effect.

It is the purpose of this communication to examine in how far these different definitions are identical, and what conclusions can be drawn from any one of them. The laws of thermodynamics tell us what must be known of any substance in order that we may completely know its thermal and dynamical properties. Without thermodynamics we may find the equation of condition of the substance, that is, a relation

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[1] Presented at the meeting of the American Physical Society, October 24, 1908.

[2] Bulletin Bureau of Standards, May, 1907.

$$f(p\cdot v\cdot T)=0 \tag{1}$$

between the pressure, volume and absolute temperature. This involves dynamical experiments with a manometer at various constant temperatures, and thermometric experiments on the volume at various constant pressures. Thermodynamics puts no restriction on the form of the function f. Now appealing to thermodynamics the first law

$$dQ=dU+pdv=C_x dT+L_x dx \tag{2}$$

in the last member of which C_x and L_x are defined as the specific heat for the parameter x constant, and the latent heat with respect to x, respectively, gives

$$\left(\frac{\partial U}{\partial T}\right)_x=C_x-p\left(\frac{\partial v}{\partial T}\right)_x,\quad \left(\frac{\partial U}{\partial x}\right)_T=L_x-p\left(\frac{\partial v}{\partial x}\right)_T. \tag{3}$$

If the specific heat and latent heat are known completely as functions of any two variables, such as T and p or v, then we can find U the energy, and the properties of the body are known. This involves calorimetric experiments, with two variables. Eliminating U, we have

$$\frac{\partial C_x}{\partial x}-\frac{\partial L_x}{\partial T}=\frac{\partial p}{\partial x}\frac{\partial v}{\partial T}-\frac{\partial p}{\partial T}\frac{\partial v}{\partial x}. \tag{4}$$

The second law of thermodynamics gives

$$dQ=TdS=C_x dT+L_x dx, \tag{5}$$

so that we have

$$C_x=T\left(\frac{\partial S}{\partial T}\right)_x,\quad L_x=T\left(\frac{\partial S}{\partial x}\right). \tag{6}$$

Eliminating the S by differentiation,

$$\frac{\partial C_x}{\partial x}-\frac{\partial L_x}{\partial T}=-\frac{L_x}{T}, \tag{7}$$

and combining this equation with the last, we have

$$(8) \qquad \frac{L_x}{T} = \frac{\partial p}{\partial T}\frac{\partial v}{\partial x} - \frac{\partial p}{\partial x}\frac{\partial v}{\partial T},$$

so that L_x is obtained from (8), and need not be investigated experimentally. Knowing thus the partial derivatives of U and S we may obtain both these functions, when all questions about the substance can be answered. In order to fully characterize a substance, then, we require dynamical, thermometric and calorimetric experiments.

Let us now consider the energy. Combining both laws

$$(9) \qquad TdS = dU + pdv.$$

Subtracting $d(TS)$ from both sides,

$$(10) \qquad d(U - TS) = -pdv - SdT = d\psi,$$

where ψ is one of Gibbs's functions, and Helmholtz's free energy. The condition for a perfect differential gives

$$(11) \qquad \frac{\partial S}{\partial v} = \frac{\partial p}{\partial T}.$$

Using this relation in the equation (9), otherwise written,

$$(12) \qquad dU = \frac{\partial U}{\partial T}dT + \frac{\partial U}{\partial v}dv = T\left(\frac{\partial S}{\partial T}dT + \frac{\partial S}{\partial v}dv\right) - pdv,$$

along with (6),

$$T\left(\frac{\partial S}{\partial T}\right)_v = C_v$$

gives

$$(13) \qquad dU = C_v dT + \left\{ T\left(\frac{\partial p}{\partial T}\right)_v - p \right\} dv.$$

If we put

$$(14) \qquad p_c = T\left(\frac{\partial p}{\partial T}\right)_v - p, \quad dU = C_v dT + p_c dv,$$

then $p_c dv$ is the work necessary to be done in expanding the gas at constant temperature and stored as energy, and as this is positive if $p_e > 0$, we may call p_e the cohesion pressure of the gas, due to attraction of the molecules. If $p_e < 0$ the molecules repel each other.

Consider now the porous plug experiment. The conditions of this are that the work of driving the gas into the plug is taken from the energy, the conditions being adiabatic, so that $U + pv$ remains constant through the process of passing through the plug. Now $U + pv$ is Gibbs's function χ. Adding vdp to both sides of (9) we obtain

$$(15) \qquad vdp + TdS = d(U + pv) = d\chi.$$

Choosing as variables T, p, the equation for the plug experiment is

$$(16) \qquad d\chi = vdp + T\left(\frac{\partial S}{\partial T}dT + \frac{\partial S}{\partial p}dp\right) = 0.$$

Now replace $\partial S/\partial p$ by its equal $-\partial v/\partial T$ obtained by adding $d(pv)$ to (10) to get

$$(17) \quad d\zeta = d(U - TS + pv) = vdp - SdT, \quad \frac{\partial \zeta}{\partial p} = v, \quad \frac{\partial \zeta}{\partial T} = -S$$

and eliminating ζ. Using the definition of C_p in (6),

$$(18) \qquad C_p = T\left(\frac{\partial S}{\partial T}\right)_p$$

we have from (16)

$$(19) \qquad \left\{v - T\left(\frac{\partial v}{\partial T}\right)_p\right\}dp + C_p dT = 0$$

giving as the measure of the Joule-Kelvin effect

$$(20) \qquad K = \frac{dT}{dp} = \left\{T\left(\frac{\partial v}{\partial T}\right)_p - v\right\}\Big/ C_p.$$

This equation was obtained by Jochmann in 1859.

Let us now consider the three equations:

$$\text{I.} \qquad pv = RT,$$

$$\text{II.} \qquad p_e = T\left(\frac{\partial p}{\partial T}\right)_v - p = 0,$$

$$\text{III.} \qquad C_p K = T\left(\frac{\partial v}{\partial T}\right)_p - v = 0,$$

or the less inclusive than I expressing Boyle's law,

$$\text{I}_a. \qquad pv = f(T).$$

If I is true we easily show that II and III are also. Accordingly I is a *sufficient* definition of an ideal gas, but, as has been stated, involves the previous knowledge of the absolute temperature. On the other hand suppose that we have *only* Boyle's law I_a. Then we have

$$(21) \qquad p_e = \frac{1}{v}(Tf'(T) - f(T))$$

$$(22) \qquad C_pK = \frac{1}{p}(Tf'(T) - fT).$$

Consequently *either* of II or III involves the other, and in addition, since we obtain by integration of

$$(23) \qquad Tf'(T) - f(T) = 0, \quad f(T) = RT$$

the product *pv does* measure the absolute temperature. Accordingly Buckingham's definition is both necessary and sufficient, and defines the absolute temperature, but may be replaced by the equivalent specification of Boyle's law and no cohesion, or, what is the same thing, the energy depends wholly on the temperature.

Finally, suppose we disregard the gas laws altogether, and consider only II. Integrating on the supposition $v = const.$, we obtain

$$(24) \qquad \frac{T}{p} = g(v).$$

Such a gas then is suitable for measuring the absolute temperature on a *constant volume* thermometer, but will not, in general, have a zero Joule-Kelvin effect. In like manner, if we suppose III, integrating on the hypothesis of $p = const.$

$$(25) \qquad \frac{T}{v} = h(p),$$

so that such a gas is suited to measure the absolute temperature on a *constant pressure* thermometer, but will, in general, have its energy depend on the volume or pressure as well as on the temperature.

Finally, suppose that II and III hold simultaneously, then by multiplication of (24) and (25)

$$\frac{h(p)}{p}=\frac{g(v)}{v},$$

and as a function of p alone cannot equal a function of v alone, *both* must be constant $1/R$, so that the product pv *does* measure the absolute temperature, and the gas is ideal. Consequently the pair, I_a II, I_a III, or II, III is equivalent to the single equation I and we have three possible definitions of an ideal gas.

Thus it appears that the disappearance of the Joule-Kelvin effect in the plug experiment is *not* a test of absence of cohesion, unless the gas is ideal, and conversely. As an example consider the case instanced by van der Waals as being true for hydrogen,

$$p(v-b)=RT.$$

This evidently comes under (24) so that there is no cohesion and according to this criterion alone the gas would be ideal. Calculating K we have

$$\left(\frac{\partial v}{\partial T}\right)_p=\frac{R}{p}=v-b, \quad C_pK=-b,$$

so that the plug effect does not vanish, but is $-b/C_p$, which is of the sign actually found for hydrogen at ordinary temperatures.

No particular originality is claimed for the results here stated, as the far more important problem of determining the absolute temperature when K is known as a function of p, T has been treated in extenso by many authors, but it is believed that the points here made in the last pages have been nowhere explicitly stated.

Clark University,
October 23, 1908.

ON THE ABSOLUTE MEASUREMENT OF ELECTROMAGNETIC QUANTITIES.

BY V. BJERKNES.

1. The measurement of any physical quantity consists in a comparison with a quantity of the same kind used as unit. Thus every measurement is relative. But using the intrinsic relations existing among quantities of different kinds, we can make the choice of a unit for one quantity dependent upon that for other quantities. Thus the choice of primary units may be reduced to a minimum. As primary units there have been chosen those of length, mass and time, and a quantity is very inadequately said to be measured in absolute measure if the unit of this quantity is fully determined by the choice of units for length, mass and time.

From this it will be evident that the necessary and sufficient condition for the absolute measurement of n independent quantities, in the stated conventional sense of the expression, will demand a knowledge of n independent relations connecting these quantities with others which have already been measured absolutely. If we take any group of physical quantities, introduced to describe a certain class of phenomena, it cannot of course be postulated beforehand that the required number of equations exists. The necessity of introducing new primary units may present itself. But in the following discussion of absolute measurement of that group of quantities which have been introduced to describe the phenomena of the electromagnetic field, we proceed under the explicitly stated hypothesis that in this case no new primary unit is required.

2. To describe electromagnetic phenomena in a homogeneous isotropic dielectric under the simplest conditions, the following six quantities can be used:[1] α and β, respectively the electric and mag-

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[1] The notation and terminology are the same as in my Columbia lectures, "Fields of Force," New York, 1906, or the revised German edition of these lectures, "Die Kraftfelder," Braunschweig, 1909. Heaviside's rational units are used consistently.

netic inductivity of the dielectric supporting the field; **A** and **B**, respectively, the electric and magnetic flux (induction); **a** and **b**, respectively, the electric and magnetic field intensity. These quantities satisfy the following equations:

The equations of connection between a flux and the corresponding field intensity

$$\mathbf{A} = \alpha\mathbf{a}, \quad \mathbf{B} = \beta\mathbf{b}. \tag{a}$$

The equations describing the geometrical structure of the electromagnetic field

$$\begin{aligned} \frac{\partial \mathbf{A}}{\partial t} &= \operatorname{curl} \mathbf{b}, & \frac{\partial \mathbf{B}}{\partial t} &= -\operatorname{curl} \mathbf{a}, \\ \operatorname{div} \mathbf{A} &= 0, & \operatorname{div} \mathbf{B} &= 0. \end{aligned} \tag{b}$$

The equations for the dynamical properties of the electromagnetic field, which for our present purpose can be given the following form:

$$\begin{aligned} \varphi_e &= \int \tfrac{1}{2}\mathbf{A} \cdot \mathbf{a}\, dt, \\ \varphi_m &= \int \tfrac{1}{2}\mathbf{B} \cdot \mathbf{b}\, dt, \end{aligned} \tag{c}$$

φ_e representing the electric and φ_m the magnetic energy of the field, the volume integration being extended to the whole field.

As is well known the equations (*a*) and (*b*) determine uniquely the geometrical structure of the electromagnetic field at any time, if this structure be given at an initial time, in connection with suitable conditions at the boundary, if the field be limited. Thus from a geometrical point of view the system of equations is perfectly determinate.

Bringing also the equations for the energy into application, we can deduce all known phenomena relating to the mechanical effects or transformations of energy in the field. Thus even in this respect the system of equations seems to be perfectly determinate.

But this determinateness no more exists if the problem be to measure in absolute measure each of the quantities which have been introduced in order to form these equations containing our knowledge of the electromagnetic field.

3. Of the six quantities α, β, $\mathbf{A}$, $\mathbf{B}$, $\mathbf{a}$, $\mathbf{b}$ two can be eliminated by the equations of connection (*a*). Thus there remain four quantities which should be measured absolutely by use of the relations (*b*) and (*c*).

As is immediately seen, each of the equations (*c*) makes possible one absolute measurement. Considering here only the intrinsic principle of the measurement, not its practical realization, we can imagine the equations to be resolved with respect to the scalar products $\mathbf{A} \cdot \mathbf{a}$ and $\mathbf{B} \cdot \mathbf{b}$, or what is in this case the same thing, the arithmetical products Aa and Bb. Thus measurements of energy combined with space-measurements lead to absolute measurements of the products Aa and Bb. More than this cannot be deduced from the equations (*c*).

Passing to equations (*b*) we see that none of them can be used immediately as the basis of any absolute measurement. They only show how the electromagnetic quantities vary from place to place and from time to time, but give no relations of these quantities to quantities of known nature. One consequence can, however, be drawn from the equations, which gives rise to an absolute measurement. As is well known, it follows from these equations that electromagnetic disturbances are propagated at the speed $1/\sqrt{\alpha\beta}$. Thus the measurement of a velocity of propagation leads to the measurement of the product $\alpha\beta$, the value of this product being equal to the reciprocal of the square of the measured velocity of propagation.

Subject to absolute measurements there are thus only the three products Aa, Bb, $\alpha\beta$; and absolute measurement of each of these quantities separately will not be possible before we have succeeded in measuring absolutely a fourth independent expression formed by the same quantities. This requires the discovery of a fourth independent relation, and until this relation be discovered, we can only define systems of *relative* units for the electrical quantities, obtained by a supplementary choice in addition to the choice of the units of mass, length and time.

4. In order to determine suitable systems of relative units two different choices have been made. Either the electric or the magnetic inductivity of empty space (the free ether) has been given the numerical value 1. The first choice defines the system of electric,

the second the system of magnetic units. Both have by international convention been called systems of absolute units. In what follows we shall avoid this misleading terminology and denote them as the two systems of *relative* units.

However the system of units be defined, the numbers expressing the products Aa, Bb, and $\alpha\beta$ must remain unchanged. Remarking that the products Aa and Bb can also be written $1/\alpha \cdot A^2$ and $1/\beta \cdot B^2$, respectively, αa^2 and βb^2, we find the following rules for passing from the numbers expressing any of our quantities in *one* to the numbers expressing the same quantity in *another* system of units.

1. The numbers expressing electric and magnetic inductivity are changed in inverse proportion to each other.

2. The numbers expressing a flux and the corresponding field intensity are changed in inverse proportion to each other.

3. The number expressing an inductivity is changed in direct proportion to the square of the corresponding flux and in inverse proportion to the square of the corresponding field intensity.

Combining the three rules we finally deduce:

4. The numbers expressing electric flux are changed in the same proportion as those expressing magnetic field intensity, and the numbers expressing electric field intensity are changed in the same proportion as those expressing magnetic flux.

To express the same rules in formulæ, let the index q denote that the quantities are expressed in one and the index r that they are expressed in another system of units, and let it be known that the number expressing an electric flux in the system r is n times greater than the number expressing the same flux in the system q. We then get the following scheme for passing from the numbers expressing our quantities in one to those expressing them in the other of these systems of units

$$(a) \qquad \begin{array}{lll} A_q = nA_r, & \alpha_q = n^2\alpha_r, & B_q = \dfrac{1}{n}B_r, \\ a_q = \dfrac{1}{n}a_r, & \beta_q = \dfrac{1}{n^2}\beta_r, & b_q = nb_r. \end{array}$$

If the index q is referred to the electric and index r to the magnetic system of relative units, we have for free space α_q and β_r both numerically equal to unity, and α_r and β_q both equal to the reciprocal of the square of the velocity of light. For this case we thus find $n = 3 \times 10^{10}$. If on the other hand q be referred to the magnetic and r to the electric system of units, we have $n = \frac{1}{3} \times 10^{-10}$. The same system of equations will allow us to pass from the numbers expressing our quantities in any of our present relative systems of units to those expressing them in the absolute units defined by no other arbitrary choices than those of length, mass and time.

5. The origin of the indeterminateness occurring in the system of equations 2, (*a*), (*b*), (*c*) most probably comes from an incompleteness of the proper field-equations (*b*). For these give only one instead of two equations serviceable for the purpose of absolute measurement.

I shall try here a conjecture as to a hypothetical, more complete form of these equations. I have been led to this conjecture by a mechanical representation of the phenomena of the electromagnetic field, which I have recently made the subject of an investigation.[1] But the conjecture does not necessarily presuppose the full identity of the compared mechanical and electromagnetic phenomena. Our only supposition is that the analogy, which has been proved to hold so far as we have hitherto been able to examine it, also holds one step outside this limit.

My conjecture is then that the field equations should have the form

$$(a) \qquad \frac{\partial \mathbf{A}}{\partial t} = \text{curl } \mathbf{b}, \qquad \frac{\partial \mathbf{B}}{\partial t} + \mathbf{a} \nabla \mathbf{B} = -\text{ curl } \mathbf{a},$$

$\mathbf{a} \triangle \mathbf{B}$ denoting a vector having along the three axes o f*x*, *y*, *z* the components

$$(b) \qquad \begin{aligned} &a_x \frac{\partial B_x}{\partial x} + a_y \frac{\partial B_x}{\partial y} + a_z \frac{\partial B_x}{\partial z}, \\ &a_x \frac{\partial B_y}{\partial x} + a_y \frac{\partial B_y}{\partial y} + a_z \frac{\partial B_y}{\partial z}, \\ &a_x \frac{\partial B_z}{\partial x} + a_y \frac{\partial B_z}{\partial y} + a_z \frac{\partial B_z}{\partial z}. \end{aligned}$$

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[1] V. Bjerknes, "Die Kraftfelder," Braunschweig, 1909, Chapter XII. The mechanical representation referred to is an extension of that discussed by Wills, this REVIEW, Vol. 26, p. 220, 1908.

Thus the first of Maxwell's equations, the equation for the electric current, is left unchanged. The second, the equation for the "magnetic current," is completed by the addition of the term $\mathbf{a} \nabla \mathbf{B}$. Thus the extension consists in this: We have two kinds of magnetic current, viz., first the well-known magnetic displacement current, having the current density $\partial \mathbf{B}/\partial t$, and second a magnetic current existing wherever an electric field is superimposed upon a magnetic field. The density of this magnetic current is given by the vector $\mathbf{a} \nabla \mathbf{B}$.

6. Supposing this extended form of Maxwell's equations to be correct, we can at once determine the dimensions of the electromagnetic quantities.

From the second equation of 5, (*a*) it follows that $\mathbf{a} \nabla \mathbf{B}$ has the same dimensions as curl $\mathbf{a}$. The operations ∇ and curl both corresponding to a division by a length L, we get the dimensional equation $[a][B] = [a]$ or

$$[B] = 1.$$

Thus the magnetic flux is of dimensions zero. Remembering now that $Bb = 1/\beta \cdot B^2 = \beta b^2$ has the dimensions of energy divided by a volume or $ML^{-1}T^{-2}$, that $\alpha\beta$ has the dimensions of the reciprocal square of a velocity or $L^{-2}T^2$, and finally that $Aa = 1/\alpha \cdot A^2 = \alpha a^2$ has also the dimensions of energy divided by a volume, we find the following system of dimensions:

$$[A] = ML^{-2}T^{-1}, \quad [\alpha] = ML^{-3}, \quad [B] = M^0L^0T^0,$$
$$[a] = LT^{-1}, \quad [\beta] = M^{-1}LT^2, \quad [b] = ML^{-1}T^{-2}.$$

I leave open the question whether in a case like this the determination of the dimensions of a set of quantities involves a complete specification of their physical nature. But a possible specification is this: $\mathbf{A}$ is the specific momentum (momentum per unit volume or mass transport per square centimeter per second) and $\mathbf{a}$ the velocity of the medium supporting the field; α is the density and β the reciprocal coefficient of elasticity of this medium; $\mathbf{B}$ is the double angle of rotation of an element of this medium, and $\mathbf{b}$ one half of the corresponding specific moment of rotation (moment of rotation per unit volume). Adding that the velocities are directed along the electric, the axes of rotation along the magnetic lines of force, and that this rotation, which is directed negatively if the positive screw rule be used, is counteracted by a special kind of rotational or gyrostatic elasticity represented by the coefficient of elasticity $1/\beta$, we arrive at the mechanical representation referred to above which has led us to the change in Maxwell's second equation. Leaving open the question as to what degree the introduction of the term $\mathbf{a} \nabla \mathbf{B}$ in Maxwell's second equation involves an accordance of the mechanism of the electromagnetic field with the mechanical representation referred to, we shall in what follows use for convenience a terminology based upon an assumption that this accordance is established.

7. As the mere existence of the term $\mathbf{a} \nabla \mathbf{B}$ in the second Maxwell equation would thus lead to the determination of the dimensions of electromagnetic quantities, the quantitative experimental discovery of a phenomenon depending upon this term would lead to absolute measurement of the quantity B and thus to absolute measurement of all the fundamental quantities of the electromagnetic field.

To consider the simplest case possible, let us produce an homogeneous electric field by use of a condenser formed by two parallel circular discs. Outside this condenser let us place two equally strong cylindrical magnets, facing each other, with poles of the same name. The whole system shall be symmetrical with respect to the common axis of the magnets and the condenser-discs, as well as with respect to a plane normal to this axis, passing through the middle point between the discs. This point which is a neutral point of the magnetic field, may be chosen for origin of coördinates, the axis of symmetry being used as the axis of x, and the axes of y and of z being contained in the plane of symmetry already defined.

We can now use Taylor's theorem in finding the components B_x, B_y, B_z of the magnetic flux in the space surrounding the neutral point. Neglecting terms of the second and higher orders, each component is represented by a linear expression in x, y, z. The origin of coördinates being a neutral point, the constant terms will be equal to zero. The linear expressions can be reduced further by the condition that the coördinate planes are planes of symmetry of the field. At any two points situated symmetrically with respect to one of these planes the components of the vector tangential to the plane should therefore be directly and the components normal to the plane oppositely equal to each other. Noting this we easily see that the three components are given by

$$(a)\qquad B_x = \left(\frac{\partial B_x}{\partial x}\right)_0 x,\quad B_y = \left(\frac{\partial B_y}{\partial y}\right)_0 y,\quad B_z = \left(\frac{\partial B_z}{\partial z}\right)_0 z.$$

As there is further symmetry round the axis of x, we must have $(\partial B_y/\partial y)_0$ equal to $(\partial B_z/\partial z)_0$. Using this result and the solenoidal condition div $\mathbf{B} = 0$, or

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0$$

we find that

$$\left(\frac{\partial B_z}{\partial z}\right)_0 = \left(\frac{\partial B_y}{\partial y}\right)_0 = -\tfrac{1}{2}\left(\frac{\partial B_x}{\partial x}\right)_0,$$

showing that the field in the neighborhood of the neutral point is fully specified by the value of the derivatives of the components of B at the neutral point.

Proceeding now to form the expressions 5, (*b*), we remark that the electric field intensity has no component along the axes of y and z. On the other hand it is seen from the expressions (*a*), that B_y and B_z have no derivatives with respect to x. Thus only one term of the scheme 5, (*b*) will be different from zero, namely,

$$(b)\qquad a_x\left(\frac{\partial B_x}{\partial x}\right)_0.$$

Thus in the space between the two condenser-plates we have a magnetic current directed from one plate to the other, and having a current density equal to the product of the field intensity of the homogeneous electric field into the value of the derivatives $(\partial B_x/\partial x)_0$ of the magnetic flux. If this derivative is positive, *i. e.*, if the magnetic poles are both south poles, the magnetic current will have the same direction as the electric field. If both poles be north poles the magnetic current will be directed against the electric field.

Now Maxwell's second equation expresses that a magnetic current surrounds itself with an electric field exactly in the same way as an electric current surrounds itself with a magnetic field, the only difference being in the rule of signs. The current density (*b*) in all points of a circle round the axis x and of radius r gives a total magnetic current $\pi r^2 a_x(\partial B_x/\partial x)_0$. This current produces an electric field intensity a' having the line integral $2\pi r a'$ along the circumference of this circle. Identifying these two expressions we get

$$(c) \qquad a' = \tfrac{1}{2} r a \left(\frac{\partial B_x}{\partial x}\right)_0 .$$

The direction of the electric field intensity a' relatively to the magnetic current is given by the negative screw rule.

The field intensity produced by the magnetic current will combine with that of the primary homogeneous electric field. The lines of force of this field will thus be twisted *so as to become screw-lines, forming the angle*

$$(d) \qquad \theta = \frac{a'}{a_x} = \tfrac{1}{2} r \left(\frac{\partial B_x}{\partial x}\right)_0$$

with their primary course. As seen from this expression, the angle of twist is independent of the strength as well as of the direction of the electric field, and dependent only upon the derivative $(\partial B_x/\partial x)_0$ of the magnetic flux. Remembering what has been said above on the signs we see that the lines of force should become left-handed screw-lines if both magnetic poles be south-poles and right-handed if both poles be north-poles. Further it is seen from formula (*d*) that the angle of twist θ will increase proportionally to the distance r from the axis of rotation, reaching its maximum near the edge of the discs.

Taking into consideration this twist of the electric lines of force, it is easily seen how the magnetic current continues its course: Having arrived at the second condenser-plate, it continues as a surface-current radially outwards; then through the exterior space where the electric field is no more homogeneous, and where the magnetic field can no more be represented by the linear expressions (*a*), it returns to the first plate, where it goes radially inwards, closing its course.

Equation (*d*) can now be used for the absolute measurement of magnetic polarization. Let B be the value of this polarization on the axis of symmetry at the distance l from the neutral point. We then have

$$\left(\frac{\partial B_x}{\partial x}\right)_0 = \frac{B}{l}.$$

Introducing this in equation (d) and solving with respect to B, we get

$$B = 2\theta\frac{l}{r}.$$

This formula is interesting inasmuch as it directly represents the magnetic polarization as a quantity of the nature of an angle. $\frac{1}{2}B$ representing the angle of rotation round a magnetic line of force, we find, putting $l = r$,

$$\tfrac{1}{2}B = \theta.$$

In the field considered therefore the following simple relation exists: The angle of rotation round a magnetic line of force at a point on the axis of symmetry situated at any distance r from the neutral point is equal to the angle of twist of those electric lines of force which are at the distance r from the axis of symmetry.

Measuring the angle of twist θ of the electrical lines of force, we thus get an absolute measurement of $\frac{1}{2}B$ and thus of B. Supposing that we measure this same magnetic flux in relative magnetic measure we get a comparison of this measure with the absolute measure. Let B_m be the number which expresses the flux in magnetic measure, and let this number be n times greater than the number B expressing it in absolute measure. We then have

$$B = \frac{1}{n}B_m.$$

Afterwards by equation 4, (a) we find the corresponding relations for the other quantities.

8. Using the means which I had at hand, I have performed the experiment of which I have thus indicated the principle. The result being negative inasmuch as no angle of twist θ of the electric lines of force could be detected, it will be of no interest to enter into a detailed description. I therefore only give these summary indications.

Two cylindrical electromagnets were used, the iron cores of which had a length of about 40 and a diameter of 7.5 cm. On these cores was coiled a 3 cm. thick and 34 cm. long layer of copper wire. The current was furnished by a generator delivering 30, or, when forced, 50 amperes at the pressure of 4 volts. The condenser-discs had the same diameter as the cores of the magnets and were placed 2 cm. apart. At the upper edge of the discs, at 3.5 cm. distance from the axis, was suspended a light aluminium needle carrying a mirror. At the distance of 2 meters scale and telescope were placed.

The condenser plates were charged to opposite potentials varying from 1,000 to 2,000 volts. In the field thus produced the needle was held strongly in a position of equilibrium, being directed along the lines of force. The magnetic field was then put on, alternately, so that two south-poles or two north-poles were facing each other. The change from the one case to the other should cause a change of direction of the electrical lines of force of 2θ, and the needle pointing always in the direction of the lines of force should show the same deflection.

All causes of disturbance being carefully removed, or their effect being eliminated by special measurements (direct magnetic effect on the needle, small disturbances of the electric field), it was established that the hypothetical double-deflection could not reach two tenths of a millimeter. Thus the angle θ did not exceed $\frac{1}{40000}$ radian. Hence the rotation round a magnetic line of force at a point situated on the axis of symmetry at the distance of 3.5 cm. from the neutral point could not exceed $\frac{1}{40000}$. The value of B at this point being equal to twice this angle, the result of the measurement gives for the magnetic flux at the place indicated

$$B < \frac{1}{200000} \text{ cm.}^0 \text{ gr.}^0 \text{ sec.}^0$$

The ends of the magnets were at the distance of 1.75 cm. from the neutral point. Using the Lenard spiral the flux at the surface of the magnets was found equal to about 180 irrational magnetic units of flux. Passing to Heaviside's rational units, which we have used consistently in the preceding, we have to divide this number by $\sqrt{4\pi}$. Thus the flux at the surface of the magnets was about

50 rational magnetic units. If the field had extended to double the distance of 3.5 cm. from the neutral point, we should have had the field-intensity 100. Thus

$$B_m = 100 \text{ rational magnetic units.}$$

The number expressing a magnetic flux in magnetic measure is thus at least two million times greater than that expressing the same flux in absolute measure. We then have to introduce in equations 4, (*a*) $n > 2 \times 10^6$, and get as a result the scheme

$$A > 2 \times 10^6 A_m, \quad \alpha > 4 \times 10^{12} \alpha_m, \quad B < \tfrac{1}{2} \times 10^{-6} B_m,$$

$$a < \tfrac{1}{2} \times 10^{-6} a_m, \quad \beta < \tfrac{1}{4} \times 10^{-12} \beta_m, \quad b > 2 \times 10^6 b_m.$$

Using the scheme 4, (*a*) once more, to pass from the numbers expressing our quantities in magnetic to those expressing them in electric units, we get $n = \frac{2}{3} \times 10^{-4}$ and thus

$$A > \tfrac{2}{3} \times 10^{-4} A_e, \quad \alpha > \tfrac{4}{9} \times 10^{-8} \alpha_e, \quad B < \tfrac{3}{2} \times 10^4 B_e,$$

$$a < \tfrac{3}{2} \times 10^4 a_e, \quad \beta < \tfrac{9}{4} \times 10^8 \beta_e, \quad b > \tfrac{2}{3} \times 10^{-4} b_e.$$

To consider more closely the content of some of these equations, we remark that in the free ether $\alpha_e = 1$. Thus the absolute electric inductivity or the density of the ether is greater than $\frac{4}{9} \times 10^{-8}$ grams per cubic centimeter. Further in the free ether we have $\beta_m = 1$. Thus the absolute magnetic inductivity or the reciprocal coefficient of elasticity of the free ether is smaller than $\frac{1}{4} \times 10^{-12}$ gr.$^{-1}$ cm. sec.^{2} Or the coefficient of elasticity of the ether is greater than 4×10^{12} gr. cm.$^{-1}$ sec.$^{-2}$, *i. e.*, dynes per square centimeter. Putting $a_e = 1$, we get $a < \frac{3}{2} \times 10^4$, *i. e.*, the velocity corresponding to the rational electric unit of electric field intensity is smaller than $\frac{3}{2} \times 10^4$ centimeters per second. $A_e = 1$ gives the corresponding value of A greater than $\frac{2}{3} \times 10^{-4}$, *i. e.*, the specific momentum corresponding to the rational electric unit of electric flux is a transport of mass greater than $\frac{3}{2} \times 10^{-4}$ grams per square cm. per second. Putting $B_m = 1$ we get the corresponding value of B smaller than $\frac{1}{2} \times 10^{-6}$, *i. e.*, the double angle of rotation corresponding to the rational magnetic unit of magnetic flux is smaller than $\frac{1}{2} \times 10^{-6}$ radian. Setting finally $b_m = 1$ we get the corresponding value of b greater than 2×10^6, *i. e.*, the half torque per unit volume, which represents the rational magnetic unit of field intensity, is greater than 2×10^6 dynes-centimeter per cubic centimeter.

9. Repeating the experiment described there would be no difficulty in increasing the accuracy far beyond the limit reached in this first attempt. Coiling more wire upon the magnets, and using the strongest current which the coil can support for a short interval of time, it would probably be possible to increase ten times or more the value of the derivative $(\partial B_x / \partial x)_0$ and thus of the angle θ. On the other hand, if the experiment be performed at a place perfectly free from shaking (I only succeeded in getting satisfactory readings thanks to a strong snow-fall, after which the shaking produced by the traffic on the streets was very much reduced), it ought to be possible, using a powerful telescope, to increase ten times the accuracy of the readings. Finally, we can get, theoretically, infinite increase of the effect by increasing the diameter of the iron-cores and the condenser-discs. Far more delicate methods for detecting the electric field intensity a' than that of measuring the angle of twist θ can also be used. Finally, new arrangements can be devised, which may have advantages over the arrangement used in this experiment, which was chosen especially on account of its theoretical simplicity. To mention only one example, an experiment analogous to that of Hall could be performed. The magnets should then have opposite poles facing each other, and the lines of magnetic force should cut those of electric force at right angles. This arrangement would be much more advantageous for the production of a powerful magnetic field.

The higher the accuracy brought into application not leading to the discovery of the effect, the greater must be the value of the density of the ether. With this increasing density follows a decreasing value of the velocity corresponding to the present unit of electric field-intensity and an increasing value of the specific momentum corresponding to the present unit of electric flux. Further, there would follow with increasing density increasing values of the coefficient of elasticity $1/\beta$ of the ether, increasing values of the half torque corresponding to the present unit of magnetic field intensity and decreasing values of the double angle of rotation corresponding to the present relative unit of magnetic flux.

Sir Oliver Lodge has recently produced interesting arguments in favor of an ether-density of the order of magnitude of 10^{12} grams per cubic centimeter, thus 10^{20} times greater than the limit arrived at above. If this estimate be correct, the angle of twist of the electric lines of force really occurring in our experiment would be 10^{10} times smaller than the limit found by us. To discover this angle under the supposition of ten times increased values of $(\partial B_x/\partial x)_0$ and ten times increased accuracy of the reading, we should have to use iron-cores and condenser-discs having a diameter equal to the radius of the earth. Under these conditions the prospect of discovering the effect using the stationary or static fields produced by ordinary means would be very small. The only way out would then probably be the application of the enormously intense magnetic field existing in the immediate neighborhood of a moving negative corpuscle. The first thing to investigate would then be the motion of the corpuscle itself, the new term introduced in Maxwell's equations being taken into account.

10. It should be emphasized in this connection that the view of the mechanical nature of the electromagnetic field which has led Sir Oliver Lodge to his conclusions is different from ours, inasmuch as he explains the magnetic field by motions along the magnetic lines of force and the electric field by rotations round the electric lines of force. Nevertheless his arguments in favor of the ether density 10^{12} will be equally legitimate in both cases. But the method of bringing about an experimental test of the question would be somewhat different. From Lodge's theory, which is essentially the same as that of Larmor and Sommerfeld, we find that Maxwell's equations should have the form

$$(a) \qquad \frac{\partial \mathbf{A}}{\partial t} + \mathbf{b}\nabla\mathbf{A} = \text{curl } \mathbf{b}, \quad \frac{\partial \mathbf{B}}{\partial t} = -\text{curl } \mathbf{a}.$$

The term $\mathbf{b}\nabla\mathbf{A}$ represents a new form of electric, not of magnetic current. While, according to the previous equations 5, (*a*), a static or stationary electric field will be distorted when a magnetic field is superimposed upon it, we derive from these new equations (*a*) that a static or stationary magnetic field should be distorted if an electric field be superimposed upon it.

In attempting to discover this distortion, we can use the same experimental arrangement as above, only letting the magnetic field be a homogeneous one, produced by opposite poles *N*, *S* of the magnets, while an electric field of the same character as the previously used magnetic one could be produced by charging both discs to the same potential. A slightly magnetic needle might be used, held strongly in equilibrium along the magnetic lines of force. These lines should be twisted into screw lines when the electric field is produced, the screw lines having positive twist if both discs be charged positively. Changing from a positive to a negative charge, the needle should get a deflection equal to the double angle of twist. I have performed this experiment qualitatively, without being able to discover any deflection. I mention it in order to emphasize that the two different views as to the correspondence between electromagnetic and mechanical quantities lead to parallel methods for solving the problem of absolute measurements. One of the experiments having given a positive result, we should arrive at a decision regarding the two different views.

THE MOLECULAR AND THE FRICTIONAL FLOW OF GASES IN TUBES.

By Willard J. Fisher.

IN 1866 O. E. Meyer[1] integrated approximately the equations for the flow of a viscous gas through a circular capillary tube, obtaining the following equation

$$F=\frac{\pi R^4}{16\eta Lp}\left(1+\frac{4\zeta}{R}\right)(p_1^2-p_2^2),$$

in which F is the volume of gas passing per second, measured at pressure p, R is the radius, L the length of the tube, ζ the slip coefficient, p_1 the entrance, p_2 the exit pressure, η the coefficient of viscosity. He also showed[2] that ζ is equal to the mean free path of the molecule. This varies inversely as the pressure, which, of course, is not the same throughout the tube; an average value of ζ may be used, computed from the kinetic theory equations for pressure and viscosity, and the two measured end pressures. A not very satisfactory derivation of the value of ζ was given by E. Warburg,[3] a better one by A. Bestelmeyer,[4] who showed that the proper pressure to use is the arithmetical mean of the entrance and exit pressures. Using this value of the mean pressure and the kinetic theory relations, O. E. Meyer's equation reduces to

$$F=\left[\frac{\pi R^4}{16\eta L}+\frac{\pi\sqrt{3}}{2}\,\frac{R^3}{L(p_1+p_2)\sqrt{\rho_1}}\right](p_1^2-p_2^2),$$

in which ρ_1 denotes the density of the gas at a pressure of 1 dyne per square centimeter and the temperature of the tube. The numerical factor $\pi\sqrt{3}/2$ has the value 2.72.

M. Knudsen,[5] of Copenhagen, has recently published a theoretical and experimental study of the flow of gases through capillary tubes, with pressures ranging from ordinary to one or two thousandths of a millimeter, or, in other words, under conditions such that the mean free path is varied from values very small in relation to the radius of the tube to others very large in relation to the radius. The flow of the gases was then in part the well-known viscous flow, at the other extreme a flow in which the *slip* on the walls is the whole motion, and between an intermediate, mixed state. The two distinct states he calls *molecular flow* and *frictional flow*, which seem to be well-selected and descriptive names. To represent his results he uses the following formula :

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[1] O. E. Meyer, Pogg. Ann., 127, p. 269, 1866.

[2] O. E. Meyer, Kinetic Theory of Gases, English by R. E. Baynes, p. 211.

[3] E. Warburg, Pogg. Annalen, 159, p. 399, 1876.

[4] A. Bestelmeyer, Ann. d. Physik, 13, p. 980, 1904; see also Phys. Rev., 28, p. 73, 1909.

[5] M. Knudsen, Ann. d. Physik, 28, p. 75, 1909.

$$Q_t = \left[ap + b\frac{1 + c_1p}{1 + c_2p}\right](p_1 - p_2),$$

in which

$$Q_t = pF,$$

$$a = \frac{\pi R^4}{8\eta L},$$

the coefficient of frictional flow,

$$b = \frac{4\sqrt{2\pi}}{3}\frac{R^3}{L\sqrt{\rho_1}},$$

the coefficient of molecular flow, derived from his theory; c_1 and c_2 are constants for a given tube-radius and temperature. They have the ratio 2/2.47, as found by Least Squares. $p = (p_1 + p_2)/2$, L and R are the length and radius of the tube.

The equation may readily be compared with that of O. E. Meyer if written

$$Q_t = \left[\frac{\pi R^4}{16\eta L} + \frac{4\sqrt{2\pi}}{3}\frac{R^3}{L(p_1 + p_2)\sqrt{\rho_1}}\frac{1 + c_1p}{1 + c_2p}\right](p_1^2 - p_2^2).$$

The numerical factor $4\sqrt{2\pi}/3$ has the value 3.34. The fraction

$$\frac{1 + c_1p}{1 + c_2p}$$

is designed so as to equal 1 for large values of p, c_1/c_2 for small and vanishing values; it therefore varies from 1 to about 0.8.

Knudsen's curve for CO_2 is plotted with ordinates $T = Q_t/(p_1 - p_2)$, abscissas $p = (p_1 + p_2)/2$, and shows a decided minimum in the region where p becomes comparable with the saturation pressure of mercury vapor. While others have not studied the flow at as low pressures as Knudsen, and so their results are not strictly comparable, it is interesting to note that previous results on mixtures of light and dense gases as those of J. Puluj[1] and P. Breitenbach[2] on H and CO_2, and of P. Tänzler[3] on He and Ar, show that the viscosity coefficient has a maximum value not for either pure gas but for a mixture of the two, the position of the maximum depending on the temperature. Hence for such mixtures Knudsen's T would show a minimum, if we imagine an experiment carried on in which $(p_1 + p_2)/2$ would be continually diminished, while at the same time the per cent. of heavy gas were increased. This imaginary case and Knudsen's experiments are not exactly parallel, but show the necessity of caution and a more careful elimination of the mixture effect, even at low pressures.

It will be observed that for large pressures the two equations of O. E. Meyer and Knudsen are identical, for $3.34 \times 2.00/2.47 = 2.70$, and in all other respects they are exactly alike. For lower pressures the factor

$$\frac{1 + c_1 p}{1 + c_2 p}$$

comes into play; but as shown above, this is of relatively small importance. It is not at all unlikely that the need of this factor is due to a defect in the experimental method.

The experiments were carried on at practically constant temperature; gas passed directly from contact with a free mercury surface through the tube, carrying with it saturated mercury vapor. At ordinary pressures this usually forms a negligible factor, but when the pressures approach or even fall below the pressure of saturated mercury vapor it is certain that a very important part of the substance passing through the tube is not the gas in question at all; Knudsen's observations at low pressure relate then, not to the molecular flow of gases but to the flow of varying mixtures of gases and mercury vapor. Were the vapor disturbance eliminated it is not at all improbable that the results would be very well expressed by the long-known formula of O. E. Meyer.

It is to be hoped that improvements in the method of observation may be made, as investigations in this region are of much promise.

GÖTTINGEN, June, 1909.

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[1] J. Puluj, Carls Rep., 15, pp. 518, 633, 1879.

[2] P. Breitenbach, Wied. Ann., 67, p. 820, 1899.

[3] P. Tänzler, Verh. d. Phys. Ges., p. 222, 1906.

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PHYSICAL REVIEW.

APPLICATION OF GIBBS-HELMHOLTZ EQUATION TO CONCENTRATION CELLS.

BY FRANK J. MELLENCAMP.

INTRODUCTION.

UP to the present time there is no theory that satisfactorily accounts for the E.M.F. of concentration cells. The Nernst equation, which is based upon the gas law, applies only to very dilute solutions — millinormal and under. For somewhat greater concentrations it has to be modified, and for strong solutions it no longer holds. It has been pointed out[1] that the Gibbs-Helmholtz equation applies when the heats of reaction and dilution are so small that they may be neglected. This also restricts the solutions to very dilute ones. The present research has for its object, in general, the application of the Gibbs-Helmholtz equation to concentration cells, and in particular, to cells in which the heat of dilution is not negligible, and to show how to take this factor into account. The class of cells to which application is made is that in which the two electrodes are composed of amalgams of the same metal immersed in a salt solution of the metal, the concentration of the amalgams being the same, but that of the solution being different at the two electrodes.

HISTORICAL.

Robert Mayer[2] was one of the first to point out that there existed a relation between the change in chemical energy and the electrical energy of the current. Helmholtz[1] was of the opinion that all the chemical energy lost reappeared in the electrical form. This opinion was also held by Sir William Thomson,[2] J. Bosscha[3] and others. Later investigations by Raoult,[4] Thomsen[5] and Braun[6] showed experimentally that the electrical energy may be either greater or less than the heat of chemical reaction. Gibbs[7] and Helmholtz[8] theoretically came to the same conclusion. Helmholtz finally put this theory into the form in which it is usually found. He called that portion of the whole heat energy which can be transformed into electrical energy, "free energy," that portion which cannot be transformed, "bound energy" and the sum of these two "total energy." The theory stated in this form is known as the Gibbs-Helmholtz equation.

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[1] Trans. Am. Electrochem. Soc., 1, 32, 1906.

[2] Mayer, Die Organische Bewegung in ihrem Zusammenhange mit dem Stoffwechsel.

Numerous examples[9] of the Gibbs-Helmholtz equation have been worked out for voltaic cells, but no general application has been made to concentration cells. Richard and Lewis[10] applied the equation to concentration cells for determining the heats of amalgamation from the temperature coefficient when amalgam was used for one of the electrodes and pure metal for the other.

Carhart,[11] in his thermoelectric theory of concentration cells, maintains that concentration cells are devices for converting heat of their surroundings into electrical energy, and that whenever the heat of reaction and dilution is negligible, the last term only of the Gibbs-Helmholtz equation, $T \cdot dE/dT$, remains and represents the E.M.F.

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[1] Ostwald, Klassiker der exakten Wissenschaften, Nr. 1.

[2] Phil. Mag., 2, 429, 1851; 2, 551, 1851.

[3] Pog. Ann. Phys., 101-517, 1857; 103, 1858; 105, 396, 1858.

[4] Ann. de Chimie et de Phys., 4, 392, 1856.

[5] Wied. Ann. Phys., 11, 246, 1880.

[6] Wied. Ann. Phys., 5, 183, 1878; 16, 561, 1882; 17, 593, 1882.

[7] Trans. of Connecticut Acad., 3, 588, 1878.

[8] Berl. Sitsungs., 22, 1884.

[9] Czapski, Wied. Ann. Phys., 21, 209, 1884. Gockel, Wied. Ann. Phys., 24, 618, 1885. Jahn, Wied. Ann. Phys., 28, 21, 1886; 28, 491, 1886; 28, 498, 1886. Nernst & Bugarsky, Zeit. f. anorg. Chem., 14, 145, 1890. Poincaré, Ann. de Chimie et de Phys., 21, 344, 1890. Oswald & Miller, Zeit. Phys. Chem., 34, 62, 1900; 34, 612, 1900. Cohen, Zeit. Phys. Chem., 60, 707, 1901. Lorenz, Electrolysis of Fused Salts, Vol. 3. Lorenz, Zeit. Phys. Chem., 63, 121, 1908.

[10] Am. Acad. of Arts and Sciences, 34, 87, 1898.

[11] Trans. Am. Electrochem. Soc., 2, 123, 1902.

of the cell. Application[1] was made to two cells of the type, Ag — $AgNo_3$ conc. — $AgNo_3$ dil. — Ag, and Cd.Hg — *n*/10 $CdSO_4$ — *n*/100 $CdSO_4$ — Cd.Hg. The agreement between the calculated values and the observed values was very good but in both cases the calculated values were low because the heat of dilution was not taken into account.

In 1907 Richards and Forbes[2] reported work done on concentration cells in which the difference of concentration was in the amalgams comprising the electrodes. Heats of dilution of cadmium and zinc amalgams were measured with a specially devised calorimeter. The heat of dilution of a 3 per cent. cadmium amalgam was so small as to be negligible. In this case the last term of the Gibbs-Helmholtz equation was found to represent the E.M.F. of the cell. For the zinc amalgam, the heat of dilution is not negligible. No application of the Gibbs-Helmholtz equation was made to such cells.

For the last two decades numerous examples of concentration cells have been worked out for the purpose of verifying the equations of Nernst and Planck, but as far as is known to the writer the Gibbs-Helmholtz equation has not been applied to concentration cells of any type so as to take into account the heat of dilution.

THEORETICAL.

According to the Gibbs-Helmholtz equation wich is based upon fundamental principles of thermodynamics,

$$E = \frac{H}{nF} + T\frac{dE}{dT}, \tag{1}$$

in which E represents the E.M.F. of the cell; H the heat of reaction; n the valence of the ion; F the number of coulombs transported by a gram equivalent of the ion; T the absolute temperature.

Equation (1) when written

$$nFE = H + nFT\frac{dE}{dT} \tag{2}$$

gives the conditions under which the electrical energy nFE, is a measure of the heat of reaction of a cell. It is to be observed that $nFE = H$ whenever $dE/dT = 0$, or $T = 0$. In other words, the electrical energy of a cell is equal to the heat of reaction, (1) when the temperature coefficient is zero, (2) when the absolute temperature is zero. If $dE/dT > 0$, or the temperature coefficient is positive, the electrical energy is greater than the heat of reaction. If $dE/dT < 0$, or the temperature coefficient negative, the electrical energy is less that the heat of reaction. In the former case the cell absorbs heat from the surroundings; in the latter, it evolves heat. The total electrical energy of the cell may be calculated by determining the heat of reaction and the temperature coefficient.

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[1] Trans. Am. Electrochem. Soc., 1, 32, 1906.

[2] Zeit. Phys. Chem., 58, 683, 1907.

If equation (2) is written

$$H = nF\left(E - T\frac{dE}{dT}\right) \qquad (3)$$

an expression is obtained by means of which the heat of reaction may be calculated by merely measuring the E.M.F. at different temperatures. It is to be noted in this case that $H = 0$ whenever $E = T\cdot dE/dT$, or the heat of reaction is equal to zero whenever the E.M.F. of the cells equals $T\cdot dE/dT$ as is approximately the case for concentration cells with dilute solutions; $H > 0$ whenever $E > T\cdot dE/dT$, or the heat of reaction is positive whenever the E.M.F. of the cell is greater than $T\cdot dE/dT$; $H < 0$ whenever $E < T\cdot dE/dT$, or the heat of reaction is negative whenever the E.M.F. is less than $T\cdot dE/dT$.

These equations were applied to cells of the following general type: Amalgam — concentrated salt solution of metal in amalgam — dilute salt solution of metal in amalgam — amalgam. In all cases the amalgam used for the two electrodes was of the same kind and concentration; the solutions in the two legs of the cell were of the same salt but of different concentration. Only those solutions were used for which the heats of dilution were known.

For cells of the above type the heat of dilution becomes the heat of reaction. If we dilute from any initial concentration to the concentrations of the solutions in the two legs of the cell, the heats of dilution, in general, are different. This difference is available as energy to produce E.M.F. represented by the first term of the

Gibbs-Helmholtz equation. The electrode in the more dilute solution is the anode, that is, the electrode by which the current enters the cell. Whenever the heat of dilution is positive, that is, when dilution generates heat, then more heat is generated for the entrance of a gram-equivalent of the electrode at the anode than is absorbed by the removal of an equal amount at the cathode. If the heat of dilution is negative, that is, if dilution absorbs heat, then more heat is absorbed at the anode than is generated at the cathode. Let us suppose the quantities of the two solutions so great that the entrance of a gram-equivalent at the anode and the removal of an equal amount from the solution about the cathode do not change the concentration of the solutions. Now when the gram-equivalent of metal at the anode goes into solution it becomes a salt and is diluted down to the same concentration as that of the solution surrounding the anode; at the same time an equal amount of the metal is removed from the more concentrated solution surrounding the cathode. When the heat of dilution is positive, this latter operation absorbs less energy than is derived from the former; when negative the former absorbs more energy than is generated by the latter. This difference of energy is available to produce E.M.F. When it is positive the E.M.F. is directed within the cell from the more dilute to the concentrated solution; when negative, from the concentrated to the more dilute.

DESCRIPTION OF APPARATUS.

For the accurate determination of the E.M.F. of concentration cells in which the electrolytes on the two sides of the cell are of different concentration two things are especially desired in the construction of the apparatus. First, that the temperature of the two legs of the cell be kept the same and remain constant for any desired length of time; and second, that during this time there shall be no mixing of the electrolytes. To satisfy the first condition, three thermostats were employed, one for the warm bath, the second for the intermediate, and a third for the cold. The temperature in these could be maintained constant to within 0.02° C. by means of a toluene-mercury regulator. Beckmann thermometers were employed for reading the temperatures. The E.M.F. was measured by means of an Otto Wolff potentiometer which reads directly to 0.00001 volt. A Weston normal cell was used as a standard of comparison.

The concentration cell was set up as shown in Fig. 1. A_1 and A_2 represent the two legs of the cell, which are connected by means of a bent tube containing a filter paper plug *h* and provided with a glass stop-cock *S*; *a, a*, the two electrodes; *b, b*, the two electrolytes. The electrodes *a, a* are composed of glass cups *c, c* about 2 cm. in diameter, into which are fused glass tubes. Through the tubes near the bottom are sealed short platinum wires *d, d*, which make connection with the amalgams *e, e* in the cups and the mercury *f, f* inside the tubes. Copper wires *g, g* lead from the mercury in the tube direct to the potentiometer.

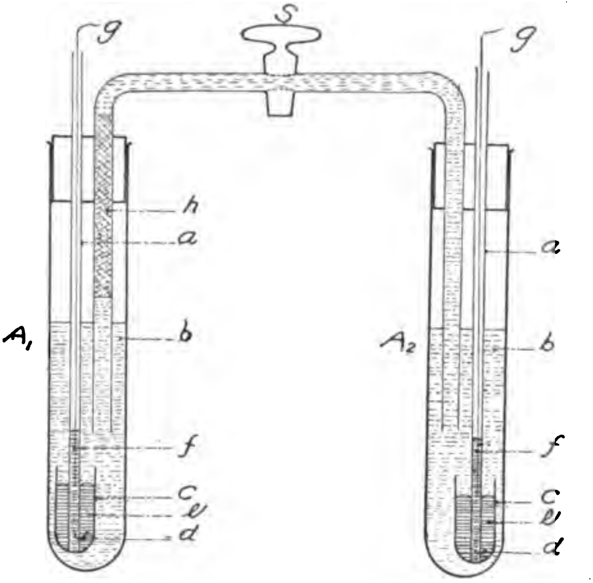

Fig. 1.

The filter paper plug *h* in the arm of the tube making connection with the two solutions deserves special attention, as it is this that prevents the mixing of the electrolytes, and thus meets the requirements of the second condition. The solutions in the two legs of the cell were maintained at the same level and the stop-cock kept open only while readings were being taken. The cell was suitably supported in the thermostat so that the temperature remained the same for both legs. The filter paper plug was near the end of the tube so that the junction of the liquids might be maintained at the same temperature as the electrodes.

PREPARATION OF MATERIALS.

Solutions.

Zinc and cadmium sulphates were prepared in practically the same manner. The chemically pure salt was dissolved in twice distilled water to form a saturated solution, which was allowed to evaporate until about three fourths of the salt had crystallized out. The crystals were washed several times with distilled water and then, before drying, transferred to a closed vessel where they were kept until ready for use. It is essential that the salt be dry but at the same time lose none of its water of crystallization; so a portion was carefully dried each time just before making up the solutions. The solutions were prepared as follows: $ZnSO_4 \cdot 50H_2O$, means that for every gram-molecule of anhydrous salt 50 gram-molecules of water, or 900.76 grams of water are required. One gram-molecule of $ZnSO_4 \cdot 7H_2O$ weighs 287.67 grams; of this 161.46 grams are anhydrous salt, and 126.11 grams water of crystallization. If hydrated salt be used, then 126.11 grams must be subtracted from 900.76 grams, thus leaving 774.65 grams of water to be added to the gram-molecule of the hydrated salt.

The lead nitrate was recrystallized from the pure salt, dried on a watch glass and kept in a desiccator until ready for use. The solutions were prepared by weight similar to the sulphate.

The preparation of the zinc chloride solutions will be described in connection with the zinc chloride cell.

Amalgams.

The mercury used for the amalgams was prepared by double distillation. For most of the work the amalgams were prepared, electrolytically from an acid solution of the purified salts; however, the observations showed no difference between these and others prepared by the fusion of the metal and mercury. Since the exact concentration of the amalgams is not essential, they were kept under an acid solution until ready for use, and then, just before using, thoroughly washed with distilled water.

EXPERIMENTS AND RESULTS.

Zinc Sulphate Cells.

Amalgam electrodes are best adapted for work on electrode potential. Since with pure metals it is very difficult to prevent oxidation or exclude occluded gases, variable and unsatisfactory results must necessarily follow. To obtain consistent results the electrodes must remain reasonably constant for days. Amalgams best fulfill these conditions. Preliminary tests were made to see what amalgams were best adapted for zinc sulphate cells. A series of amalgams ranging from .1 to 15 per cent. were used for this purpose. The amalgams must not change phase in the temperature interval employed, that is, they must remain either solid or liquid throughout the test. Change of phase causes a break in the temperature coefficient. Amalgams below one per cent. and above 10 per cent. fulfill these conditions, the former remaining liquid throughout; the latter, solid. Amalgams of a little less than one per cent. were finally chosen as best adapted to the following work because an amalgam which is much more dilute oxidizes readily, and the solid is less convenient to handle. The solutions used were prepared from the freshly dried $ZnSO_4 \cdot 7H_2O$ as described. High temperature could not be employed because zinc sulphate at 39° changes from the heptahydrate to the hexahydrate salt, and irregularities in readings occurred when this temperature was approached. These irregularities were probably due to the surface condition of the amalgam changing with the least possible current.

The cell was set up in the following manner: All parts were washed with nitric acid, then treated with cleansing solution, after which they were thoroughly rinsed with distilled water and carefully dried. The two legs of the cells were filled about two thirds full with their respective electrolytes. Into one end of the connecting tube was packed chemically pure filter paper, as shown in Fig. 1. The tube was then filled with the two electrolytes, the denser solution in the shorter arm and the more dilute in the longer arm, the two meeting at about the middle of the filter paper plug.

The amalgam, which had previously been prepared and kept under a dilute solution of sulphuric acid, was washed with distilled water and then transferred to the small cups by means of a clean pipette. The electrodes were at once put into the solutions of the two legs, leaving them exposed to the air but a few seconds; the connecting tube was put in place and the legs of the cell closed with rubber stoppers, which had been soaked in distilled water for several days. When all had been adjusted as shown in Fig. 1, the stoppers were covered with paraffin.

The following combination was first used:

$$Zn \cdot Hg - ZnSO_4 \cdot 50H_2O - ZnSO_4 \cdot 400H_2O - Zn \cdot Hg.$$

Before recording observations, the cell was allowed to stand a few hours, until equilibrium had been reached. In general it took a few hours before the readings of the cells became constant. There was no regularity in the shifting of the preliminary readings; some cells assumed a high initial value and gradually fell to constancy, others started with a low initial value and rose, while still others remained constant from the very first.

TABLE I.

Date.	Temp.	E.M.F.
August 30, 8:00 P. M.	0	0.01581
" " 8:30 "	0	0.01580
" " 8:40 "	0	0.01580
" 31, 9:00 A. M.	25	0.01708
" " 9:15 "	25	0.01708
" " 10:00 "	0	0.01578
" " 10:30 "	0	0.01578
" " 11:00 "	25	0.01711
" " 1:45 "	25	0.01712
" " 3:30 "	25	0.01713
" " 3:55 "	0	0.01580
" " 4:05 "	0	0.01577
" " 4:30 "	0	0.01578
" " 7:00 "	0	0.01579
" " 7:30 "	25	0.01708
" " 8:00 "	25	0.01710

Table I. gives the readings of cell 1*b* for a period of twenty-four hours after beginning to record observations. This is a fair sample of the constancy of the cells used. The readings show that the changes due to oxidation or other causes are so small as not to materially affect the results. The readings also show that the cell practically assumes the same value when brought back to the same temperature, otherwise it was discarded as in all subsequent work. Irregularities in nearly all cases were traced to some defect in the construction of the cells, such as a crack in the glass where the platinum was sealed through, or to change of phase for some of the materials. In fact, it seems that irregularities due to change of phase became evident several degrees before the temperature was reached at which this change nominally takes place. In most cases the temperature coefficient used was the mean of three different sets of readings as shown in Table I.

Table II. gives the results of the investigation of zinc sulphate cells. E_h is the E.M.F. due to the heat of dilution and $T \cdot dE/dT$ is the E.M.F. derived from the heat of the surroundings. All cells were set up and manipulated in the general manner just described. The solutions used were those for which the heats of dilution had been determined by Thomsen.[1] The most concentrated solution used was that of $ZnSO_4 \cdot 50H_2O$. $ZnSO_4 20H_2O$ was tried in the preliminary work, but the salt began to crystallize out before the lower temperature was reached, thus restricting the range for determining the temperature coefficient to higher readings. This fact made it undesirable as an electrolyte; consequently this solution was rejected. For the remaining solutions all possible combinations were made.

Two sets of comparisons are given in the table — one consisting of two columns of E.M.F.'s and the other consisting of two columns of heats of dilution. The computed E.M.F. for the first set is obtained in the following manner. According to Thomsen, for one gram-molecule of $ZnSO_4 \cdot 20H_2O$ diluted to $ZnSO_4 \cdot 50H_2O$, 318 calories of heat are generated. Also, $ZnSO_4 \cdot 20H_2O$ diluted to $ZnSO_4 \cdot 400H_2O$ generates 400 calories. Hence one gram-molecule of $ZnSO_4 \cdot 50H_2O$ diluted to $ZnSO_4 \cdot 400H_2O$ generates 82 calories. Since this heat of dilution is positive, it is free energy and gives rise to

$$\frac{82 \times 4.189}{2 \times 96{,}500} = 0.00178 \text{ volt E.M.F.}$$

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[1] Thermochem. Untersuchungen, J. Thomsen (see Table X.).

TABLE II.

Solutions.	Cells.	dE/dT	$T \cdot dE/dT$ at 25°.	E_k	Comp. E.M.F.	Obs. E.M.F.	Comp. Heat of Dil. in Cal.	Obs. Heat of Dil. in Cal.
$ZnSO_4 \cdot 50H_2O$	1*a*	0.0000528	0.01573	0.00178	0.01751	0.01713	65	82
	1*b*	0.0000524	0.01561	"	0.01739	0.01710	69	"
$ZnSO_4 \cdot 400H_2O$	1*c*	0.0000510	0.01520	"	0.01698	0.01701	83	"
	1*d*	0.0000510	0.01520	"	0.01698	0.01711	88	"
	1	0.0000518	0.01544	0.00178	0.01722	0.01709	76	82
$ZnSO_4 \cdot 50H_2O$	2*a*	0.0000352	0.01029	0.00125	0.01193	0.01162	53	67
$ZnSO_4 \cdot 200H_2O$	2*b*	0.0000344	0.01045	"	0.01170	0.01159	62	"
	2*c*	0.0000340	0.01013	"	0.01158	0.01139	58	"
	2	0.0000345	0.01029	0.00125	0.01174	0.01153	57	67
$ZnSO_4 \cdot 100H_2O$	3*a*	0.0000340	0.01013	0.00072	0.01085	0.01090	35	33
	3*b*	0.0000344	0.01025	"	0.01097	0.01090	30	"
$ZnSO_4 \cdot 400H_2O$	3*c*	0.0000344	0.01025	"	0.01097	0.01081	26	"
	3	0.0000343	0.01021	0.00072	0.01093	0.01087	30	33
$ZnSO_4 \cdot 50H_2O$	4*a*	0.0000174	0.00518	0.00106	0.00624	0.00628	51	49
$ZnSO_4 \cdot 100H_2O$	4*b*	0.0000160	0.00477	"	0.00583	0.00607	60	"
	4	0.0000167	0.00498	0.00106	0.00604	0.00618	56	49
$ZnSO_4 \cdot 100H_2O$	5*a*	0.0000172	0.00512	0.00039	0.00551	0.00541	13	18
$ZnSO_4 \cdot 200H_2O$	5*b*	0.0000180	0.00536	"	0.00575	0.00552	7	"
	5	0.0000176	0.00524	0.00039	0.00563	0.00547	10	18
$ZnSO_4 \cdot 200H_2O$	6*a*	0.0000164	0.00489	0.00033	0.00522	0.00543	25	15
$ZnSO_4 \cdot 400H_2O$	6*b*	0.0000180	0.00536	"	0.00569	0.00556	9	"
	6	0.0000172	0.00513	0.00033	0.00546	0.00550	17	15
2+6	7	0.0000519	0.01542	0.00178	0.01720	0.01703	74	82
4+3	8	0.0000510	0.01519	0.00178	0.01697	0.01708	86	82
4+5+6	9	0.0000515	0.01535	0.00178	0.01713	0.01715	83	82
Mean of 1, 7, 8, 9	10	0.0000515	0.01535	0.00178	0.01713	0.01709	80	82

The temperature coefficient of (10) is 0.0000515, as a mean of the four independent determinations. Hence $T \cdot dE/dT$ at 25° is 298 × 0.0000515 equal to 0.01535. Then E computed equals 0.00178 + 0.01535 = 0.01713 volt, as compared with 0.01709 volt, the observed value at 25°.

To obtain the computed heat of dilution of the second set of comparisons, the process is reversed. The observed E.M.F. at 25°

equals 0.01709 volt, and $T \cdot dE/dT$ equals 0.01535 volt as before. Then 0.01709 — 0.01535 equals 0.00174 volt as the difference of the observed E.M.F. and that due to the heat from the surroundings. This surplus energy arises from the heat of dilution and is equivalent to 80 calories, as compared with 82 calories, observed by Thomsen.

By applying the principle of a cyclic system we obtain three checks on cell 1; the sum of 2 and 6 should equal 1; 3 and 4, equal 1; 4, 5 and 6, equal 1. Thus we obtain independently, four values for the combination $Zn \cdot Hg - ZnSO_4 \cdot 50H_2O - ZnSO_4 \cdot 400H_2O - Zn \cdot Hg$; (10) gives the mean of these four. From a comparison of these four it will be seen that the agreement is very satisfactory. If we compare the computed and observed E.M.F.'s the maximum difference is about one per cent., while for the mean it is less than one fourth of one per cent. Since the heats of dilution account for only about ten per cent. of the total E.M.F., the second set of comparisons is not so favorable from a percentage basis. Comparison of results on a percentage basis does not signify much unless we take into account all the factors which enter the quantities compared. To illustrate: one per cent. difference, between the computed and observed E.M.F. in cell 1, is equivalent to ten per cent. difference between the computed and observed heats of dilution; in cell 6 one per cent. difference in E.M.F. is equivalent to seventeen per cent. difference in heats of dilution.

For ten of the sixteen cells recorded in Table II. the calculated E.M.F. exceeds the observed; the mean difference, as shown above, is less than one fourth of one per cent. With the exception of *2c*, *4a* and *4b* the observed E.M.F. differs from the mean value by less than one per cent. The difference between the observed E.M.F. and that due to $T \cdot dE/dT$ is from five to eighteen per cent. of the whole E.M.F. of the cell and bears no direct relation to either, but is proportional to the heat of dilution as observed calorimetrically. The Gibbs-Helmholtz equation applies rigorously in this case, the deviation being less than the probable experimental error.

Cadmium Sulphate Cells.

For zinc sulphate cells the heat of dilution is relatively small, accounting for only a small portion of the total E.M.F. of the cell, the remainder being due to $T\cdot dE/dT$, the heat absorbed from the surroundings. Extending the investigation to solutions of cadmium sulphate we obtain a cell in which the heat of dilution is relatively large. The study of this cell was taken up in the general manner just described. The amalgam, however, proved more troublesome. With dilute amalgams, the E.M.F. did not consistently return to the same value when brought back to the same temperature. Evidently some change had taken place either in the amalgam or the electrolyte. Since there is no transition point in cadmium sulphate below 74° these irregularities cannot be ascribed to the salt. They might be due to oxidation, as the electrode in the more dilute solution gradually lost its brilliancy and became covered with a thin film. A "time test" made for a period of eighteen days, as given in Table III., shows that the cell remained remarkably constant. After reaching equilibrium, the maximum change in E.M.F. was only 0.00003 volt. Next, cells were set up from which the air had been thoroughly removed, and then saturated with hydrogen. These remained constant at 25° as before, but showed the same irregularities with change of temperature. Thus it seemed altogether unlikely that oxidation was the disturbing factor. Richards and Forbes[1] found that oxidation became evident only with very dilute amalgams.

The potential difference between cadmium amalgam and cad-

Table III.

Cell set up November 30, 1908.		25°	0.00532
December	1, 10:00 A. M.	"	0.00520
"	2, 9:00 "	"	0.00526
"	2, 1:20 P. M.	"	0.00525
"	3, 2:00 "	"	0.00527
"	5, 9:00 A. M.	"	0.00524
"	5, 4:00 P. M.	"	0.00526
"	6, 2:00 "	"	0.00527
"	9, 9:00 A. M.	"	0.00527
"	10,	"	0.00526
"	17,	"	0.00525

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[1] Zeit. Phys. Chem., 58, 683, 1907.

mium sulphate does not change for amalgams between five and fifteen per cent. Amalgams lying within this interval would therefore seem most favorable; hence six per cent. amalgam was chosen. This amalgam has the further advantage of being a liquid at ordinary temperature. Cells set up with it gave readings which were more constant, consequently more trustworthy. The temperature interval for determining the temperature coefficient was taken from 25° to 40°; at lower temperatures the solid phase appears.

TABLE IV.

Solutions.	Cells.	dE/dT	$T \cdot dE/dT$ at 25°.	E_h	Comp. E.M.F.	Obs. E.M.F.	Comp. Heat of Dil. in Cal.	Obs. Heat of Dil. in Cal.
$CdSO_4 \cdot 30 \cdot 6H_2O$	1*a*	0.0000235	0.00700	0.01565	0.02265	0.02205	694	722
$CdSO_4 \cdot 400H_2O$	1*b*	0.0000244	0.00727	"	0.02292	0.02213	685	"
	1*c*	0.0000210	0.00626	"	0.02191	0.02213	732	"
	1	0.0000230	0.00684	0.01565	0.02249	0.02210	704	722
$CdSO_4 \cdot 30 \cdot 6H_2O$	2*a*	0.0000080	0.00238	0.00960	0.01199	0.01176	432	443
$CdSO_4 \cdot 100H_2O$	2*b*	0.0000078	0.00231	"	0.01191	0.01170	434	"
	2	0.0000079	0.00235	0.00960	0.01195	0.01173	433	443
$CdSO_4 \cdot 100H_2O$	3*a*	0.0000155	0.00462	0.00604	0.01066	0.01025	260	279
$CdSO_4 \cdot 400H_2O$	3*b*	0.0000156	0.00465	"	0.01069	0.01038	265	"
	3*c*	0.0000160	0.00477	"	0.01081	0.01052	266	"
	3	0.0000157	0.00468	0.00604	0.01072	0.01038	264	279
$CdSO_4 \cdot 50H_2O$	4*a*	0.0000192	0.00572	0.01082	0.01654	0.01638	492	499
$CdSO_4 \cdot 400H_2O$	4*b*	0.0000200	0.00596	"	0.01678	0.01637	481	"
	4*c*	0.0000185	0.00551	"	0.01633	0.01615	491	"
	4	0.0000192	0.00573	0.01082	0.01655	0.01629	488	499
$CdSO_4 \cdot 50H_2O$ $CdSO_4 \cdot 200H_2O$	5	0.0000090	0.00268	0.00747	0.01115	0.01107	387	391
$CdSO_4 \cdot 200H_2O$ $CdSO_4 \cdot 400H_2O$	6	0.0000095	0.00283	0.00234	0.00517	0.00496	98	108
2 3	7	0.0000236	0.00703	0.01565	0.00268	0.02211	697	722
5 6	8	0.0000185	0.00551	0.01082	0.01633	0.01603	485	499

Table IV. gives the results of thirteen different cells. As before 2 and 3 should equal 1, and 5 and 6 should equal 4. While the agreement is quite satisfactory, it is not so good as for the zinc sulphate. It will be seen that for all but one, 1*c*, of the thirteen cells, the computed E.M.F. exceeds the observed value; the difference between the two being about 2 per cent. It is further to be noted that this difference is comparatively constant, indicating an experimental error, rather than some defect in the principles involved.

The heat of dilution accounts for from fifty to eighty per cent. of the total E.M.F. of the cell.

Zinc Chloride Cells.

In the preceding experiments the cells have a positive heat of dilution and a positive temperature coefficient. Three other variations may arise, namely, (1) a positive heat of dilution with a negative temperature coefficient; (2) a negative heat of dilution with a positive temperature coefficient; (3) a negative heat of dilution with a negative temperature coefficient. In the third case the E.M.F. would be reversed, that is, the electrode in more dilute solution would become the cathode. In the other two cases the direction of the E.M.F. depends upon the relative magnitude of the quantities. For the solutions used the zinc chloride cell satisfies case (1) — a positive heat of dilution with a negative temperature coefficient — and will be next studied.

Upon diluting zinc chloride solutions hydrolysis takes place causing a change in concentration. The addition of the amalgam electrode sometimes brings about this change; even a change in temperature may produce it. Taking these things into account, it is difficult to prepare a solution of known concentration and to maintain it throughout the test.

The amalgam for this cell was a portion of that originally prepared for the zinc sulphate cells, and diluted to about one per cent. Solutions of approximately $ZnCl_2 \cdot 20H_2O$ and $ZnCl_2 \cdot 400H_2O$ were used for the preliminary tests. These were carefully filtered, then subjected to temperature changes and refiltered. Cells from these solutions remained reasonably constant but were not entirely free from hydrolysis (especially the more dilute solution) when subjected to temperature changes. The observed E.M.F. was higher than the computed value; this was probably due to the fact that the dilute solution hydrolyzed, thereby decreasing its concentration.

After trying solutions of different concentration under various conditions it was found that hydrolysis could be avoided by allowing the solutions to stand for some time; accordingly the two solutions from which the final determinations were made, $ZnCl_2 \cdot 18.84H_2O$ and $ZnCl_2 \cdot 180H_2O$, were allowed to stand for about six weeks. These solutions were carefully filtered before using. Great care was taken that all parts of the cell were perfectly dry and the solutions made contact in the middle of the filter paper plug in the connecting tube. Table V. gives the readings of the cell with these two solutions.

TABLE V.

Date.	Cell Set Up.	Temp.	E.M.F.
August 3,	5:00 P. M.	25	0.06737
" 4,	8:00 A. M.	25	0.06722
" 4,	8:45 "	25	0.06722
" 5,	9:00 "	25	0.06722
" 5,	3:00 P. M.	25	0.06720
" 6,	8:30 A. M.	25	0.06718
" 6,	11:30 "	25	0.06717

The total change, after equilibrium had been reached, was only 0.00005 volt. During this time, the temperature was lowered to 0° twice, and raised to 40° three times. The concentration of the solution was determined by titration against $AgNO_3$ using potassium chromate as an indicator. Heats of dilution were obtained by interpolation by means of a curve plotted from Thomsen's values. The curve as shown in Fig. 2 appears perfectly smooth, and when plotted on a large scale gives values which can be but little in error. To insure greater accuracy the solutions chosen were approximately those for which the heats of dilution had been determined.

Table VI. gives the results of three different cells from these solutions.

TABLE VI.

Solutions.	Cells.	dE/dT	$T \cdot dE/dT$ at 25°.	E'_h	Comp. E.M.F.	Obs. E.M.F.	Comp. Heat of Dil. Cal.	Obs.[1] Heat of Dil. Cal.
$ZnCl_2 \cdot 18.8H_2O$	*a*	−0.000097	−0.02891	0.09832	0.06941	0.06722	4429	4530
$ZnCl_2 \cdot 180H_2O$	*b*	−0.000098	−0.02920	0.09832	0.06912	0.06723	4443	4530
	c	−0.000097	−0.02891	0.09832	0.06941	0.06712	4435	4530

[1] Thomsen (see Table IX.).

The agreement is very satisfactory, and as in the case of the cadmium sulphate cells, the computed E.M.F. is greater than the observed value.

The zinc chloride cell furnishes a specially instructive illustration of the correctness of the principle involved, because the temperature coefficient is negative, as in most voltaic cells. The E.M.F. is accordingly less than the value derived from the heat of dilution.

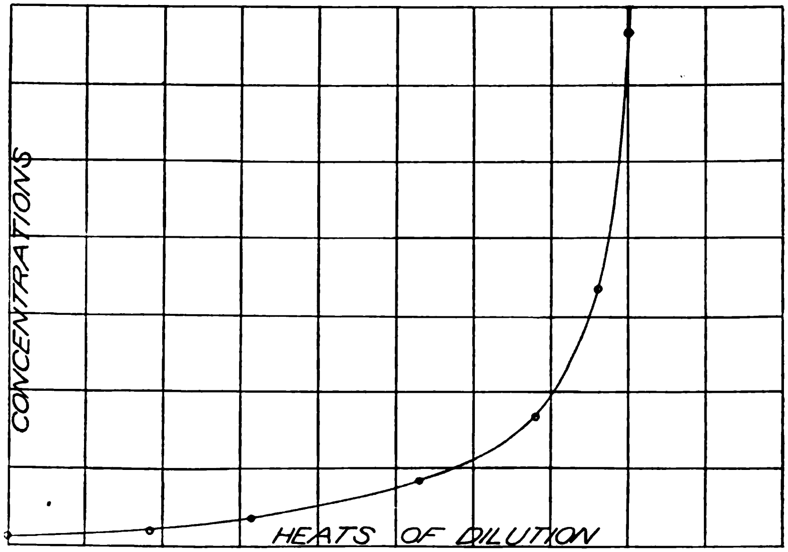

Fig. 2.

Lead Nitrate Cells.

Two more possible variations remain — a negative heat of dilution with either a positive or negative temperature coefficient. It becomes therefore a matter of great interest to know if the Gibbs-Helmholtz principle will apply to these. The lead nitrate cell fulfills one of these conditions, a negative heat of dilution with a positive temperature coefficient.

The solutions for this cell were prepared by weight from the recrystallized salt, similar to the sulphate; the amalgams, electrolytically, from an acid solution of the purified salt, and they were kept under an acid solution. Lead amalgams when left exposed to the air or in contact with aqueous solutions oxidize readily.

As a preliminary step, tests were made with reference to the constancy of the electrodes. With dilute amalgam and concentrated solutions the cell remained fairly constant, but when dilute solutions were used the readings were more variable. The electrodes in the dilute solutions became covered with a grayish film probably due to oxidation. Tests were made with $Pb(NO_3)_2 \cdot 400H_2O$, the most dilute solution used, to determine under what conditions oxidation might be prevented. It was found that by slightly acidulating the solution with nitric acid the film did not form and the readings became reasonably constant. Next, this test was extended to see what effect the acid had upon the initial readings of the cell. Equal portions of the two solutions from which the cell was set up were treated with the same quantity of nitric acid. The amount used was just enough to prevent the film from forming, and was determined beforehand. To make certain that both solutions were treated with the same amount, the conductivity water from which the solutions were made was acidulated before preparing the solutions. Such cells gave concordant values with change of temperature and remained reasonably constant. Furthermore, the readings were almost identical with the initial readings when no acid was used. For example, a cell without acid started with 0.01616 volt and reached apparent equilibrium at 0.01630 volt in about two hours, after which it slowly dropped in value. Table VII., which embodies the result of the final test, shows that this is in exact agreement with the mean of the three cells examined. Thus it is fair to suppose that the addition of the nitric acid did not affect the initial readings. It might be noted in this connection that all cells without the acid attained about the same initial readings at apparent equilibrium, and then slowly dropped in value. Constant agitation seemed to lessen this change, but did not completely eliminate it.

TABLE VII.

Solutions.	Cells.	dE/dT	$T \cdot dE/dT$ at 25° C.	E_h	Comp. E.M.F.	Obs. E.M.F.	Comp. Heat of Dil.	Obs.[1] Heat of Dil.
$Pb(NO_3)_2 \cdot 100H_2O$	*a*	0.000146	0.04351	−0.02756	0.01595	0.01620	−1659	−1270
$Pb(NO_3)_2 \cdot 400H_2O$	*b*	0.000145	0.04321	−0.02756	0.01565	0.01632	−1240	−1270
	c	0.000141	0.04202	−0.02756	0.01446	0.01636	−1183	−1270

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[1] Thomsen (see Table IX.).

Referring to Table VII., it will be noted that the computed E.M.F. is now less that the observed value. This was found to be true for all the preliminary determinations. The difference between the computed and observed values in cells *a* and *b* is well within the limits of experimental error. No reason can be assigned for the greater variation in cell *c* as it gave a good initial reading and remained fully as constant as the others.

Referring again to comparisons by percentage difference, it might be pointed out that if $T\cdot dE/dT$ observed be compared with $T\cdot dE/dT$ computed the percentage difference is only about one third of what it is for the E.M.F.'s and less than two thirds of what it is for the heats of dilution. For example, take cell *a*, $T\cdot dE/dT = 0.0435$ volt; $T\cdot dE/dT$ computed $= 0.04373$ volt; percentage difference is 0.5. The percentage difference between the observed and computed E.M.F. is about 1.4; between observed and computed heats of dilution, 0.8. The observed and computed E.M.F.'s do not give a fair comparison of the quantities involved unless the temperature coefficient and heat of dilution are both of the same sign.

This last case is particularly interesting since the chemical process is endothermic and the cell when in operation uses only a portion of the heat absorbed from the surroundings to be converted into electrical energy. It furnishes a striking proof of the incorrectness of the assumption that the heat of reaction is a measure of the electrical energy of a cell.

Discussion of Results.

One more variation remains — a negative heat of dilution with a negative temperature coefficient. As previously stated such a cell must have its E.M.F. reversed. This combination does not seem probable, for taking a current from such a cell would increase the difference in concentration of the solutions, thereby causing its E.M.F. to rise.

However, as it is, the results obtained are a very satisfactory confirmation of the principle involved. The deviation from the computed value in each case is comparatively constant indicating experimental errors. It now becomes a matter of interest to attempt an explanation of the probable cause of these small discrepancies.

The last term of the equation $T \cdot dE/dT$, is proportional to the absolute temperature, and therefore purely thermo-electric. According to Carhart's thermo-electric theory, the coefficient dE/dT is the algebraic sum of all the thermo-electric forces per degree arising at the several junctions. The temperature between the junction of the two liquids was considered the same as that at the electrodes. This perhaps is not strictly true because there would be more or less diffusion between the two solutions and the construction did not admit of immersing the whole cell. The thermo-electromotive force at the junction is accordingly less than the value which would be obtained with the junction at the same temperature as the electrodes. This assumption that the temperature of the junction is not always the same as that of the electrodes, and furthermore that it is variable, is borne out by the fact that the cells always remained the most constant at 25° or about room temperature. Results based on this assumption would indicate that for the zinc sulphate cell the thermo-electromotive force at the junction was practically negligible; for the cadmium and zinc chloride cells, it was directed from the concentrated to the more dilute solution; and in the lead nitrate cell, from the dilute to the more concentrated solution.

Secondly, the small difference may be explained on the assumption that the electrode in the more dilute solution oxidized to a greater extent than the other, thus establishing a counter E.M.F. In the case of the lead nitrate the observed E.M.F. is higher than the calculated value, which would indicate the reverse had taken place. This may be caused, however, by the addition of the nitric acid affecting the surface condition of the electrodes.

Thirdly, this difference between the observed and computed values may be due to the calorimetric determination of the heats of dilution. The heats of dilution as given may be inaccurate and are in need of revision because of modern facilities and a better knowledge of absolute units. Assuming the Gibbs-Helmholtz equation to apply rigorously in all cases, the results would indicate that the heats of dilution for the zinc sulphate solutions are correct as determined, that the heats of dilution for the cadmium sulphate and zinc chloride solutions are too large, and that the heats of dilution for the lead nitrate solutions are too small.

SUMMARY.

1. Simple concentration cells, made up of various solutions for which the heats of dilution have been determined, were investigated with reference to the application of the Gibbs-Helmholtz equation. The total heat of reaction becomes the heat of dilution and answers for the first term of the equation.

2. The four following cells were investigated: (1) Cadmium amalgam electrodes with cadmium sulphate of different concentration as electrolytes; (2) zinc amalgam electrodes with zinc sulphate of different concentration as electrolytes; (3) zinc amalgam electrodes with zinc chloride of different concentration as electrodes; (4) lead amalgam electrodes with lead nitrate of different concentration as electrolytes.

TABLE VIII.

Solutions.	E_h	$T \cdot dE/dT$	Comp. E.M.F.	Obs. E.M.F.	Comp. Heat of Dilution.	Obs. Heat of Dilution.
$ZnSO_4 \cdot 50H_2O$— $ZnSO_4 \cdot 400H_2O$	0.00178	0.01535	0.01713	0.01709	80 Cal.	82 Cal.
$CdSO_4 \cdot 30.6H_2O$— $CdSO_4 \cdot 400H_2O$	0.01565	0.00693	0.02258	0.02211	701 "	722 "
$ZnCl_2 \cdot 18.8H_2O$— $ZnCl_2 \cdot 180H_2O$	0.09832	−0.02901	0.06931	0.06719	4435 "	4530 "
$Pb(NO_3)_2 \cdot 100H_2O$ $Pb(NO_3)_2 \cdot 400H_2O$—	0.02756	0.04336	0.01580	0.01626	−1250 "	−1270 "

3. A filter paper plug in the connecting tube was used to prevent the mixing of the two solutions and to reduce diffusion to a minimum.

4. A method was found for successfully preventing oxidation of the lead amalgams under lead nitrate solution without vitiating the values of the readings of the cell. Zinc chloride solutions were also used without hydrolysis.

5. Table IX. gives a brief summary of the four cells investigated. For the lead nitrate cell the mean of cells *a* and *b* only are used.

The first cell has a large positive temperature coefficient with a relatively small heat of dilution; the second, a small temperature coefficient with a relatively large heat of dilution; the third, a negative temperature coefficient with a large positive heat of dilution; the fourth, a positive temperature coefficient with a relatively large negative heat of dilution. Of the two variables E_k and $T \cdot dE/dT$ which go to make-up the E.M.F. of the cell, three of the four possible variations worked out.

6. The Gibbs-Helmholtz equation applies rigorously in all cases, the difference being less than the probable experimental error.

TABLE IX.

m n	$ZnSO_4$ n 20	$CdSO_4$ n 30.6	$ZnCl_2$ n 5	$Pb(NO_3)_2$ n 40
10			1850	
20			3150	
50	318	223	5320	
100	367	443	6810	−1230
200	385	614	7630	−1980
400	400	722	8020	−2500

In conclusion the writer wishes to express his thanks to Professor Carhart under whose direction this investigation was conducted for his interest and helpful suggestions.

For convenience of reference a table of the heats of dilution for the solutions used is appended. The heats of dilution are given in gram calories. *m* is the number of gram-molecules of water added to the solution when it already contains *n* gram-molecules of water to one gram-molecule of the anhydrous salt.

University of Michigan,
Ann Arbor, Mich.,
May, 1909.

ANODE DROP WITH HOT CaO CATHODE.

By C. D. Child.

THE drop in potential at the anode of the electric arc in a vacuum has been found to be much smaller than the corresponding drop in a vacuum with small currents and unstriated discharge. The value of the former is about 6 or 7 volts,[1] while that of the latter is about 20 volts.[2] The discharge from hot CaO in a vacuum is in some respects intermediate between these two forms of discharge and the investigation of which the following is an account was undertaken for the purpose of examining this drop with such intermediate currents.

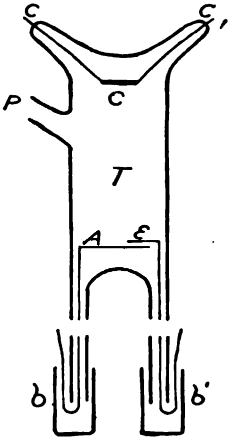

Fig. 1.

Apparatus. — The apparatus in the first experiments here described is shown in Fig. 1. *T* is a tube 3 cm. in diameter, through which the discharge passes. *C* is the cathode consisting of a piece of platinum foil, approximately 2 mm. in width, 6 mm. in length and .02 mm. in thickness, the bottom being covered with CaO. This was welded to aluminum wires and was heated by a storage battery connected at *c* and *c'*. *A* is the anode. This was an iron disk 2.4 cm. in diameter, soldered to an iron wire which ran through a tube containing mercury into a cup *b*. The wire was there bent and brought out of the cup. By this means the anode could be conveniently raised and lowered, while the air was exhausted from the tube. The potential difference between *A* and *C* was maintained by a battery of small storage cells. *e* is the exploring electrode brought out at *b'* in a similar way.

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[1] Wills, Phys. Rev., 16, 65, 1904. Child, Phys. Rev., 20, 365, 1905. Stark, Retschinsky and Schaposchihnoff, Ann. d. Phys., 19, 243, 1905. Pollak, Ann. d. Phys., 19, 217, 1906.

[2] Skinner, Phil. Mag. (6), 8, 387, 1904.

This was also iron. The upper vertical part was covered with a small glass tube for several centimeters, in order to insulate this part of the wire from the conducting gas about it. The wire was .4 mm. in diameter. The length of the horizontal part was 4 mm. (Experiment showed that this length might be varied greatly without affecting the readings.) P is the connection to the vacuum pump, McLeod gauge, and tube containing P_2O_5. e was connected to a quadrant electrometer which was standardized by comparison with a Weston voltmeter. The current through the tube was measured by a Weston ammeter, or when too small for that instrument, by a galvanometer which was compared with the ammeter. The part of the tube containing the cathode and the leading-in wires was a separate piece of glass which was sealed into the other tube with sealing wax. This enabled one to replace the platinum foil in case it was melted.

Anode Drop with Different Distances between Electrodes. — The first experiment given below is a repetition of one given by Skinner,[1] showing the relation between the anode drop and the distance between the electrodes. In this the current through the tube was kept as nearly as possible at 1.4×10^{-4} ampere, and the pressure at .035 mm. of mercury. The distance between A and e was approximately 1 mm. and slight variations of this distance made no appreciable difference in the readings. The gas in these experiments was air. The current remained practically constant as long

TABLE I.

Distance Between Electrodes in cm.	Anode Drop in Volts.	Remarks.
.5	23	In Crookes dark space.
1	15.5	" " " "
2	14.5	" " " "
3	5.5	In negative glow.
4	6	" " "
6	9	" " "
7	21	Faint glow on anode.
8	21	" " " "
9	25	Glow on anode.
10	13	First striation extending 1.5 cm. above anode.

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[1] Phil. Mag. (6), 2, 637, 1901.

as the temperature of the cathode was constant, irrespective of the distance between the electrodes. One hundred cells of a storage battery were used in series with a water resistance of approximately 2,000 ohms. The data found are given in Table I.

These data are plotted in Fig. 2. The large values of the anode drop occurred in Crookes dark space and just before a new striation began to appear. This was quite in harmony with what was found by Skinner.

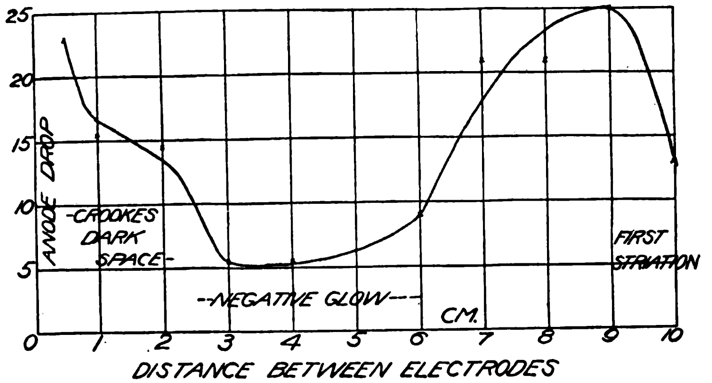

Fig. 2.

In addition it was found that increasing either the current or the pressure of the gas shortened the Crookes dark space greatly. This is shown by comparing the preceding table with the data given in Table II., where the current was .011 ampere and the pressure .07 mm. of mercury.

TABLE II.

Distance Between Electrodes in cm.	Anode Drop in Volts.	Remarks.
.5	5.5	In negative glow.
1	3.6	" " "
2	3.5	" " "
4	3.7	" " "
6	3.7	" " "
7	4.4	" " "
8	3.4	" " "
9	17.5	Beginning of bright glow.
10	16.5	One cm. of glow appearing above anode.

The curve given by these data is plotted in Fig. 3.

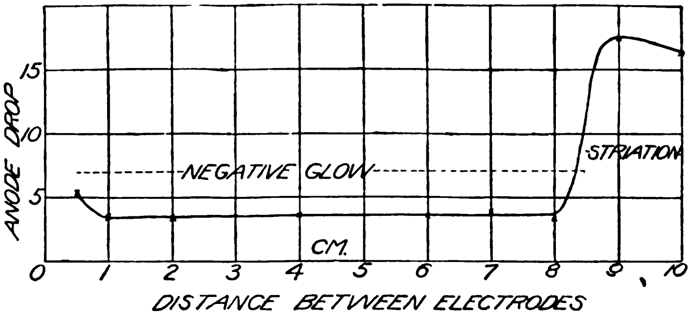

Fig. 3.

Anode Drop and Striation. — It was found also that the anode drop varied as the position of the anode in the striation was varied. In order to show this well by having wide striations, a very low pressure was necessary, and with the tube used in the preceding experiment it was not possible to get all of a striation at such a low pressure, except when the cathode rays were deflected to the side of the tube by a magnet. Bringing a magnet near to the discharge is equivalent for the purpose in hand to lengthening the tube. The following observations were, therefore, taken with changes in position of striation produced by changing the position of a magnet. The distance between the electrodes was 8 cm. The pressure was .019 mm. and the current was constant at .022 ampere.

Table III.

Anode Drop in Volts.	Remarks.
6.4	No glow at anode. No magnetic field.
19	Faint glow appearing at anode as magnet is brought nearer.
16	Top of glow 8 mm. above anode.
13	" " " 12 mm. " "
12	" " " 18 mm. " "
12	" " " 25 mm. " "
12	" " " 30 mm. " "
18	" " " 35 mm. " " Second dark space at anode.
17	Second glow 8 mm. above anode.

These data are plotted in Fig. 4. The fact is here more clearly shown that the largest value of the anode drop was at the end of the dark region just as a faint glow was about to appear on the anode. Many cases of this sort were examined in the course of the investigation and this was found to hold true in all cases.

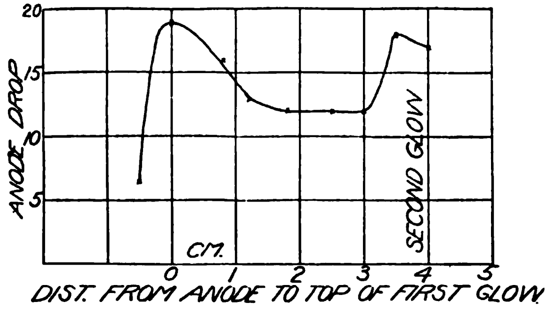

Fig. 4.

Anode Drop with Different Currents. — Data are given in Table IV. for the anode drop with different currents. The distance between the electrodes was 5 cm. The pressure of the gas was .15 mm. The other conditions were the same as in the preceding experiments. The current was varied by changing the temperature of the cathode.

TABLE IV.

Current in Amperes $\times 10^{-3}$.	Anode Drop in Volts.	Current in Amperes $\times 10^{-3}$.	Anode Drop in Volts.
.01	34	2.5	3.3
.02	26	9	4.6
.034	8	20	7
.051	5	250	14
.10	3.5	500	18
.60	2.5	700	20
.95	2.2		

Part of these data are plotted in Fig. 5. With the two smaller currents the only luminosity was a faint glow about one third of the way from the anode to the cathode. With larger currents this appeared to rise toward the cathode. With currents larger than those given in the table the beginning of a striation appeared on the anode.

The form of this curve was changed greatly by changing either the pressure of the gas, or the distance between the electrodes. It was apparently impossible to repeat the series of observations and get the same values, because the cathode continually gave out a slight quantity of occluded gas. However, the general character of the curve was always the same. There were always large values of the anode drop with very large and with very small currents.

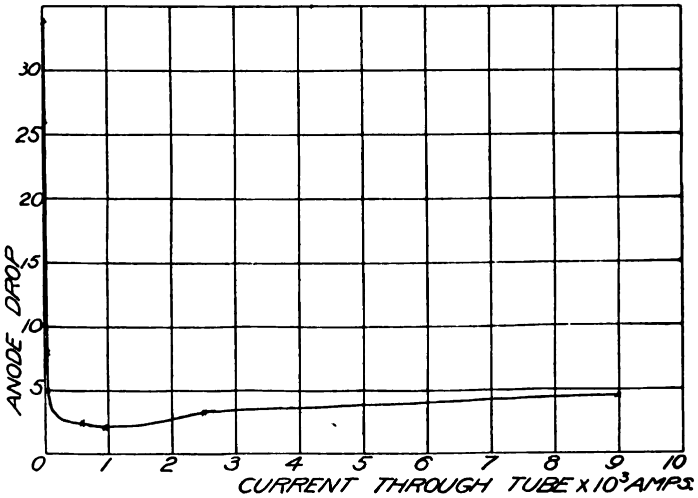

Fig. 5.

Larger values of the current approach those which may be had with the mercury arc and these will be considered in a later part of this article.

Anode Drop in Magnetic Field. — It would appear from what has preceded that the anode drop depended largely on whether the anode was in the direct path of the cathode rays or not. This was perhaps shown more clearly by deflecting the rays by means of a magnet. The change thus produced was especially noticeable when the anode was placed so that without the magnet the anode drop was small, as is shown in the following table.

Deflecting the rays by a magnet is equivalent to moving the anode further from the cathode as has been stated, but it is instructive to show the experiment in this form. In the following table the first reading is that given when there was no magnetic field. In each succeeding one the magnet was moved nearer. The number of cells was 120. The distance between the electrodes was 8 cm. The pressure was .008 mm., and the temperature of the cathode remained constant.

TABLE V.

Current in Amperes × 10^{-4}.	Anode Drop in Volts.
1.95	2.4
2.05	9.3
2.07	18
2.14	23

Anode Drop and Path of Cathode Rays.— That this increase in the anode drop was caused by the deflection of the cathode rays and not by any direct effect of the field was shown by taking the cathode out of the tube *T* and placing it in a side tube, which was made to open into *T* two or three centimeters above *e*. It was so placed that under ordinary conditions no rays could go directly to the anode. When this occurred there was a glow above the anode at all pressures which could be attained. The lowest pressure was .001 mm. and the least drop was 22 volts.

By placing a magnet near the tube and in the right position the cathode stream could be made to curve down into the tube *T*, so as to strike directly on *A*. Under such conditions the glow disappeared and the anode drop decreased to 2.6 volts, while the current remained practically constant.

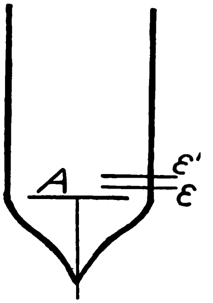

Fig. 6.

Anode Drop and the Electric Force near the Anode. — Skinner[1] calls attention to the fact that the region between the cathode and the first striation, where the anode drop is greatest, is also that where the electric force is greatest. It was found, however, on examining this force experimentally, that this statement would not hold under all conditions. In making measurements on this point the lower part of the tube was replaced by one shown in Fig. 6. *A* is the anode as before, *e* and *e'* are two exploring electrodes brought through the side of the tube, the distance between them being 1.2 mm. These could not be moved, but since the different parts of the discharge could be brought to the anode either by changing the current or by bringing a magnet near the tube, this arrangement was considered sufficient. The distance between the electrodes was 10 cm.

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[1] Phil. Mag. (6), 2, 621, 1901.

The fact that the anode drop was not always greatest when the electric force was greatest was best shown by changing the current from quite small to very large values. This was done by changing the temperature of the cathode. There is given in Table VI. a series of readings taken in this way.

Table VI.

Current Through Tube in Milliamperes.	Anode Drop in Volts.	Potential Difference Between e and e' in Volts.
.007	18	+2.6
.035	4.4	– .4
.7	4.4	– .25
2.7	5	– .25
112	24	–2.0
112	15	+2.5

The two last readings were taken with the same current, but in the last one a magnet was brought near the tube, causing the top of a striation to appear two or three millimeters above the anode.

The negative sign before some of the values in the third column indicates an electric force in the opposite direction to that in which the current was flowing. Very careful tests indicated that a force existed at times in such a direction. The current is carried against this by the momentum of the negative ions.

A very great change in the elecric force took place between the first and second readings, while the change in the anode drop was small. Between the second and third a large change took place in the anode drop while the change in the electric force was small.

A similar lack of agreement in the changes was found as a striation was about to appear. This, however, is less significant, since the electric force changes very rapidly at the beginning of a striation and the electric force measured by the electrodes may have been quite different from that between the lower electrode and the anode. However, this objection can hardly hold in the case of the first three readings, so that we have good reason for saying that the anode drop does not depend on the electric force in its neighborhood.

The conductivity of the gas in the neighborhood of the anode was also examined. This was done by placing the two electrodes *e* and *e′* parallel to the anode and connecting them to an E.M.F. of about 20 volts and to a galvanometer. The conductivity increased quite rapidly with increase of current and seemed to depend but very little on whether there was a glow at the anode or not. If the conductivity was measured when there was no glow at the anode and a glow was then produced by bringing a magnet near the tube, the conductivity changed but very little, while the anode drop changed greatly under these conditions. In fact no relation could be found between the anode drop and the conductivity of the gas near the anode. This was found to be true both with iron and with mercury for the anode.

Anode Drop with Higher Pressures. — As the pressure of the gas was increased there came to be a glow at the anode at all times, whether it was a dark or bright part of the striation. The variations of the anode glow were then much smaller than those given above.

At still higher pressures there were no striations and no glow except in the immediate neighborhood of the electrodes. Under these conditions the anode drop scarcely varied at all when the current was changed. It did, however, increase as the pressure of the gas increased. Thus when the distance between the electrodes was 6 cm. the anode drop changed from 19.5 volts when the pressure was .2 mm. to 23 volts when it was 2 mm. This increase was approximately proportional to the increase in pressure. With these higher pressures the luminous discharge could not be started with the 160 cells used, except by passing a spark through the tube from an induction coil. At the higher pressures there was no change in the anode drop which could be detected when the current through the tube was varied.

With hydrogen in the tube it was possible to have a discharge with greater pressures and this increase in the anode drop with increase in the pressure was then still more noticeable. Thus it increased from 10.5 volts when the pressure of the hydrogen was 1.2 mm. to 28.5 volts when it was 5.3 mm. At this pressure it varied approximately .7 volt as the current was changed, there being a minimum value when the current was .003 ampere. This variation was similar to that shown in Table IV., except that the value of the anode drop was here much larger. The distance between the electrodes was here 9 cm.

Anode Drop in Hydrogen. — The anode drop was also examined with the discharge through hydrogen using iron for the anode, and it was found to have the same general characteristics as that with the discharge through air. The currents which passed through the tube were larger than those which passed under the same conditions with air. The disappearance of striations occurred with greater pressures and it was possible to pass the discharge with greater pressures, but the anode drop showed the same dependence on the condition of ionization in the tube, and has approximately the same values.

Anode Drop with Mercury and Aluminum Anodes. — These experiments were repeated with mercury for the anode instead of iron. A film always formed on the surface of the mercury when a glow appeared in the tube and this made the anode drop somewhat greater. For the purpose of keeping a fresh surface on the mercury, the bottom of the tube shown in Fig. 6 was replaced by a long tube running into a cistern of mercury, which was open to the air. When the surface became coated, the cistern was lowered a few centimeters and then raised, bringing a comparatively clean surface of the mercury to the top. With such a surface the anode drop was, as nearly as could be determined, the same as with iron.

With an aluminum anode the anode drop was found to be much larger than with iron, although it showed the same general characteristics. The higher values for aluminum quite agree with those found by Skinner.[1]

No further comparison of the drop with different metals was made, since it is necessary to have much more complicated apparatus in order to make accurate comparisons.[2]

Large Currents Through Tube. — None of the currents used in the preceding experiments approach in size those of the ordinary form of electric arc. While it is possible to get such currents, it is difficult to maintain them long enough to secure satisfactory measurements.

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[1] Wied. Ann., 68, 757, 1899.

[2] See Phil. Mag. (6), 8, 387, 1904.

Three different substances were used for heating the CaO when attempting to get large currents, namely, platinum, tungsten and carbon. It was possible to get momentary currents as large as 1.5 amperes with the platinum foil, but in every case the foil was melted before measurements with the anode drop could be made, but as far as could be observed the discharge was similar to that in the preceding experiments. There were striations and the anode drop was in the neighborhood of 20 volts.

The tungsten wire which was used in these experiments was very kindly furnished by the General Electric Company. It was .32 mm. in diameter. It was possible with this to get larger currents through the tube, but they were nearly as transitory as when the lime was heated on platinum. In this case the trouble was due to the vaporization of the CaO. At higher temperatures it was entirely vaporized, leaving the tungsten as bright as when new.

Before this point was reached the CaO appeared to melt and collect in globules on the cooler parts of the wire. For some reason which I have not discovered, it was more difficult to secure the glow from CaO on tungsten than on platinum. It appeared to be necessary to raise the temperature of the tungsten every few seconds in order to maintain the discharge. The wire also gave out occluded gases in much greater abundance than platinum, so that it proved much less satisfactory than platinum for the smaller currents through the tube, and not any more satisfactory for the large ones.

The carbon also gave out a large amount of occluded gas and the CaO was driven off from this at higher temperatures, but it proved more satisfactory than either of the other substances for large currents. The carbon used was 1.6 mm. in diameter and the current passed through it was approximately 18 amperes. The current through the tube was furnished by a 110 volt dynamo, and was regulated by resistance in series with the dynamo. The tube was 2 cm. in diameter and the distance from the cathode to anode was 6 cm.

It was found that with currents in the neighborhood of .7 ampere a change occurred in the appearance of the anode. The glow then became concentrated in two or three spots and the anode drop became smaller. With currents in the neighborhood of 1 ampere the glow was confined to one spot and this was quite luminous.

The striations then became indistinct or disappeared entirely and the anode drop became still smaller. This change from the form where there was no concentration of the glow at the anode to the one where there was but one spot of light was liable to occur without any intermediate condition.

With this change the potential difference between the cathode and the anode became smaller and the current became much larger. In other words the discharge had become an arc. When the change had occurred resistance could be inserted in the circuit and the current lowered a considerable extent without changing back to the striated form.

A series of readings showing the relation between the current and the anode drop is shown in Table VII. The pressure of the gas was approximately .013 mm. These are only approximate values, for the carbon was continually giving out gas and the anode drop was largely affected by the amount of gas present.

Table VII.

Current Through Tube in Amperes.	Anode Drop in Volts.	Remarks.
.2	19.3	No concentration of glow at anode.
.7	15	Three bright spots at anode.
1.2	12	One bright spot at anode.
2	10	" " " " "
2.3	9.8	" " " " "
3	8.2	" " " " "

The discharge from CaO on carbon was also tried through a larger tube, one 3.5 cm. in diameter and it was found that there was no change in the form of discharge, until the current was more than one ampere,. and then it suddenly changed to the arc form with a current of 5 amperes.

These experiments were with a mercury anode. Similar phenomena were observed with iron and carbon anodes. With the carbon different sizes and lengths were used and it was found that the larger the surface of the carbon exposed, the larger the amount of current flowing when the change from the striated to the arc form occurred. Apparently the point where this change occurs depends on the density of the current rather than on the total current.

The attempt was also made to use the luminous vapor rising from the mercury arc as a source of negative ions. This was largely a repetition of the work of Pollak[1] but it seemed worth while to go over the work again with the knowledge of the preceding experiments in hand. It would seem at first sight as if this method would easily afford an opportunity to pass from small to very large currents by continuous changes and without trouble caused by the vaporization of CaO or by change of pressure due to occluded gas. But other difficulties made it harder to secure constant results by this means than by the hot lime on carbon. As has been shown in a preceding article[2] the luminous vapor rising from a mercury arc is highly conducting and the part of this in the boundary between the hot mercury vapor and the residual gas in the tube is the most conducting of all. This is an excellent source of negative ions but it is continually moving, so that it is impossible to keep the conditions steady.

In the second place the pressure in the vapor arising from the arc is much higher than in the remaining part of the tube and with the increase in pressure the striations become less distinct. The pressure appears also to be continually changing. As a result of both of these and possibly of others the striations were not nearly as distinct as with hot lime, and it was impossible to get any satisfactory measurements by this means.

It was evident that the discharge in this case went through the same general changes as with the discharge from hot lime. There were striations which changed with larger currents into a continuous glow with a concentration of the glow at one point on the anode. With this concentration there was a decrease in the anode drop as with the other case.

The lowest anode drops with the arc in a vacuum occur when the anode is in the direct path of the cathode rays, as has been pointed out by Stark, Retschinsky and Schaposchnikoff.[3] This was the cause of the low anode drop found by myself in a preceding article.[4] This condition is quite distinct from that given above, where there is a concentration of the current at one point, but is probably identical with that in the preceding work where the anode was in the direct path of the cathode rays.

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[1] Ann. d. Phys., 19, 239, 1906.

[2] PHYS. REV., 22, 221, 1906.

[3] Ann. d. Phys., 18, 213, 1905.

[4] PHYS. REV., 20, 365, 1905.

Explanation of the Variation in the Anode Drop. — The anode drop is due primarily to an excess of negative ions in the immediate neighborhood of the anode. Negative ions come to the anode from the gas, and if no positive ions come from the anode, there must of necessity be an excess of negative ions in its neighborhood. Evidently this excess will be smaller for a given current the greater the velocity of the negative ions moving through this region. It will also be smaller, if positive ions come from the metal or are produced at the surface of the anode. Some of the phenomena mentioned above appear to depend on the velocity of the negative ions as has been pointed out by Skinner; others on the production of positive ions at the surface of the anode.

In saying that the anode drop depends on the velocity of the negative ions, one must remember that the average velocity of the ions is meant. Not only the velocity of the cathode rays but also that of the negative ions produced in the gas between the electrodes must be considered. No quantitative explanation of this drop can be tested until the average velocity of the negative ions in the immediate neighborhood of the anode can be measured. It is, however, evident that the lowest anode drop occurred where we should expect this velocity to be greatest. Thus we should expect the velocity of the cathode rays a few centimeters from the cathode to be great and in this region the anode drop is indeed small as was shown in Table I.

As this distance is increased the ions coming from the cathode and having high velocity are stopped by collision. Others are produced by these collisions and, since the field where they are produced is weak, the velocity which is given to them is small, so that the average velocity of the ions is diminished. This is the region just above a striation and here we find the anode drop to be large.

When the velocity of the negative ions has become very small, they are crowded together and a strong field is produced beyond them. Why there should be this periodic change in the field is not known, and we can here only take account of the fact that it exists. Where the field again becomes strong, the velocity of the negative ions is again large. This is the region of the luminous part of a striation and here again we find the anode drop to be smaller.

This is true whether the cathode rays are stopped by collision with other molecules or by hitting the side of the tube, as when deflected by a magnet, or when the cathode was placed in a side tube. In any case we have the formation of a striation and a rise again of the anode drop.

As one passes downwards through the striation the negative ions are again stopped and others produced, which move more slowly, since they are in a weak field. In this region again we find the anode drop to be greater.

That the negative ions produced in the tube, or secondary cathode rays as one might call them, go for some distance through the gas before being stopped, was shown by a phenomenon which was noticed when the cathode was in a tube at right angles to the main tube. With this arrangement the top of the striation was concave downward and there was a conical region immediately under this which was brighter than the rest. That this was caused by the projection of rays from the top of the striation was shown by changing the current through the tube. As the current was decreased the top became more concave and the cone of light was brought to a focus nearer to the top.

The cone of light could also be deflected by a magnet and the direction in which it was deflected was the correct one for cathode rays. The length of this cone was about 2 cm. A similar phenomenon could at times be detected when the cathode was above the anode, but the striation was then flatter and the cone could not then be readily detected.

At first sight this explanation appears to fail when the electrodes are near together, for there we should expect to find the velocity of the negative ions large and the cathode drop to be small, while in fact the anode drop was here large. However, Skinner called attention to the fact that with short distances between the electrodes more cathode rays are reflected. Whether we are to consider the cathode rays as being reflected, or as producing secondary rays, there is in either case a large number of negative ions flying away from the anode. These would increase the density of negative electricity near the anode. Or looking at it from another standpoint, a larger drop in the neighborhood of the anode would be necessary to draw back again these negative ions to the anode. There was evidence that more cathode rays were reflected when the electrodes were near together in the fact that then the part of the tube above the cathode was more luminous, as if rays were reflected from the anode into that region.

With very small currents the only luminosity was in the lower part of the tube near the anode. One might, therefore, expect that a greater number of ions would be formed in that region and would have less velocity when they came to the anode. This would make the anode drop large, as it was indeed found to be.

In what has preceded no account has been taken of ionization at the surface of the anode, such as might be produced by the impact of negative ions. While such ionization may occur at low pressures, there was apparently no evidence of it. At higher pressure it would, however, appear to be the controlling factor. With pressures in the neighborhood of a millimeter, there was no glow in the region between the electrodes, but there was one at the anode with all distances and currents. The anode drop was then quite constant, scarcely varying at all with change of current and increasing but slowly with increase of pressure. Apparently under these conditions the drop at the anode gives the negative ions sufficient velocity to ionize the molecules at the surface of the anode, or else in its very immediate neighborhood, irrespective of what their velocity may have been before reaching the anode. When this critical velocity was reached, changes in the current or in other conditions would have but small effect, for if any change should momentarily increase the anode drop, there would immediately be an increase in the number of positive ions produced at the surface of the anode and this would again lower the anode drop.

With currents in the neighborhood of .5 ampere the decrease in the anode drop appears to be quite analogous to that which occurs in the electric force in a vacuum tube with much smaller currents. With very small currents from hot lime the electric force through the tube may be large. As the current is increased the conductivity of the vapor is greatly increased and the electric force through the vapor decreases. Whether this is caused by the high temperature of the gas or by the repeated bombardment of an atom by different electrons, until it is broken in two, is perhaps debatable. Whichever is the correct explanation in that case, a similar one in this case should probably be given. So that we would need to look on the arc as an instance where the normal anode drop has been diminished by the concentration of current.

Summary. — The anode drop was examined with the luminous discharge produced by hot CaO when pressures of the gas were low. It varies greatly as the distance between the anode and cathode is varied. It is greatest when they are near together and least when a few centimeters apart. As the distance between the electrodes is further varied, the anode appeared to move through the different parts of a striation and the anode drop depended on the part of the striation in which the anode is. The anode drop passes through a maximum value just before a new striation begins to appear. The same thing occurs when the striation is displaced by means of a magnet.

The anode drop varies greatly as the current through the tube is varied. It is large with very small and with very large currents. The amount of current corresponding to the minimum anode drop depends on the pressure of gas and the distance between the electrodes.

It is least when the anode is in the direct path of the cathode rays. It does not appear to depend on the electric force near the anode nor on the conductivity of the gas near the anode.

With pressures of the gas in the neighborhood of 1 mm. or more the anode drop is approximately independent of the current and of the distance between the electrodes and increases slightly as the pressure of the gas increases.

In hydrogen it is approximately the same as in air. It is the same for iron and mercury anodes, but an aluminum anode gives a higher value.

With currents in the neighborhood of .5 ampere there was a concentration of glow in one spot on the anode and with this a decrease in the anode drop. The discharge then appears to have all the characteristics of an arc.

The phenomena can be explained by assuming that at low pressures and with small currents the anode drop depended on the velocity of the negative ions at the anode, that with higher pressures it depended on the potential difference required to produce positive ions at the surface of the anode, and that with larger currents this ionization potential difference is modified by the concentration of current at one spot.

Part of the work described above was performed in the Cavendish Laboratory and I desire to express my thanks to Sir J. J. Thomson for allowing me the use of the laboratory and for his suggestions in regard to the work.

Colgate University,
June, 1909.

THE RESIDUAL OF INDUCTANCE AND CAPACITY IN RESISTANCE COILS. A STANDARD RESISTANCE WITH BALANCED INDUCTANCE AND CAPACITY.

By S. LeRoy Brown.

I. Introduction.

A CAREFULLY constructed resistance coil can be standardized with direct current and its resistance accurately determined. The two principal errors attendant upon the use of such a coil with direct current are due to variations in its resistance caused by changes in temperature and thermo-electromotive force in the connections. The error caused by thermo-electromotive force can often be eliminated by reversing the current through the coil and the variations in temperature are reduced by making the coil of large wire, thereby reducing the heating effect of the current.

If a coil is used with interrupted or alternating current, two other errors must be guarded against. These are due to the inductance and the capacity of the coil. If no particular precaution has been taken to reduce both the inductance and the capacity to very small values, the coil may be entirely satisfactory when used with direct current but not at all satisfactory when used with alternating current. Some of the features in the design of resistance coils to be used with direct current are really disadvantages if the coils are used with alternating or interrupted current. For instance, making the coils of large wire and immersing them in oil or coating them with shellac, greatly increases their capacity. Without going further into the construction of resistance coils, it is clear that coils which are designed for direct current are not always adapted for use with alternating or interrupted current.

The wide application of interrupted and alternating current in measuring and comparing capacities and coefficients of induction necessitates, in many methods, the use of resistance coils which are measurably free from inductance and capacity. The inductance and capacity of a resistance coil can be reduced to small values but neither can be reduced to zero. There will always be some magnetic area enclosed by the wires of the coil and some electrostatic capacity between the adjacent wires; but the capacity reduces the effective inductance. In well-constructed alternating-current coils the residual effect of inductance or capacity is made as small as possible. Since the effect of the capacity is equivalent to a negative inductance, the residual effect of inductance and capacity can be described as a positive or negative residual of inductance. The usual method of reducing this residual of inductance is to make both the inductance and capacity small, but apparently no attempt has been recorded to make the inductance exactly balance the capacity.

The double-wound resistance coil can be constructed nearly free from inductance, but the capacity of the wires is more difficult to eliminate. If the two wires of the coil enclose equal magnetic areas there remains only the small area between the wires to produce induction. The capacity depends upon the size of the wires and the closeness of winding. This capacity may be negligible in low-resistance coils, but for some purposes it may not be negligible in coils of higher resistance, especially if they are connected in parallel. By crowding together the two wires of the double-wound coil the inductance is reduced, but the capacity may be greatly increased.

Another method of constructing coils is to wind a single wire on a thin sheet of card-board or mica. The magnetic area per turn is reduced to the cross-sectional area of the sheet and the capacity may be made very small by separating the turns. Coils wound on this plan have a residual of inductance, depending upon the cross-sectional area of the material on which they are wound, since the capacity is reduced to a negligible quantity much smaller than the inductance.

Possibly the best method of winding coils is that devised by Chaperon,[1] and used in some of the resistance boxes manufactured by Hartmann and Braun. Several turns of a single wire are wound on a spool and then the direction of winding is reversed. The magnetic area enclosed after reversing the direction of winding is made equal to the area enclosed before reversing. Many sections, like the one described, make up the entire coil, which is then practically free from inductance. The coil is built in several divisions instead of having the layers carried the entire length of the spool. Beginning at one end, each division is completed before proceeding to the next adjacent division, so that the ends of the wire are at opposite ends of the spool when the coil is finished. Adjacent wires are therefore at a small difference of potential and the capacity is greatly reduced. In order to test any resistance coil for a residual of inductance or capacity a standard is required, free from inductance and capacity, or having one balanced by the other, or showing a known excess of one over the other. Coils could then be compared with the standard and described as having, for instance, 1,000 ohms resistance and 0.00001 henry residual inductance, or 1,000 ohms resistance and 0.001 microfarad residual capacity. Rosa and Grover,[1] of the Bureau of Standards at Washington, have called attention to the necessity of knowing the residuals of inductance of resistance coils when they are used with alternating current.

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[1] G. Chaperon, Comptes Rendus, 108, p. 799.

Such standards of comparison have been constructed already by reducing both the inductance and the capacity of the standards to values as small as possible. In this paper it is proposed to employ a different principle in the construction of the standard. Instead of seeking to make the inductance and capacity both small, it is proposed to offset one against the other, making the residual smaller than can be measured. If measurements of residuals less than a few hundred micromicrofarads are required, the latter plan offers noticeable advantages. In the standards hitherto constructed, the residuals are usually considerably greater than the amount indicated.

II. A STANDARD RESISTANCE FOR USE WITH INTERRUPTED CURRENT.

Two long parallel wires, joined at one end, are to be separated to such a distance that the inductance and distributed capacity may balance. Suppose first a localized capacity, like a condenser, shunted by an inductive resistance. If a constant electromotive force be suddenly applied, current will flow as if the condenser were absent and the resistance non-inductive, provided that the so-called time-constant of the capacity C, shunted by the resistance R, equals the time-constant of the inductance L associated with the resistance R.

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[1] E. B. Rosa and F. W. Grover, Bulletin, Bureau of Standards, I., p. 291.

That is,

$$\frac{L}{R} = CR. \tag{1}$$

Stated more exactly, the defect in quantity passing through the inductive resistance during the period of variable flow, caused by the electromotive force of self-induction, is equal to the charge of the condenser, provided the time-constants are equal. In this sense the condenser-capacity balances the inductance.

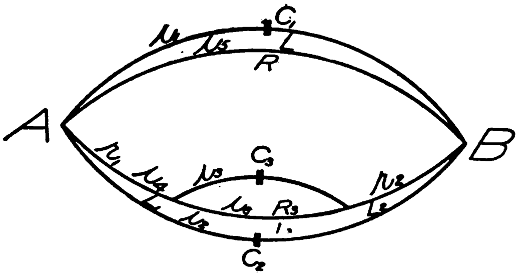

Fig. 1.

Suppose next, distributed capacity between the long parallel wires joined at one end. What is the time-constant of this arrangement? Consider the net-work of conductors shown in Fig. 1, divided into two systems, one consisting of the resistances r_1, r_2, R_3, capacities C_2, C_3 and inductances L_1, L_2, L_3, the other consisting of the resistance R, capacity C_1 and the inductance L. What is the condition that the one system may be equivalent to the other when a difference of potential is suddenly applied and maintained constant between A and B?

If the quantities of electricity q_1, q_2, etc., correspond to the currents i_1, i_2, etc., the equivalence of the two systems is expressed by

$$q_1 + q_5 = q_4 + q_2 = q, \tag{2}$$

where the integration covers the period of variable flow.

$$R_3 + r_1 + r_2 = R' + R_3 = R, \tag{3}$$

$$L_3 + L_1 + L_2 = L_3 + L' = L, \tag{4}$$

$$Ri_5 + L\frac{di_5}{dt} = R_3 i_6 + R' i_4 + L_3\frac{di_6}{dt} + L'\frac{di_4}{dt}, \tag{5}$$

$$Rq_5 + LI_5 = R_3 q_6 + R' q_4 + (L_3 + L')I_5,$$

$$Rq_5 = R_3 q_6 + R' q_4. \tag{6}$$

From equation (2), expressing the quantities ot electricity which flow into the condensers as the products of the capacities and the differences of potential at the terminals,

$$q_5 = q - q_1 = q - C_1 R I_5, \tag{7}$$

$$q_6 = q_4 - q_3 = q_4 - C_3 R_3 I_6 = q_4 - C_3 R_3 I_5, \tag{8}$$

$$q_4 = q - q_2 = q - C_2 R I_5. \tag{9}$$

I_5 and I_6 are the final values of the currents whose variable values are denoted by i_5 and i_6. Substituting these values of the quantities q_5, q_6 and q_4 in equation (6),

$$C_1 R^2 = C_2 R^2 + C_3 R_3^2. \tag{10}$$

This result can be extended by considering any number of condensers symmetrically distributed along a resistance which is equal to the resistance R, giving

$$C_1 R^2 = C_2 R^2 + C_3 R_3^2 + \cdots C_n R_n^2. \tag{11}$$

Therefore C_1 is a condenser, which if placed at the terminals of a resistance R, would be equivalent to the condensers C_2, C_3, etc., distributed along the resistance.

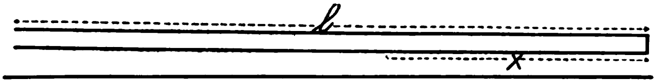

Fig. 2.

Let R be the resistance of two parallel wires which are connected at one end (Fig. 2). C_1 is the capacity per unit length of the parallel wires and C is a capacity, which, if placed at the terminals of the resistance R, would be equivalent to the distributed capacity.

The length of the double wire is l and x is the distance from the connected ends to any element of the parallel wires. The capacity for an element of length of the wires is $C_1 dx$ and the resistance in parallel with this element of capacity is Rx/l. From equation (11)

$$CR^2 = \int_0^l \frac{C_1 R^2 x^2}{l^2} dx = \frac{C_1 R^2 l}{3} \tag{12}$$

or $CR = C' \cdot R/3$ where C' is the entire capacity of the wires when they are not connected at either end. Therefore the effective capacity or the so-called time-constant of a capacity uniformly distributed as in Fig. 2 is one third as much as it would be if the entire capacity of the wires were located at the terminals of the resistance.

Let D be the distance between the wires and r their radius. Assume that the specific inductive capacity of the medium between the wires and the permeability of the medium, including the wires, are both unity. The inductance of the wires is given by the formula,

$$L = 4l\left(\log_e \frac{D}{r} + \frac{1}{4}\right) \text{ electromagnetic units}$$

and the capacity is given by the formula,

$$C' = \frac{l}{4 \log_e (D/r - 1)} \text{ electrostatic units.}$$

The equivalent capacity C is $C'/3$ and the effective capacity and inductance, expressed in practical units, are

$$C = \frac{l 10^{-11}}{9 \cdot 12 \cdot 2.3026 \log_{10}(D/r - 1)} \text{ farads} \tag{13}$$

and

$$L = 4 \times 10^{-9} l \left[2.3026 \left(\log_{10} \frac{D}{r}\right) + \frac{1}{4} \right] \text{ henrys,} \tag{14}$$

the length l being measured in centimeters.

If an electromotive force is suddenly applied at the terminals of these parallel wires and maintained constant, the residual inductance will be zero when the inductance and capacity have equal time-constants. Substituting the values of C and L in the equation of the time-constants (equation (1)),

$$4\left(2.3026 \log_{10} \frac{D}{r} + \frac{1}{4}\right) 12 \cdot 2.3026 \log_{10} \left(\frac{D}{r} - 1\right) 9 \cdot 10^2 = R^2. \tag{15}$$

The first attempt to satisfy the conditions expressed in equation (15) led to the construction of a parallel-wire standard having the following dimensions. Two number 35 german silver wires were placed 0.88 cm. apart; their common radius was 0.0067 cm.; and their length was 2,751 cm. This length yielded a resistance of 1,040 ohms as required by the substitution of the calculated inductance and capacity in equation (15).

Several resistance coils were compared with the standard by means of a Wheatstone network. The connections of a Leeds and Northrup moving-coil galvanometer and the battery were made through a secohmmeter. The bridge was balanced in the usual way before the secohmmeter was started. The deflection of the galvanometer, when the secohmmeter was rotating, was thus a measure of the residual inductance of the resistance coil which was being compared with the standard. In order that the residual inductances in the ratio-arms of the bridge might not produce a deflection of the galvanometer, they were selected so that when interchanged the balance was not disturbed. The galvanometer was calibrated by determining the capacity which must be added in parallel with the standard to produce 1 mm. deflection.

Measurements of resistance-coils used in the laboratory showed that in coils wound according to the Chaperon system, the effects of capacity and inductance were nearly balanced, but in coils wound in the ordinary bifilar fashion, the capacity over-balanced the inductance. That is, the double-wound coils would be described as having a negative residual of inductance. The residuals of inductance in the double-wound coils were balanced by capacities ranging from 0.0005 to 0.001 microfarad connected across the terminals of 1,040 ohms.

III. A Standard Resistance for Use with Alternating Current.

The next problem undertaken was the investigation of the conditions for balanced inductance and capacity when alternating current was used, instead of interrupted, steady current. The effect of inductance in general is to cause the current in a resistance to lag in phase behind the impressed electromotive force, while the effect of capacity alone is to cause the current to precede the electromotive force in phase. There is neither lag nor advance when inductance is balanced by capacity.

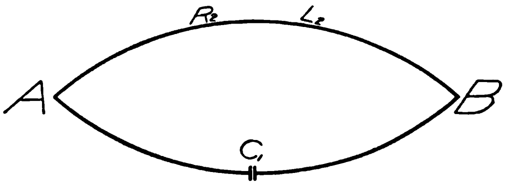

Fig. 3.

Fig. 3 represents an inductive resistance in parallel with a capacity. The circuit between A and B consists of two branches; the resistance, capacity and inductance of the branches are respectively $R_1 = 0$, C_1, $L_1 = 0$ and R_2, $C_2 = \infty$, L_2. Let an electromotive force which follows the simple harmonic law of variation with time ($e = E \sin \omega t$) be impressed between A and B, and consider the relations among C_1, R_2 and L_2 when the current flows in phase with the electromotive force.

Let the resistance, capacity and inductance of a single circuit which is equivalent to the two branches be R, C, L. Bedell and Crehore[1] have deduced the following equations between the resistances, capacities and inductances of parallel circuits and the resistance, capacity and inductance of a single circuit which is equivalent to the parallel circuits. The symbols without the subscripts refer to the equivalent, single circuit and the symbols with similar subscripts refer to the same branch of the parallel circuits.

$$\frac{1}{C\omega} - L\omega = \frac{B\omega}{A^2 + B^2\omega^2}, \tag{16}$$

$$A = \Sigma \frac{R_1}{R_1^2 + (1/C_1\omega - L_1\omega)^2}, \tag{17}$$

$$B\omega = \Sigma \frac{1/C_1\omega - L_1\omega}{R_1^2 + (1/C_1\omega - L_1\omega)^2}, \tag{18}$$

$\omega = 2\pi$ times the frequency.

Substituting the values of the resistances, capacities and inductances given for the circuit of two branches (Fig. 3),

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[1] Bedell and Crehore, "Alternating Currents," 1895, p. 303.

$$A = \frac{R_2}{R_2^2 + L_2^2\omega^2}, \tag{19}$$

$$B\omega = C_1\omega - \frac{L_2\omega}{R_2^2 + L_2^2\omega^2}, \tag{20}$$

$$\frac{1}{C\omega} - L\omega = -\frac{C_1\omega(R_2^2 + L_2^2\omega^2) - L_2\omega}{1 + C_1^2\omega^2(R_2^2 + L_2^2\omega^2) - 2C_1L_2\omega^2}. \tag{21}$$

If a single circuit with a resistance R, a capacity C and an inductance L has impressed on it an electromotive force $e = E \sin \omega t$, the current is

$$i = \frac{E}{\sqrt{R^2 + (1/C\omega - L\omega)^2}} \sin\left[\omega t + \tan^{-1}\left(\frac{1}{CR\omega} - RL\omega\right)\right]. \tag{22}$$

Putting $1/C\omega = L\omega$; then $i = E/R \sin \omega t$ and the current and electromotive force are in phase. When $1/C\omega = L\omega$, the relation among R_2, C_1 and L_2 is determined by equation (21).

$$R_2^2 + L_2^2\omega^2 = L_2/C_1. \tag{23}$$

In a standard similar to the one described for interrupted current, the term $L_2^2\omega^2$ is negligible in comparison with the term R_2^2 for moderate frequencies and equation (23) becomes

$$R_2C_1 = L_2/R_2. \tag{24}$$

In order to secure a zero residual of inductance, the relation among the resistance, capacity and inductance is therefore approximately the same when either interrupted steady current or alternating current of moderate frequency is applied to a coil of small inductance.

The effective value of the uniformly distributed capacity of two parallel wires connected at one end was determined by experiments made with alternating current as described in Section V. of this paper. The results indicate that the effective capacity was one third of the total distributed capacity of the two parallel wires when disconnected at both ends. The standard proposed for interrupted current can therefore be used with alternating current of moderate frequency. When the parallel-wire standard is used with alternating current, the values of the capacity and inductance expressed in equation (24) are given by equations (13) and (14).

IV. DESCRIPTION OF A GALVANOMETER FOR ALTERNATING CURRENT.

The experimental work mentioned above required an instrument to indicate small differences of alternating potential. A disturbance in the bridge balance produced by only a few micromicrofarads of capacity added in parallel with the standard had to be detected in order to make accurate measurements, since the effective capacity of the 1,040-ohm resistance was only 52 micromicrofarads. That is, the distributed capacity, found by calculation from the dimensions of the wires was equivalent to no more than 52 micromicrofarads of capacity connected across the free ends of the wires in the form of a condenser. The instrument used for this purpose was an alternating-current galvanometer, similar to the one described and used by Stroud.[1] A disturbance in balance could be detected with this galvanometer when a capacity as small as 2 micromicrofarads was added in parallel with the 1,040 ohms.

This galvanometer, as described by Stroud, is similar to an ordinary moving-coil galvanometer, but it has a wire-wound field-magnet with laminated, soft iron poles and yoke instead of a permanent magnet. The poles and yoke were built of U-shaped stampings of soft iron insulated from each other by a thin coat of shellac. The dimensions of these stampings were 12.8 cm. by 25.4 cm. by 0.039 cm. with a gap 3.85 cm. by 16.6 cm. These stampings were piled 3.85 cm. thick and cemented together with shellac. Each leg of the U was wound with 625 turns of number 22 insulated copper wire forming a coil 12.8 cm. long. The armature was a rectangular coil 3.85 cm. by 3.12 cm. by 0.78 cm. wound with 1,500 turns of number 40 silk-covered, copper wire. It was suspended between the upper faces of the U-shaped field magnet by a number 40 phosphor-bronze wire 30 cm. long. Connection to the armature was made above through the suspension and below by a wire dipping into mercury.

The galvanometer is not sensitive to differences of potential which are in phase with the electromotive force which excites the field magnet, but it is extremely sensitive to differences of potential which are in quadrature with the exciting electromotive force. When the same electromotive force which excites the field magnet is applied to a Wheatstone bridge, the galvanometer is very sensitive to differences of potential due to capacities or inductances in the arms of the bridge. The resistances of the bridge were balanced by using direct current and a moving-coil galvanometer, and then, without changing the resistances, the capacities and inductances were balanced by using alternating current and the alternating-current galvanometer. Leakage in the condensers or lack of accurate balance among the resistances does not appreciably affect the balance between the capacities and inductances. The alternating-current galvanometer is not sensitive to differences of potential due to small leakage currents or a slight inaccuracy of balance among the resistances, since the differences of potential are in phase with the applied electromotive force.

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[1] W. Stroud, Phil. Mag., Vol. 6, 1903, p. 707.

V. Connections and Measurements.

The connections of the galvanometers and sources of electromotive force to the bridge are shown in Fig. 4.

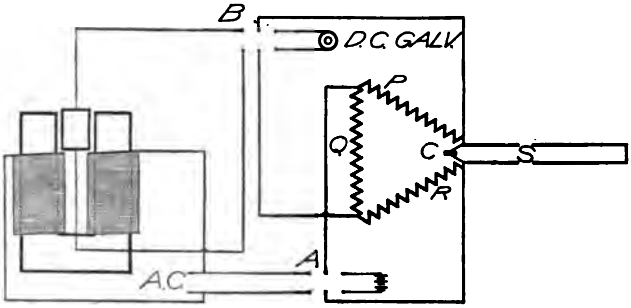

Fig. 4.

Switches *A* and *B* are thrown to the right and the bridge is accurately balanced with direct current. Then switch *B* is thrown to the left and the equilibrium position of the moving coil of the alternating-current galvanometer is noted with telescope and scale. Switch *A* is then thrown to the left and the deflection noted if there is lack of balance between the capacities and inductances of the bridge. The ratio arms *P* and *Q* are selected so that when they are interchanged, the balance is not affected. If *R* has a greater negative residual of inductance than *S*, a balance can be obtained by adding a small known capacity at C. If R has a greater positive residual of inductance than S, the capacity is added in parallel with R instead of S. C is a variable capacity made of tin-foil on plates of glass with a range from 2 to 7,000 micrimicrofarads. The source of electromotive force was an inverted rotary converter, driven by a storage battery. The rotary converter produced a constant electromotive force and prevented any deflection of the galvanometer due to changes in its field. The frequency of the alternating current was 40 cycles per second.

The following experimental determinations were made with this apparatus. Pairs of parallel wires, placed at different distances apart, some less and some greater than the distance calculated for balance of inductance and capacity, were compared with the standard resistance. The pairs of wires had equal resistances so that the same resistance-coil in the arm R could be used with each pair. This avoided any error which might be introduced by inserting successively coils having different residuals of inductance when balancing R against successive pairs of wires. The ratio-arms were chosen so that the balance was not disturbed by interchanging them. The standard was first placed in the arm of the bridge denoted by S in Fig. 4 and capacity added at C until a balance was obtained. The standard was then replaced by another pair of parallel wires and C changed until a balance was again obtained.

TABLE I.

Resistance (Ohms).	Dist. Apart (cm.).	Calc. L (mil. h.).	Calc. C (cm.).	C (mmf.) Necessary to Add	
				By Cal.	By Exp.
1,037.2	.88	.056	52	0	0
1,037.2	.335	.046	65	−22	−20
1,037.2	1.35	.061	48	8.5	10
1,037.2	6.67	.082	38	40	35

The calculations were made on the assumption that the effective capacity was one third of the total capacity of the two wires when not connected at either end. The results expressed in Table I. show a close agreement between theoretical and experimental values.

The effective value of a uniformly distributed capacity was also determined experimentally by measuring the effect of 2,500 micromicrofarads capacity when connected successively at different points along the parallel wires. The deflections of the galvanometer were first taken as a measure of the effect of the 2,500 micromicrofarads capacity. Suspecting that the calibration curve of the alternating-current galvanometer might not be a straight line, the amount of capacity required in parallel with the arm *R* to bring the armature of the galvanometer to its equilibrium position was taken as the measure of the effect of the capacity connected at different points along *S*.

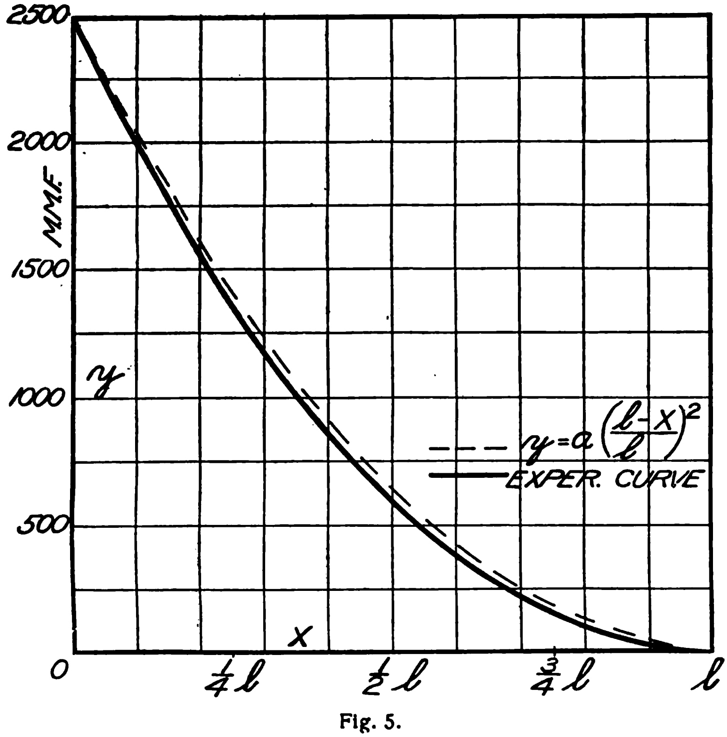

Fig. 5.

A curve was then plotted with abscissas representing the distances of the connections of the capacity from the terminals of *S* and ordinates representing the capacities required in parallel with *R* to balance the capacity in *S*.

Fig. 5 shows the close agreement between this curve, obtained experimentally, and the curve $y = a(l - x)^2/l^2$, wherein l is the length of the parallel wires and a is the greatest ordinate of the curve plotted from experimental data. The measure of the effect of the capacity connected at different points along S decreases with the square of the fractional part of the total resistance, which shunts the capacity. Therefore, as shown by the agreement between the equation of this curve and the expression employed in equation (12), the integrated effect of a uniformly distributed capacity would be one third of the effect of the same capacity located at the terminals.

VI. THE RESIDUALS OF INDUCTANCE IN ORDINARY RESISTANCE COILS.

A balanced standard consisting of two parallel wires was next used to determine the residuals of inductance of resistance-coils from different makers. For convenience of comparison a new standard was constructed of number 35 german silver wires, placed 0.77 cm. apart and having a resistance of 1,000 ohms. A sliding contact-piece at the connected ends of the wires provided for small adjustments of resistance.

Referring again to the scheme of connections shown in Fig. 4, S is now the 1,000-ohm standard and R is the resistance-coil or series of connected coils under test. Table II. shows the results of the measurements. Positive values of C indicate that capacity predominates in the resistance R; negative values indicate that the capacity was inserted in parallel with R instead of S, that is, that inductance predominates in R. The values of C were determined within about 25 micromicrofarads. When the resistance of arm R was 2,000 ohms, two 1,000-ohm standards were used in arm S.

VII. A STANDARD OF VARIABLE RESISTANCE APPARENTLY IMPRACTICABLE.

An increase in the resistance of a standard requires a new adjustment between the capacity and the inductance if they are to remain balanced. According to equation (24), the inductance must increase faster than the capacity if balance is maintained when the resistance increases. If a variable resistance standard is constructed similar

TABLE II.

Description of Coils.	Number of Resistance Box.	Resistance (Ohms).	Residual Capacity (mmf.).
Hartmann and Braun "double-wound" coil.	3,213	1,000	1,120
	3,215	1,000	1,110
	3,213 in series	1,000	
	with 3,215	1,000	585
	4,549	400+300+200+100	1,245
	3,995	400+300+200+100	730
	2,313	2,000	2,275
	3,215	2,000	2,130
	3,995 in series	400+300+200+100	
	with 4,549	400+300+200+100	450
Hartmann and Braun "Chaperon-wound" coil.	3,028	1,000	105
	3,028	400+300+200+100	−260
	3,028	2,000	60
Wolff "double-wound" coil.	3,187	1,000	735
	3,187	1,000	760
	3,187	2,000	1,140
	3,187 in series	1,000	
	with 3,187	1,000	390
Leeds and Northrup "double-wound" coil.	4,491	1,000	610
	4,492	1,000	630
	4,491 in series	1,000	
	with 4,492	1,000	330
	4,492	2,000	835
	4,491	2,000	860
	4,491 in parallel	2,000	
	with 4,492	2,000	1,875

to the 1,040- and 1,000-ohm standards already described, the distance between the wires must increase with the distance from the terminals. Increasing the resistance by connecting the wires together farther from the terminals, would then increase the inductance faster than the capacity. The impracticability of such a construction is easily seen from a single example.

Beginning with 500 ohms of number 35 wire the uniform distance between the wires, as determined by equation (15) would be 0.0635 cm. If 100 ohms are added, the uniform distance between the wires in the added section, in order that the 600 ohms may be free from residual of inductance or capacity, is determined as follows.

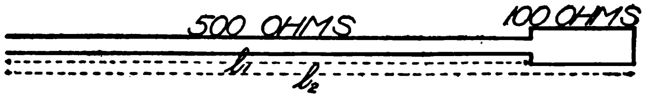

Fig. 6.

Let C_1 and C_2 be the capacity per unit length for the 500-ohm and 100-ohm sections respectively. According to equation (12) the effective capacity C of the two sections is

$$C=\int_0^{l_2-l_1}\frac{C_2x^2dx}{l_2^2}+\int_{l_2-l_1}^{l_2}\frac{C_1x^2dx}{l_2^2}, \qquad (25)$$

$$l_1=\tfrac{5}{6}l_2.$$

Equations (12) and (15) were derived on the assumption that the standard was to be used with interrupted current but experimental results have shown that the same equations apply when alternating current is used. The total inductance is the sum of the inductances of the two sections.

$$L=L_1+L_2.$$

The effective capacity of the 500-ohm section is increased about one fifth by adding the 100 ohms, but the effective capacity of the added section is small, since no part of it is shunted by much resistance and the wires are far apart in order to increase the inductance sufficiently. The relative contribution of the two sections can be determined from equation (25). Therefore the effective capacity is increased approximately one fifth and the square of the resistance is increased about one half by adding the second section. Then, according to equation (24), the total inductance must be increased to about one and four fifths times the inductance of the 500-ohm section and this would require that the wires in the 100-ohm section should be 106 cm. apart.

This emphasizes the necessity of a comparatively high inductance in a double-wound high resistance-coil if the last few turns are close together. Ordinarily the two wires of the double-wound coils are wound side by side throughout the entire coil and therefore the inductance is not sufficient to balance the capacity in coils of higher resistance.

VIII. THE RESIDUAL INDUCTANCE OF A COIL MADE BY TWISTING THE WIRES TOGETHER AND SEPARATING CONSECUTIVE TERMS.

A so-called "ideal coil" has been constructed by Taylor and Williams,[1] whose purpose was to make both inductance and capacity as small as possible. With their coil, having braided bifilar windings, the consecutive turns being separated 5 mm., they detected no residual capacity, even with a very sensitive alternating-current galvanometer, though the capacity had been increased by immersing the coil in petroleum. Since the inductance had been made very small by twisting the wires together, it was concluded that both the inductance and the capacity had been effectively reduced below measurable quantities.

From results obtained with the apparatus described in the present paper, it appeared that a twisted-wire standard is not sufficiently free from capacity if residuals of only a few micromicrofarads are to be detected and measured. A coil similar to that of Taylor and Williams was constructed and the excess of capacity over inductance was determined both by calculation and experiment.

The coil constructed by the writer was made of two number 35 double silk-covered german silver wires, soldered together at one end. The wires were twisted together and wound on a rectangular block 11 cm. by 12 cm. in section. The length of the bifilar winding was 2,750 cm., the wires having a resistance of 1,000 ohms. The distance between consecutive turns was about 0.4 cm. By the aid of a microscope, the distance between the centers of the wires was determined to be 0.021 cm. In some places, however, where the pressure between the wires was sufficient to deform the covering, the distance between the centers of the wire was probably slightly less.

Comparisons of this rectangular coil with the parallel wire standard showed a considerable excess of capacity over inductance in the former. When the coil was in air, 420 micromicrofarads of capacity were added in parallel with the standard to obtain a balance. The coil was immersed in petroleum and 685 micromicrofarads of capacity were required in parallel with the standard to effect a balance. After removing the coil from the petroleum bath, but with the covering on the wires still moistened with oil, the capacity measured 585 micromicrofarads. This value is between the values obtained when the coil was in air and when it was in oil.

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[1] A. H. Taylor and E. H. Williams, PHYS. REV., XXVI., p. 417, 1908.

Assuming unit specific inductive capacity of the dielectric between the wires and that the distance between the centers is 0.021 cm., the excess of capacity is 355 micromicrofarads by calculation. If petroleum is assumed as the dielectric the excess of capacity is about 800 micromicrofarads. This value is to be compared with the value 685 micromicrofarads obtained experimentally. The discrepancy between calculated and experimental values may be accounted for by remembering that the dielectric was not entirely petroleum in one case nor air in the other.

Comparing these results with the negative results obtained by Taylor and Williams, it should be noted that they used number 40 advance wire, while the coil here described was constructed of number 35 german silver wire. If it had been made of number 40 german silver wire, having the same resistance, 1,000 ohms, and the same distance between the twisted wires, the residual capacity would have been, by calculation, only 155 micromicrofarads.

The capacities of the 1,000- and 2,000-ohm coils of resistance box number 3,187, made by Otto Wolff of Berlin, were measured by comparison with the parallel-wire standard. The coils are made of rather large wire, wound on brass cores and heavily coated with shellac. The 1,000-ohm coil showed 750 micromicrofarads of capacity and the 2,000-ohm coil about 1,140 micromicrofarads. These values are seen to be higher than those found by Taylor and Williams.

IX. BALANCED INDUCTANCE AND CAPACITY USING WIRE WITH PERMEABILITY GREATER THAN UNITY.

In the calculation of the inductance of the parallel wire standard, the permeability of the wire was assumed to be unity. If the wires are made of a metal having a permeability greater than unity, the inductance would be greater than that already calculated. In most resistance coils of 100 ohms or more, wound in the ordinary bifilar method, there is a residual of capacity. If then, wire of sufficiently high permeability could be found, it seems to be theoretically possible to balance inductance against capacity, without separating the wires as has been done in the case of the parallel-wire standard described in this paper. The self-inductance of a return circuit consisting of two parallel iron wires, l cm. in length, r cm. in radius, D cm. apart, joined at one end, is

$$L = 4l\left(2.3026 \log_{10}\frac{D}{r} + \frac{\mu}{4} - \frac{D}{l}\right),$$ [1]

wherein μ is the permeability of the wires, that of the medium between the wires being unity. Applying this formula to the calculation of the residual of inductance in the 1,000-ohm coil, made by twisting the two wires together and separating consecutive turns, a value of μ less than 40 is found to be sufficient to make the inductance balance the capacity when the medium between the wires is air.

The magnitude of the increase of the inductance due to the permeability of the wire in the coil was experimentally investigated by comparing a coil of iron and a coil of superior wire with coils of german silver wire. The iron-wire coil was made by winding two number 32 wires, 0.15 cm. apart, on a cylindrical, wooden frame 25 cm. in diameter. The two wires were soldered together at one end and consecutive turns were separated about 1 cm. The length of the wire in the bifilar coil was about 274 meters, having a resistance of 1,000 ohms. The superior wire coil was made by winding two number 26 wires, 0.1 cm. apart, on a rectangular wooden block 9.25 cm. square. The length of the wire in this bifilar coil was 40 meters, having a resistance of 265 ohms.

These coils were compared with similar coils of german silver wire placed in one arm of the Wheatstone bridge and balanced against the parallel-wire standard according to the method described above (Section V. and Fig. 4). The german silver coil was then replaced by a coil of iron or of superior wire and balance again obtained. In no case was the increase of inductance due to the permeability of the wires sufficiently large to be measured.

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[1] E. B. Rosa and L. Cohen, Bulletin Bureau of Standards, V., p. 51, 1908.

X. MEASUREMENTS WITH ALTERNATING CURRENT AT FREQUENCIES RANGING FROM 420 TO 700 CYCLES PER SECOND.

All the measurements described heretofore were made with interrupted steady current or with alternating current at a frequency not exceeding 40 cycles per second. Higher frequencies were next used to determine to what extent the values of the residuals depend upon the frequency of the applied electromotive force. The source of electromotive force used first was a small eight-pole generator with an armature which could be rotated at speeds as high as 3,600 R.P.M. The residuals of inductance of several resistance coils were measured with this source of electromotive force at 240 cycles per second. The differences between these values and those obtained with alternating current at 40 cycles per second were within the limits of experimental error.

In order to test further the effect of higher frequencies a small twenty-pole generator with a Gramme ring armature was used as a source of electromotive force. This generator was capable of producing alternating current at frequencies ranging from 500 to 800 cycles per second but it gave a voltage insufficient to excite the field magnet of the galvanometer. A small galvanometer was then built with all of the dimensions reduced to about one fifth of those of the galvanometer described in Section IV. The results of the measurements made with this galvanometer and the twenty-pole generator did not agree with those made with alternating current at 40 cycles per second. However, the same results were obtained when the frequency was 700 cycles per second as when it was 500 cycles per second. The measured residuals of capacity of 1,000-ohm double-wound resistance-coils, obtained at either frequency with the twenty-pole generator as a source of current, were 10 or 12 per cent. higher than those obtained with alternating current at 40 cycles per second. Investigaton showed that the cause of these discrepancies was a small direct electromotive force which was superimposed upon the alternating electromotive force. When the direct electromotive force was balanced by a battery, the differences vanished and the results obtained at 40 cycles per second were repeated with current at the higher frequency.

The measurements at higher frequencies could not be made as accurately as those at 40 cycles per second. The probable error of the observations in measuring capacities of 1,000 micromicrofarads was at least 5 per cent. The results of the experiments with higher frequencies can therefore be stated as follows: With frequencies ranging from 240 to 700 cycles per second, the same values of residuals of inductance are obtained and if there is a correction depending upon the frequency, it is certainly less than 5 per cent. for this range. The values of the inductance and capacity of the coils tested were both small, never exceeding a few microhenrys or a few hundred micromicrofarads. That the balance of inductance and capacity of a resistance coil is independent of the frequency for low frequencies and small values of inductance is to be expected from equation (23). The term $L^2\omega^2$ is too small to produce a measurable effect when resistance coils of small inductance are measured.

XI. MEASUREMENTS OF LARGER VALUES OF INDUCTANCE.

When the inductance of a coil is not small, the term $L^2\omega^2$ may not be negligible in comparison with R^2. The relation among the resistance, capacity and inductance, when the current flows through the coil in phase with the electromotive force, is given by equation (23) instead of equation (24).

The validity of equation (23) was tested experimentally by applying it to coils of larger inductance. Coils having an inductance of a few hundred millihenrys were inserted in one arm of the Wheatstone network (R of Fig. 4) and a capacity in parallel with R was adjusted until it balanced the inductance. The experimental manipulation was the same as when the bridge was balanced with coils of small inductance. The capacities used were those of standard mica condensers. The inductance of the coil is found from equation (23), substituting $\omega = 2\pi n$, n being the frequency of the applied electromotive force.

An inductance coil of about 0.1340 henry was measured by this method with currents at frequencies ranging from 62 to 700 cycles per second. The values of the inductance calculated from equation (23) for this range of frequencies did not differ more than 16 parts in 1,340. A standard inductance of 0.1 henry made by Hartmann and Braun was measured with the same range of frequencies and the values did not differ from 0.1 henry more than one half of one per cent. The same method is also applicable to the measurement of capacity by comparison with standard inductance.

Below is a table showing the results of measurements of two standards of inductance made by Hartmann and Braun and inductance coil number 7 taken from the laboratory.

TABLE III.

R (Ohms).	L (Henrys).	C (mf.).	Freq. Cycles Sec.	Cal. L (Henrys).
1,000	0.1	0.0996	62	0.0996
1,000	0.1	0.0965	300	0.1000
1,000	0.1	0.0905	500	0.0994
1,000	0.1	0.08325	700	0.0995
1,000	0.2	0.197	62	0.1985
1,000	0.2	0.143	500	0.1996
1,000	0.2	0.110	700	0.1952
1,000	Coil No. 7	0.1333	62	0.1338
1,000	Coil No. 7	0.125	300	0.1332
1,000	Coil No. 7	0.124	333	0.1340
1,000	Coil No. 7	0.0995	700	0.1341
1,000	Coil No. 7	0.1000	700	0.1348

This application of the alternating-current galvanometer affords an excellent method for measuring capacities and inductances, applicable to both small and large values of inductance or capacity; and the calculations involve only simple formulæ. After the galvanometer is put into good adjustment, measurements can be made quickly and accurately; but the chief advantage of the method is that the quantities entering the calculations are only the resistance, inductance and capacity of the arm of the bridge and the frequency of the applied electromotive force.

CONCLUSIONS.

1. A parallel wire standard of resistance can be constructed which is free from residual inductance when used with interrupted, steady current or with alternating current of moderate frequency.

2. The outstanding difference between inductance and capacity in resistance coils can be measured by comparison with this parallel-wire standard.

3. A standard coil, made by twisting the wires together and

separating consecutive turns is not completely compensated for inductance and capacity when capacities of a few hundred micromicrofarads are to measured.

4. Capacities and inductances of small or large values can be measured with alternating current and an alternating-current galvanometer. The values are calculated from formulæ which involve only the resistance, inductance and capacity of a single circuit and the frequency of the applied electromotive force.

University of California,
April 15, 1909.

A STUDY OF THE REFLECTING POWER OF METALS AS DEPENDENT ON THE REFRACTVIE INDEX OF THE SURROUNDING MEDIUM.

BY L. R. INGERSOLL AND R. T. BIRGE.

INTRODUCTION.

THE object of this paper is an experimental study of the reflecting power of metals in media (*i. e.*, *liquids*) of various optical densities.

As is well known, the Fresnel reflection laws for surfaces of transparent substances apply equally well to such surfaces when overlaid or surrounded with a liquid (say) if the index of refraction of the substance is taken as relative to that of the liquid. In metals we have a more complicated case, for the formula for reflecting power at normal incidence as stated by Drude[1]

$$R = \frac{n^2(1 + k^2) + 1 - 2n}{n^2(1 + k^2) + 1 + 2n}$$

involves two constants, n the refractive index and k the absorption index of the metal. For the case of a metallic surface in contact with a transparent medium of index other than unity, the above formula is customarily extended by the assumption that the refractive index is then to be taken as relative to that of the medium, the absorption index remaining practically constant. From a somewhat different standpoint Maclaurin[2] arrives at similar conclusions.

While the above assumptions are quite reasonable, this formula as applied to metal surfaces overlaid with transparent substances has never been subjected to experimental proof. The few scattering observations which are recorded, such as that of Hagen and Rubens[3] on the reflection of glass backed by silver, and of Conroy[4] for a particular case of steel under water, are not in sufficiently good agreement with the theory to give it any support.

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[1] Lehrbuch der Optik, 1906.

[2] R. C. Maclaurin, Theory of Light, Part I., Camb., 1908, pp. 249 and 261 .

[3] Hagen and Rubens, Ann. d. Phys., I., p. 352, 1900.

[4] J. Conroy, Proc. Roy. Soc., 35, p. 26, 1883; 28, p. 242, 1879; 31, p. 486, 1881.

On the other hand Conroy, Des Coudres,[1] and a number of other investigators[2] have studied the related problem of the influence of an overlying medium on the measured optical constants of a metal. The results[3] in the case of Conroy have been discussed by Voigt[4] and Drude[5] and are in fair agreement with the theory, but somewhat low. The presence of a transition layer has been assumed to explain the remaining discrepancy, but the two additional constants[6] which are introduced to take account of a transition layer, and which are determined from the results themselves, necessarily bring about a better apparent agreement between theory and facts; hence this assumption is to be avoided if possible.

In the present work an attempt is made to show that the reflecting power of perfect metal surfaces in various liquids is in good agreement with the simple theory outlined above. Surfaces imperfect because of scratches and other defects have also been studied, and while they do not by any means afford results identical with those of perfect surfaces, an empirical law is deduced which expresses the relation between their reflecting powers in air and in another medium.

METHOD AND APPARATUS.

The method of measuring reflecting power was that of comparison with a known standard, *e. g.*, a silver surface. For this purpose a spectrophotometer of the Martens-Koenig type was used and the optical train arranged as in Fig. 1. One half the field of view in the spectrophotometer was illuminated by light from the Nernst glower G, reflected from the concave mirror M_1 and the small prism: the other half by light reflected in succession at M_2; at S, which was either the comparison mirror or the surface to be tested, each being in the form of a small plane mirror; at M_3; and from D into the instrument.

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[1] Des Coudres, Inaugural Dissertation, Berlin, 1887.

[2] G. Quincke, Pogg. Ann., 128, p. 541, 1866. Also P. Drude, Wied. Ann., 39, p. 481, 1890.

[3] The angle of principal incidence was decreased on surrounding a metallic surface with a liquid. The principal azimuth was only slightly affected, if at all; in some cases it showed a very small increase.

[4] W. Voigt, Wied. Ann., 23, p. 137, 1884.

[5] P. Drude, Wied. Ann., 39, p. 481, 1890.

[6] R. C. Maclaurin, Theory of Light, p. 251.

The focal lengths of the mirrors (between 30 and 40 cm.) were so chosen that sharp images of the glower were formed on each slit of the spectrophotometer, as well as on the surface S. The advantage of working with the surface to be tested in the focus of the beam are obvious, when it is considered that in this way a minimum of surface is required, and moreover any small departure of the surface from planeness is of least effect under such circumstances. The departure from strictly normal incidence — about 5° — was not enough to affect the reflecting power to an appreciable extent.

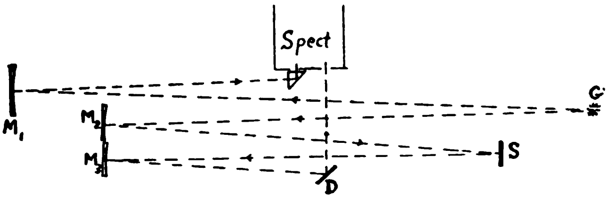

Fig. 1.

Comparison Mirrors. — For such, a large number of plate glass surfaces were silvered by the Brashear process and polished with rouge and chamois skin. Of perhaps twenty which were tested, the reflecting power of the best was, for a variety of reasons, assumed at 94.5 per cent., for the wave-length of sodium light, this being the wave-length used exclusively in this investigation. The average reflecting power of the surfaces tested was, on the assumption of the best being 94.5 per cent., about 93 per cent., which was that found by Hagen and Rubens[1] for a surface of much larger area than these. The theoretical reflecting power of silver computed from its optical constants is somewhat larger than either of these values, viz., 95.1 per cent. for sodium light.

But while the probable error of this assumption of reflecting power is not thought to exceed one half per cent., such a surface would be very difficult to use as a working standard, as it would be subject to continual change; hence it was immediately compared with three other surfaces, viz., a plate of polished steel, one of polished silicon, and an iron cathode surface. The last two apparently suffered no loss of reflecting power with age, while the steel surface changed only slightly and was frequently compared with the other two to eliminate any error from this source.

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[1] Ann. d. Phys., I., p. 352, 1900; 8, p. 1, 1902.

Observations. — In any series of observations a considerable number of settings of the spectrophotometer were first made with the comparison mirror in the position S. The surface to be tested was then introduced and a like number of settings taken, from which the relative reflecting power of the surface could be readily computed. The absolute reflecting power would, of course, follow on multiplication by that of the comparison mirror.

In measuring the reflecting power under liquids, small drops were placed on the surface and it was then covered with a slip of microscope cover glass of tested planeness, which spread the fluid out in a thin layer. To facilitate this part of the work it was found desirable to have the surface under test held horizontally. This was the arrangement used in the latter part of the work and was accomplished with the aid of an additional plane mirror (not shown in the diagram).

Correction. — The presence of the cover glass necessitated a small correction which was obtained as follows:

Let I be the intensity of the incident light, and I' the observed intensity of the reflected light. Let r be the fraction of light directly reflected at the air-glass and glass-liquid surfaces, the latter part being almost negligible. Then if R' is the fraction reflected at the liquid-metal surface, or in other words the reflecting power we desire to measure, we may write

$$I' = Ir + I(1-r)^2R' + I(1-r)^2R'\,r + \cdots.$$

The third term is so small that its value, Ic, may be computed with sufficient accuracy by assigning an approximate value to R'. Then

$$R' = \frac{(I'/I - r - c)}{(1-r)^2},$$

where r is approximately .040.

This gives at once the desired reflecting power R' from the measured apparent reflecting power I'/I.

Errors. — As a considerable number of settings of the spectrophotometer — usually twenty — were made for each measurement of reflecting power, the probable error arising from this source did not exceed one half per cent.

Inaccuracy of adjustment might easily cause an error several times as large as this and had to be carefully guarded against. A fruitful source of trouble of this sort was a tendency of the small cover slip to assume a tilted position when used with viscous liquids.

Contamination of the cover glass or surface under test might introduce considerable error. Great care was used in cleaning all surfaces with alcohol but in some cases a variation of the results is supposed to be due to this cause. Contamination of the comparison mirror or a wrong assumption as to its reflecting power would introduce errors in the determinations of the absolute reflecting powers, but would not affect our determination of the relative change of reflecting power when under the fluid, and, as will be noted later, this is of greater interest than the absolute reflecting powers.

Probable Error. — The general effect of the above sources of error is not enough, in our estimation, to cause a probable error in the determination of reflecting power in excess of one per cent. This is about the magnitude of the casual variations which are noted in the results.

RESULTS.

Metals. — The metal surfaces experimented upon were the following:

1. Silver on glass deposited chemically. Also glass-backed-by-silver mirrors on microscope cover glass.
2. A very perfectly polished steel mirror, as well as several steel surfaces ground with No. 0000 emery paper but not polished.
3. Several good silicon surfaces.
4. A rather imperfect gold surface obtained by buffing with rouge and machine oil.
5. Two cobalt surfaces gound with No. 0000 emery paper and polished with dry rouge.
6. Several cathode surfaces of platinum and iron.

With the exception of gold all the above surfaces were prepared in this laboratory. The process in the case of steel consisted in rough grinding with graded emery or carborundum on an iron plate; then fine grinding with the finest precipitated powders on a lead disk, after which it was polished in the usual way with rouge and water, using a pitch and beeswax tool. Silicon was prepared in much the same way.

Liquids. — For determining the reflecting power in media other than air, the liquids monobromnaphtalin, Canada balsam, glycerine and water were tried. It was soon found, however, that satisfactory results could not be obtained with water or Canada balsam, as the former evaporated too quickly and the latter was too viscous. The results recorded are for glycerine ($N_D = 1.466$) and monobromnaphtalin ($N_D = 1.658$).

No attempt will be made at a detailed presentation of the results which embody in all several thousand observations, but the general conclusions may be readily drawn from the table and curves which follow. It will be convenient to consider the results under two general heads, viz., those for perfect and for imperfect surfaces, the word "perfect" applying to the state of polish, rather than of planeness.

Perfect Surfaces. — As practically perfect surfaces were considered those free from casual defects and which gave reflecting powers within one or two per cent. of the theoretical, computed from the optical constants of the metal. As may be realized, such surfaces are extremely hard to prepare and only three of those used in the present investigation — silver, steel and silicon — were of this class. Fortunately, however, these metals may be regarded as typical, since they represent, as regards their optical properties, the two extremes and the mean. Thus silver has a low index of refraction, high absorption, and high reflecting power, while silicon has a high index, low absorption and low reflecting power. The optical properties of steel are more nearly like those of the majority of metals.

In Table I. the results are given for these metals. The optical constants used for steel and silver are those of Drude. For silicon they had not been previously determined and were measured by Mr. J. T. Littleton, Jr., of this laboratory, to whom acknowledgment is also due for the preparation of the silicon surfaces. The figures are all for the wave-length of sodium light, $\lambda = 5893$ Å.U

TABLE I.

Meta	Medium.	R		R (liquid) / R (air)		$1-R$ (liq.) / $1-R$ (air)	
		Theor.	Obs.	Theor.	Obs.	Theor.	Obs.
Silver $n=0.18$	air	.951	.941	.976	.969	1.462	1.490
$k=20.39$	monobrom	.929	.912				
	air	.951	.926	.981	.964	1.360	1.458
	glycerine	.934	.892				
	glass$_{(N_D=1.49)}$	.933	.912	.980	.978		
Steel $n=2.41$	air	.585	.570	.738	.730	1.368	1.358
$k=1.41$	monobrom	.432	.416				
	air	.585	.566	.800	.785	1.281	1.280
	glycerine	.468	.444				
Silicon $n=4.10$	air	.385	.377	.507	.497	1.310	1.305
$k=0.20$	monobrom	.195	.187				
	air	.385	.381	.624	.609	1.235	1.240
	glycerine	.240	.232				

The results for a glass-silver surface, included in the table, are of interest, as they show a somewhat better agreement with theory than those of Hagen and Rubens[1] for a similar case, in which they found a reflecting power of only 82–88 per cent. It is quite probable that the better agreement of the present results with theory is due to the very small area of surface used (about 1 mm. × 4 mm.), thus enabling one to select a very perfect part of the surface of such small area.

Conclusion for Perfect Surfaces. — An inspection of the above table shows that the experimental and theoretical results agree within what are probably the limits of experimental error. It is true that the observed values of R(liquid)/R(air) are uniformly a trifle low, and this might be explained by assuming a transition layer; but on the other hand, the small outstanding discrepancy could be readily accounted for by the fact that the mirrors tested only approximated perfect surfaces, as will be noted by the difference between the theoretical and observed values of R in air. In the ratio $[1-R(\text{liquid})]/[1-R(\text{air})]$, the particular significance of which will be discussed later, the disagreement is large in the case of silver; but it will be noted that this is the ratio of two very small values and would be largely affected by a small experimental error.

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[1] Ann. d. Phys., 1, p. 352, 1900.

We may then conclude for perfect metallic surfaces that the simple extension of the theory which is customarily made is experimentally justified. That is, the reflecting power when overlaid with a transparent substance of refractive index m, may be obtained by replacing in the formula for metallic reflection the index n of the metal by n/m, k remaining unchanged.

Imperfectly Polished Surfaces. — Because of the extreme difficulty, if not impossibility, of obtaining metallic surfaces with perfect polish, it becomes of interest to study these relations of optical properties under different media, for the case of imperfect surfaces. General planeness being assumed, the several causes for the departure of a surface from its "normal state," that is, a state of perfect polish, may be grouped under three heads:

(*a*) Opaque absorbing spots or masses covering some fraction of the area of the surface.

(*b*) A transparent surface layer of variable or constant index of refraction.

(*c*) Surface indentations or irregularities. This includes all such casual defects as corrugations, scratches, and "pit marks," of any size whatever.

The last class of defects is much the most common of the three, as the first two may be guarded against to a great extent by a suitable choice of abrasives for the final polishing and care in cleaning. To study briefly their effects on the reflecting power under different circumstances, it will be noted that opaque spots would result in a reduction of the reflecting power by a certain fraction which would be the same whatever the surrounding medium. This means that the ratio R(liquid)/R(air) would be constant. In other words, if points are plotted with R(liquid) as ordinates and R(air) as abscissæ they should all fall on straight lines passing through the origin. This, in general, has not been found to be the case.

The effect of a surface layer might be accounted for by introducing its hypothetical refractive index m' and calculating what this would have to be to give the observed diminished reflecting power. When such a calculation is extended to cover the case of an added overlying medium the curves of *R*(liquid) and *R*(air) are found to be lines curving with the convex side downwards. The results of the present investigation do not fit such curves any better than the straight lines mentioned above, so we may safely conclude that neither of these classes of defect was prominent in the surfaces used.

We may then study the results with the view of determining if possible the effect of the more common class of defects, such as scratches and corrugations, on the optical properties here studied. Drude[1] found that sharp scratches had, in all cases save that of steel, a very decided effect in reducing the angles of principal incidence and hence the apparent refractive indices. But when these sharp scratches are removed by rouge polishing, although the metal may then give nearly its normal optical constants, its reflecting power may

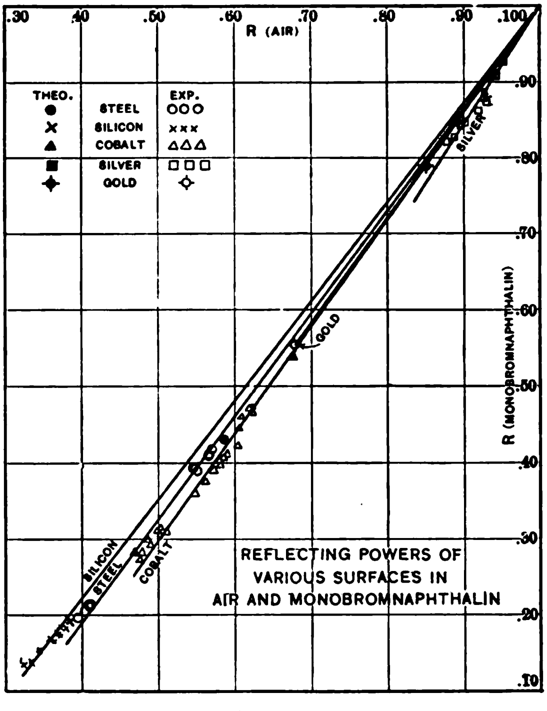

Fig. 2.

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[1] Wied. Ann., 39, p. 481, 1890.

still be considerably below the theoretical. Examination with a glass shows that the scratches have been merely rounded off into a myriad of minute corrugations with many small holes, and this, in the writers' experience, is the type of metal surface produced by the customary grinding and polishing process — buffing being excluded as it destroys the figure of the surface. In special metals, as already noted, these defects may be minimized so that a practically perfect surface results.

The results of a large number of observations on many different surfaces of this sort are given in the curves of Fig. 2. The abscissa of each point is the reflecting power of the corresponding surface in air, and the ordinate that in monobromnaphtalin — that liquid proving the most satisfactory to use. Points showing the lowest reflecting power correspond to surfaces having the most scratches or corrugations. Theoretical points for perfect surfaces, which are shown as small black circles, squares, etc., are derived from the optical constants of each metal tested.

Empirical Law.— It will be noted that for each metal the points for the different surfaces all fall fairly well on a straight line passing through the point (1, 1) and therefore having the equation $[1 - R\,(\text{liquid})]/[1 - R(\text{air})] = \text{a constant}$. Moreover, the lines for the different metals have very nearly the same slope. The small deviations of points from these lines are readily explained by the presence, in small measure, of defects of the first two classes above discussed, which could not be entirely eliminated. Not only do the points for surfaces of various qualities fall on these lines, but also those for theoretically perfect surfaces, as well as for the nearly perfect surfaces actually studied.

This then, the writers believe, is the law which expresses the relation between reflecting powers in air and in some other medium for metal surfaces of all degrees of polish, presupposing that surface layers — for example oil films — and spots such as oxidation and dirt are guarded against.

The theoretical basis of the law is not apparent. The action of scratches and corrugations on the incident light is undoubtedly twofold,— viz., to introduce in some cases more than one reflection and hence greater absorption; and to scatter in various directions a fraction of the incident light. As the present method of investigation utilized a conical beam of small solid angle, rather than strictly parallel light, it is possible that the poorer sort of surfaces might show a somewhat higher reflecting power than if measured by a method using only parallel light; for in the present case some of scattered light might still fall within the limits of the cone which reaches the measuring instrument. However, as reflecting power under these circumstances in somewhat a matter of definition, neither method has an obvious advantage.

These considerations enable us to suggest a possible explanation of one result of experiment. It will be noted in the case of the poorer sort of surfaces of any one metal, that the fractional reduction of reflecting power on overlaying with monobromnaphtalin is much greater than for the better surfaces. We may suppose that for such a surface the portion of the incident light reflected is scattered into a cone of solid angle ω', of which only ω can reach the instrument. When covered with liquid the fraction reflected — less, of course, than if the liquid were not present — is scattered into the same cone ω' which on emerging from the plane surface of the liquid becomes, following the ordinary laws of refraction, a cone of wider angle ω''.

Consequently a still smaller fraction of the returned light reaches the spectrophotometer than in the first case and this would account for the observed result. But this does not by any means explain the fairly exact relations which experiment shows to exist and which cannot reasonably be ascribed to any particular instrumental arrangement.

Cathode Surfaces. — The few cathode surfaces of iron and platinum which were tested were deposited with a rather high current density, and while they showed perfect mirror surfaces, their reflecting powers were considerably less than the normal for these metals. When experimented on with various liquids their results did not agree with those for polished surfaces, but the discrepancy was greatly reduced when the actual optical constants for these surfaces were measured and introduced into the formula. When plotted as in Fig. 2 the points for these surfaces fell on lines somewhat intermediate between those on the diagram and one through the origin. This might mean the presence of metal in a finely divided state, *e. g.*, platinum black, and some color is lent to this view by finding that the reflecting power of these surfaces could be improved by polishing with rouge and chamois.

However, as noted by all who have worked with cathode surfaces, including Skinner and Tool,[1] and one of the writers,[2] they may be produced in a variety of forms according to the conditions of deposit, varying from a state resembling that of the fused metal to states widely different from this. Accordingly, without special and more or less exhaustive study of these optical properties for this kind of surface, it is impossible to draw definite conclusions. But it may be noted in this connection that one of the writers,[3] in studying the effect on the magnetic rotation (Kerr effect) produced by overlaying iron surfaces with various liquids, found that cathode surfaces obey the theoretical law (rotation proportional to the index of the liquid) better than polished surfaces.

SUMMARY.

1. The extension of the formula for the reflecting power of a metal in terms of its optical constants, to include the case where the metal is covered by a transparent substance of any optical density, is experimentally justified for the case of perfect or practically perfect surfaces. This extension is made by replacing in the formula the actual refractive index n of the metal by the relative index n/m, where m is the index of the covering medium, the absorption index k of the metal remaining unchanged.

2. When a surface has a reflecting power lower than its normal value for that metal this condition is due in most cases to the presence of minute scratches, corrugations or indentations. While it is difficult to treat this case theoretically, the experimental results indicate that for all this class of surfaces as well as for those theoretically perfect, the relation $[1 - R(\text{liquid})] / [1 - R(\text{air})] =$ a constant, exists for any particular metal.

PHYSICAL LABORATORY,
UNIVERSITY OF WISCONSIN,
June 8, 1909.

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[1] C. A. Skinner and A. G. Tool, Phil. Mag., December, 1908.

[2] L. R. Ingersoll, Phil. Mag., July, 1909.

[3] Ibid.

ON THE PHOTO-ELECTRIC EFFECT WITH THE ALKALI METALS. II.[1]

The Dependence of the Photo-Electric Current on Light Intensity.

By F. K. Richtmyer.

IN a previous article[1] the writer has shown that the photo-electric current from a Na surface under the action of light from an incandescent lamp is proportional to the intensity of the incident light from very low intensities up to intensities in the neighborhood of .5 candle-foot. For purposes of comparison a portion of the curve previously shown is reproduced here (Fig. 1), the point marked *A* corresponding to an intensity of illumination of about .15

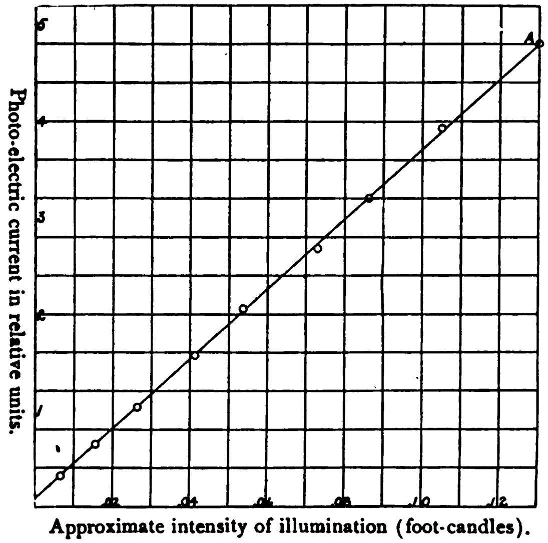

Fig. 1.

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[1] The first paper in this series was published in The Physical Review, Vol. XXIX., 1909, pp. 71–80, under the title "The Dependence of the Photo-electric Current on Light Intensity."

candle-foot. The exact intensity was not measured partly because the nature of the experiment would not permit, partly because a knowledge of the exact intensity would have no value, the photo-electric current depending quite as much on the quality of the light as on its intensity.

In view of the fact that some observers had found that this linear relation did not exist if very great ranges of intensities were used, it seemed wise to extend the above investigation to the highest obtainable light intensities consistent with accuracy. It seems quite reasonable to expect that an intensity of illumination might be reached at which a phenomenon akin to saturation might be observed.

The apparatus, a modification of that described in the previous paper, is shown in Fig. 2. The positive terminal of a 12-volt battery *E* is earthed. The negative terminal is connected to the sensitive surface (Na) of the photo-electric cell *C*. The platinum receiving wire *r* is connected to earth through the sensitive D'Arsonval galvanometer *G*, whose constant is $156 \cdot 10^{-11}$ amp./mm. at a scale distance of 138 cm. The cell and galvanometer were enclosed in a black box *abcd*, with windows at *W* and *M*, *MN* being used as before described to protect the cell from the light of the room. The source of light was a hand-feed, direct-current arc, *S*, of solid carbons placed at right angles to each other. A water cell (not shown) was placed at *M* to prevent a rise in temperature of the cell due to radiation from the arc. Behind the arc was a

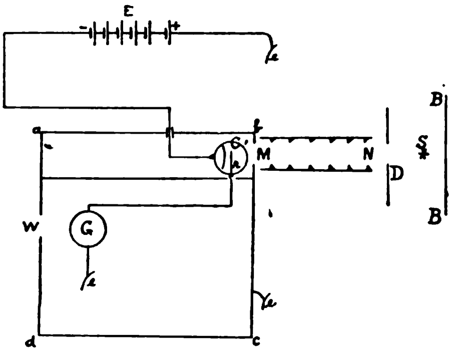

Fig. 2.

black screen *BB* and a second screen at *D* was placed so as to prevent the light from striking the walls of the box *MN* and yet not cast a shadow on the cell. The variation in intensity of illumination was obtained by varying the distance *MS* of the arc from the cell.

The difficulties noted in the previous paper, *i. e.*, the currents due to contact E.M.F. and the fact that the walls of the cell are slightly conducting, were far too small to be of any disturbance in comparison with the currents obtained in this experiment. But great difficulty was experienced in handling the arc. The well-known fluctuations not only in the intensity but in the quality of light from the arc, even though the arc length did not vary greatly, caused variations in the photo-electric current of from 20 per cent. to 150 per cent. After much time had been wasted in trying to get readings under these conditions it was noted that, for ten or fifteen seconds after lighting the arc, the galvanometer reading would not vary considerably, possibly not more than two or three per cent. And, further, if the arc was put out and then immediately relighted before the carbons had time to cool very much, the galvanometer would come back to the same place as before. It was also found that this condition was very much improved by increasing the arc length to just beyond the hissing point and running the current up somewhat above normal. Although rather difficult to work with, a constant and fairly intense light source was thus at hand.

The procedure in taking observations was then as follows : The arc was first lighted and allowed to burn until the carbons became thoroughly heated. It was then "blown out," relighted by bringing the carbons together, and the vertical carbon run down, increasing the arc length until the hissing stage was just past. The spot of light on the galvanometer scale was watched for the next few seconds and its mean position recorded. This process was repeated until several readings were obtained for each intensity of illumination used. Not all of these attempts resulted in a reading, for sometimes, after relighting, the arc would never get quiet. It would, however, generally settle down again on a second trial.

The data thus obtained are shown in Table I., and graphically in Fig. 3, the candle power of the arc being approximately 1,900 as measured by a Weber photometer.

TABLE I.

Distance of Arc from Cell.	Galvanometer Readings.				Average Galvanometer Deflection.	Photo-electric Current. Amp. × 10^9.	Illumination on Cell. Candle-feet.
10.00 ft.	3.8 mm.	3.7 mm.	3.8 mm.		3.8 mm.	5.9	19.0
8.00	5.5	5.8	5.8		5.6	8.7	30.0
6.50	8.0	8.0	8.0		8.0	12.5	45.0
5.50	10.0	10.5	11.0		10.5	16.4	63.0
4.75	15.0	15.0	14.0	14.0 mm.	14.5	22.6	84.0
4.00	21.0	19.5	20.0	20.0	20.1	31.4	118.0
3.50	26.0	24.5	23.0	26.0	24.6	38.8	155.0
3.00	32.0	34.0	34.0	35.0	33.7	52.6	210.0
2.50	51.0	50.0	52.0		51.0	79.5	304.0
2.25	63.0	61.0	63.0	63.0	62.5	97.5	375.0
2.00	79.0	78.0	80.0		79.0	123.0	475.0
1.75	100.0	107.0	95.0	105.0	103.0	160.0	620.0

Fig. 3 seems to show pretty clearly the linear relation between the photo-electric current and the intensity of illumination, even for

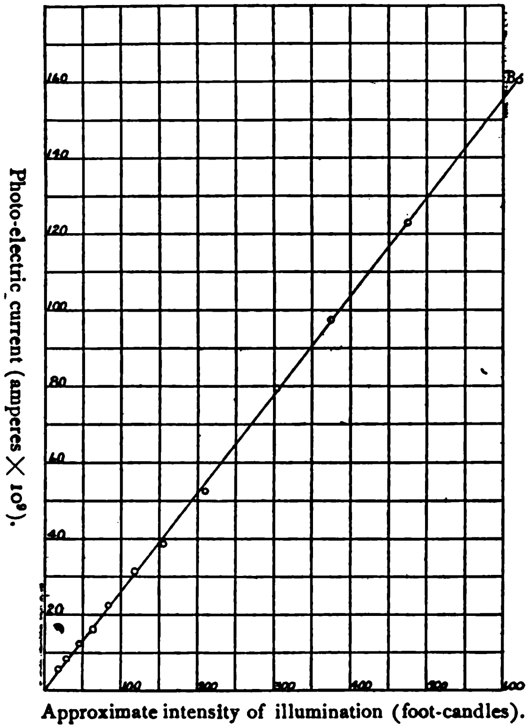

Fig. 3.

the extremely high intensities used here. The difficulty in controlling the light source accounts for the observations not giving quite so uniform a line as in Fig. 1, where an incandescent lamp was used. But even up to the highest intensities there is not the slightest suggestion of saturation.

These two curves present an interesting case not only of the extremely wide range over which a physical law holds, but also of the accuracy with which observations may be made at both extremes of the phenomenon, a fact more clearly brought out when the relative scales of the two curves are taken into account: If Fig. 3 were plotted to the same scale as Fig. 1 it would make a line 1,700 feet long!

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THE
PHYSICAL REVIEW.

VECTOR REPRESENTATION OF NON-HARMONIC ALTERNATING CURRENTS.

By Bunroku Arakawa.

PROBLEMS concerning the general theory of alternating currents are usually treated by assuming that electromotive forces and currents have wave forms that can be represented by sine curves, or they are assumed to be simple harmonic alternating quantities. But, in actual cases, the wave forms are more or less distorted from the sine curve, and some phenomena of such non-harmonic alternating quantities will be different from the corresponding phenomena produced by simple harmonic alternating quantities of equal effective values. So in these cases the necessity arises of investigating the effects upon those phenomena of wave forms of the current or electromotive force.

When the currents and electromotive forces are of simple harmonic wave form they can be represented in a well-known manner by radius-vectors, and their relations can be expressed completely by a vector diagram which in many cases greatly facilitates the solution of problems. For non-harmonic alternating currents, however, the same method of vector representation will not be generally applicable without certain modifications.

A few years ago Bedell and Tuttle[1] showed that if the electromotive force is sinusoidal and the current is non-harmonic, the higher harmonics in the current wave may be represented, collectively, by a radius-vector which is normal to radius-vectors representing the electromotive force and the fundamental wave of the current, so that the whole diagram will become a space figure. They also showed that if the higher harmonics in the current wave are to be represented separately a diagram of more than three dimensions will be required, which, therefore, cannot be shown as a geometrical figure. The purpose of the present writer is to study more fully the general method by which non-harmonic alternating quantities can be represented by radius-vectors.

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[1] Transactions, A. I. E. E., Vol. XXV., p. 615.

Since the wave form of simple harmonic alternating quantities is a definite one, such quantities are completely specified by their effective values (*i. e.*, their magnitudes) and their phase-positions, so that they can be completely represented by radius-vectors which have magnitude and direction. If, however, the alternating quantities are non-harmonic, their effective values and phase positions are not sufficient to specify them completely, and their wave forms must be given in addition. The wave form of a non-harmonic alternating quantity is determined by the magnitudes and the phase positions of higher harmonics contained in the wave. Hence if, in addition to the fundamental, only one harmonic be present, we must have four independent factors to specify a non-harmonic alternating quantity completely; therefore any method of vector representation cannot be theoretically correct, but is artificial and conventional. This is true of the method developed in this paper. While the writer does not believe this method to be simple enough for solving every practical problem of non-harmonic alternating currents, he believes it may give some help in the better understanding of the subject.

In connection with this subject consideration will also be given to the meaning of the terms "power-factor" and "reactive-factor" of a circuit carrying non-harmonic alternating currents.

Mean Value of Product of Two Alternating Quantities; Equivalent Phase Difference. — Let the instantaneous values of electromotive force and current be expressed by

$$e = E_m\{\sin \omega t + h_3 \sin (3\omega t + \theta_3) + \cdots\},$$

and

$$i = I_m\{\sin (\omega t - \varphi_1) + k_3 \sin (3\omega t + \theta_3 - \varphi_3) + \cdots\},$$

in which ω is 2π times the fundamental frequency, and h_3 and k_3 are the amplitudes (hereafter referred to as the percentage amplitudes) of the third harmonic of electromotive force and current, respectively, the amplitudes of the fundamental being taken as unity. The phase difference between current and electromotive force is φ_1 for the fundamental and φ_3 for the third harmonic. θ_3, etc., indicate the phase positions of the harmonics in the electromotive force wave with reference to the fundamental.

Let E and I be the effective values of electromotive force and current, respectively; then, as is well known,

$$E = \frac{E_m}{\sqrt{2}}(1 + h_3^2 + \cdots)^{\frac{1}{2}},$$

$$I = \frac{I_m}{\sqrt{2}}(1 + k_3^2 + \cdots)^{\frac{1}{2}}.$$

If e and i are multiplied together, the mean value W of the product during a complete period of the fundamental wave will be found to be

$$W = \tfrac{1}{2}E_m I_m(\cos\varphi_1 + h_3k_3\cos\varphi_3 + \cdots);$$

or,

$$W = EI\left\{\frac{\cos\varphi_1 + h_3k_3\cos\varphi_3 + \cdots}{(1 + h_3^2 + \cdots)^{\frac{1}{2}}(1 + k_3^2 + \cdots)^{\frac{1}{2}}}\right\}.$$

But W is the power of the circuit, and consequently the fraction within the brackets is by definition the power-factor of the circuit. Hence, if the power-factor p is denoted by $\cos\Phi$,

$$p = \cos\Phi = \frac{\cos\varphi_1 + h_3k_3\cos\varphi_3 + \cdots}{(1 + h_3^2 + \cdots)^{\frac{1}{2}}(1 + k_3^2 + \cdots)^{\frac{1}{2}}}, \tag{1}$$

and Φ may be regarded as the "equivalent phase difference" between the electromotive force and current; Φ is equal to the phase difference between two sine waves having the same effective values as E and I, respectively, and the same mean product of their instantaneous values (see Standardization Rules of the A. I. E. E., Art. 83).

The above equation for $\cos\Phi$ shows that even when the fundamentals and harmonics of current and electromotive force are, respectively, in the same phase, *i. e.*, φ_1, φ_3, etc., $= 0$, the value of $\cos\Phi$ cannot be equal to unity; or, the power cannot be expressed by the product of the effective values of electromotive force and current, unless we have $h_3 = k_3$, $h_5 = k_5$, etc., that is, the wave forms of electromotive force and current are similar to each other.

However, since the cosine assumes the same sign if the sign of the angle be changed, the foregoing equation for $\cos \Phi$ does not determine the sign of Φ; or, in other words, it is indefinite which of the two alternating quantities is ahead of the other in phase. We may give to Φ a sign equal to that of φ_1, or in case $\varphi_1 = 0$, that of φ corresponding to the lowest harmonic which happens to have a phase difference. But, if all φ's are equal to zero, there is no way of determining the sign of Φ.

On the other hand, the angle Φ is evidently not equal to the angle φ_1, which is the phase difference between the fundamentals of the two waves. In general, also, the angle Φ is not equal to either the phase angle between the zero values of the two waves or the the angle between their maximum values.

The term "equivalent phase difference" has, therefore, no definite physical meaning, and, in case of non-harmonic alternating quantities, is simply a conventional term. Hence, it may happen that the equivalent phase difference between two quantities, as A and C, is neither the sum nor the difference of the phase difference between A and a third quantity, B, and the phase difference between B and C. For example, if B lags 30° behind A, and C lags 30° behind B, C may lag, say, 50° (not 60°) behind A. The angles are related as the three angles of a triangular pyramid.

If the sign of any φ, in the equation for i, is changed, the new equation will represent a current i' of different wave form, but with the same equivalent phase difference with respect to e. The phase difference between i and i' will, however, not be zero. This will be considered more fully later.

Sum, or Difference, of Two Alternating Quantities. — Let the two alternating quantities be expressed by

$$i_1 = I_{1m}\{\sin \omega t + k_{13} \sin (3\omega t + \alpha_3) + \cdots\},$$

and

$$i_2 = I_{2m}\{\sin (\omega t - \varphi_1) + k_{23} \sin (3\omega t + \alpha_3 - \varphi_3) + \cdots\},$$

whose effective values are I_1 and I_2, respectively.

Then the sum, or difference, of the above two alternating quantities is

$$
\begin{aligned}
i &= i_1 \pm i_2 \\
&= \sqrt{I_{1m}^2 + I_{2m}^2 \pm 2I_{1m}I_{2m}\cos\varphi_1}\,\sin(\omega t - \psi_1) \\
&\quad + \sqrt{(k_{13}I_{1m})^2 + (k_{23}I_{2m})^2 + 2k_{13}k_{23}I_{1m}I_{2m}\cos\varphi_3}\,\sin(3\omega t + a_3 - \psi_3) \\
&\quad + \cdots,
\end{aligned}
$$

where

$$\psi_1 = \tan^{-1}\frac{\pm I_{2m}\sin\varphi_1}{I_{1m} \pm I_{2m}\cos\varphi_1}, \quad \psi_3 = \tan^{-1}\frac{\pm k_{23}I_{2m}\sin\varphi_3}{k_{13}I_{1m} \pm k_{23}I_{2m}\cos\varphi_3}, \quad \text{etc.}$$

The effective value of i then becomes

$$I = \sqrt{I_1^2 + I_2^2 \pm 2I_1I_2\left\{\frac{\cos\varphi_1 + k_{13}k_{23}\cos\varphi_3 + \cdots}{(1 + k_{13}^2 + \cdots)^{\frac{1}{2}}(1 + k_{23}^2 + \cdots)^{\frac{1}{2}}}\right\}}.$$

But, by reference to equation (1), the fraction within the brackets in this expression is seen to be the cosine of the equivalent phase difference Φ between i_1 and i_2, and we have

$$I = \sqrt{I_1^2 + I_2^2 \pm 2I_1I_2\cos\Phi}. \tag{2}$$

If Ψ and θ represent the equivalent phase differences between i and i_1, and between i and i_2, respectively, their values can be found from the equations for i_1, i_2, and i; expressed in terms of I_1, I_2, I, and Φ, they are:

$$\Psi = \cos^{-1}\frac{I_1 \pm I_2\cos\Phi}{I}.$$

$$\theta = \cos^{-1}\frac{I_1\cos\Phi \pm I_2}{I}.$$

It is seen that in this case

$$\theta = \Phi \mp \Psi.$$

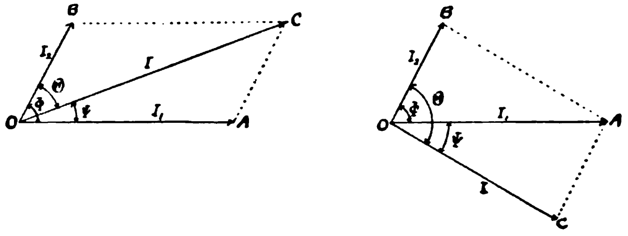
From this result we may conclude that, so far as the sum, or difference, of two alternating quantities are concerned, any non-harmonic alternating quantities can be represented by equivalent simple harmonic alternating quantities which have the same effective values and have a phase difference equal to the equivalent phase difference Φ, between i_1 and i_2, as shown above. Thus, for i_1 and i_2 may be substituted

$$i_1' = \sqrt{2}I_1\sin\omega t,$$

and

$$i_2' = \sqrt{2}I_2 \sin(\omega t - \Phi).$$

Vector Representation of the Sum, or Difference, of Two Alternating Quantities. — According to equation (2) for I already obtained, the relation of I_1, I_2, and I, may be represented by three sides of a triangle. In Fig. 1, if two lines OA and OB are radius-vectors representing I_1 and I_2, then OC is the radius-vector representing their resultant, as is well known in the case of harmonic alternating quantities, and



Sum. Difference.

Fig. 1.

$$OC = \sqrt{\overline{OA}^2 + \overline{OB}^2 \pm 2 \cdot OA \cdot OB \cdot \cos \angle AOB.}$$

The angle between OA and OB is made equal to Φ. It can easily be shown that the angle $AOC = \Psi$, and the angle $BOC = \theta$.

Therefore, by using the conventional term "equivalent phase difference," any two non-harmonic alternating quantities can be represented by radius-vectors, and their addition or subtraction may be performed in the same manner as with simple harmonic alternating quantities. In such cases, therefore, the vector diagram is a plane figure.

From the above result we can also conclude that any alternating quantity can be resolved into two components which have an equivalent phase difference of 90 degrees between them. From the equation (1) for equivalent phase difference, it is also seen that if not only the fundamentals but all the higher harmonics of two alternating quantities have a phase difference of 90 degrees separately, the equivalent phase difference between the two quantities is also equal to 90 degrees.

Sum of Three or More Alternating Quantities. — Let us take any three alternating quantities the instantaneous values of which are expressed by

$$i_1 = I_{1m}\{\sin \omega t + k_{13} \sin (3\omega t + \alpha_3) + \cdots\},$$

$$i_2 = I_{2m}\{\sin (\omega t - \varphi_1) + k_{23} \sin (3\omega t + \alpha_3 - \varphi_3) + \cdots\},$$

and

$$i_3 = I_{3m}\{\sin (\omega t - \psi_1) + k_{33} \sin (3\omega t + \alpha_3 - \psi_3) + \cdots\},$$

their effective values being I_1, I_2, and I_3, respectively. The effective value of their sum is

$$I = \sqrt{I_1^2 + I_2^2 + I_3^2 + 2I_1I_2 \cos \Phi + 2I_1I_3 \cos \Psi + 2I_2I_3 \cos \theta},$$

where Φ, Ψ, and θ, denote the equivalent phase differences between i_1 and i_2, between i_1 and i_3, and between i_2 and i_3, respectively. If the equivalent phase differences between i_1 and i, between i_2 and i, and also between i_3 and i, are $(\widehat{I_1I})$, $(\widehat{I_2I})$, and $(\widehat{I_3I})$, respectively, they can be computed from equations for i_1, i_2, and i_3; and their values are as follows:

$$(\widehat{I_1I}) = \cos^{-1} \frac{I_1 + I_2 \cos \Phi + I_3 \cos \Psi}{I};$$

$$(\widehat{I_2I}) = \cos^{-1} \frac{I_1 \cos \Phi + I_2 + I_3 \cos \theta}{I};$$

$$(\widehat{I_3I}) = \cos^{-1} \frac{I_1 \cos \Psi + I_2 \cos \theta + I_3}{I}.$$

In a similar manner, any number of alternating quantities can be added together. If I_1, I_2, I_3, etc., are their respective effective values, and $(\widehat{I_1I_2})$, $(\widehat{I_1I_3})$, $(\widehat{I_2I_3})$, etc., are the equivalent phase differences between i_1 and i_2, i_1 and i_3, i_2 and i_3, etc., respectively, the effective value of the resultant will be

$$I = \{I_1^2 + I_2^2 + I_3^2 + \cdots + 2I_1I_2 \cos (\widehat{I_1I_2}) \\ + 2I_1I_3 \cos (\widehat{I_1I_3}) + \cdots + 2I_2I_3 \cos (\widehat{I_2I_3}) + \cdots\}^{\frac{1}{2}}.$$

Vector Representation of Three or More Alternating Quantities. — Take three alternating quantities i_1, i_2, and i_3, and let Φ, Ψ, and θ, be the equivalent phase differences between each pair of them, as shown before. If the values of Φ, Ψ, and θ, are computed, it will be found that θ is generally not equal to $(\Phi - \Psi)$ or $(\Phi + \Psi)$. The only relations which can be obtained from equations for them are:

$$\theta \nless \Phi - \Psi, \quad \text{and} \quad \theta \ngtr \Phi + \Psi.$$

Hence, if I_1 and I_2 be represented by radius-vectors OA and OB, respectively, making the angle between them equal to Φ, as in Fig. 2, the third radius-vector OC representing I_3 must be placed outside of the plane containing OA and OB, in order to make the angle AOC equal to Ψ, and the angle BOC equal to θ. In this case, therefore, the vector diagram becomes a space figure, and the diagonal OD of a parallelopiped with OA, OB, and OC, as its three edges meeting at O, will represent the resultant I.

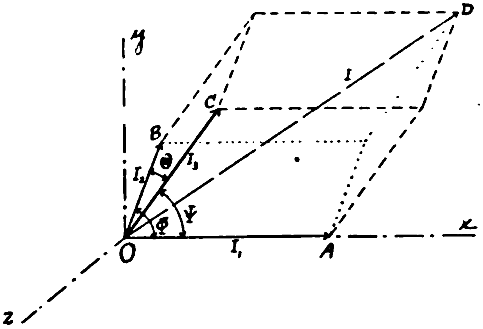


Fig. 2.

$$OD = \sqrt{\begin{aligned}&\overline{OA}^2 + \overline{OB}^2 + \overline{OC}^2 + 2 \cdot OA \cdot OB \cos \angle AOB \\ &\qquad + 2 \cdot OA \cdot OC \cos \angle AOC + 2 \cdot OB \cdot OC \cos \angle BOC,\end{aligned}}$$

$$= \sqrt{I_1^2 + I_2^2 + I_3^2 + 2I_1I_2 \cos \Phi + 2I_1I_3 \cos \Psi + 2I_2I_3 \cos \theta.}$$

It can also be shown that equivalent phase differences $(I_1\hat{}I)$, $(I_2\hat{}I)$, and $(I_3\hat{}I)$, are represented by angles AOD, BOD, and COD, respectively. It is seen, then, that any three alternating quantities can be represented by radius-vectors arranged in space, so that the vector diagram becomes a space figure.

Four or more alternating quantities can be treated in a similar manner, but in this case an idea of a vector diagram in four or more dimensions must be introduced in order to represent them simultaneously by radius-vectors. The vector diagram, therefore, can no longer be expressed by any geometrical figure. In general, if there are n independent alternating quantities to be represented simultaneously by radius-vectors, the vector diagram must become a figure of n dimensions.

However, in some special cases radius-vectors representing n independent alternating quantities can be arranged in a figure of less than n dimensions. Taking the case of three alternating quantities mentioned before, if θ is equal to either $(\Phi - \Psi)$ or $(\Phi + \Psi)$, the radius-vector OC will fall in the plane AOB, so that the vector diagram becomes a figure of two dimensions or a plane figure. Let us see what are the conditions for such special cases.

Let the three alternating quantities be i_1, i_2, and i_3, as shown before, and suppose i_2 and i_3 are so resolved into i_2' and i_2'', and i_3' and i_3'', respectively, that i_2' and i_3' have the same phase and wave form as i_1, while i_2'' and i_3'' are their remaining parts of which the fundamentals are in quadrature with i_1. Then

$$i_2' = I_{2m} \cos \varphi_1 \{\sin \omega t + k_{13} \sin (3\omega t + a_3) + \cdots \} ;$$

$$\begin{aligned} i_2'' = I_{2m} \{ & \sin \varphi_1 \cos \omega t + k_{23} \sin \varphi_3 \cos (3\omega t + a_3) + \cdots \\ & + (k_{23} \cos \varphi_3 - k_{13} \cos \varphi_1) \sin (3\omega t + a_3) + \cdots \} ; \end{aligned}$$

$$i_3' = I_{3m} \cos \psi_1 \{\sin \omega t + k_{13} \sin (3\omega t + a_3) + \cdots \} ;$$

$$\begin{aligned} i_3'' = I_{3m} \{ & \sin \psi_1 \cos \omega t + k_{33} \sin \psi_3 \cos (3\omega t + a_3) + \cdots \\ & + (k_{33} \cos \psi_3 - k_{13} \cos \psi_1) \sin (3\omega t + a_3) + \cdots \} . \end{aligned}$$

The resultant of i_1, i_2, and i_3, is evidently equal to the resultant of i_1, i_2', i_2'', i_3', and i_3''. But i_1, i_2', and i_3', are in phase and have similar wave forms, and therefore the equivalent phase difference between any pair of them is zero, so that the radius-vector representing the sum of these three is simply the algebraic sum of the radius-vectors representing them separately. And if i' is the sum of these three components i_1, i_2', and i_3', and if I_1, I_2', I_3', and I', are radius-vectors representing i_1, i_2', i_3', and i', respectively, then

$$I' = I_1 + I_2' + I_3'.$$

In case i_2'' and i_3'' have similar wave forms, the radius-vector I'' representing the sum of the two is equal to the algebraic sum of the radius-vectors I_2'' and I_3'' representing i_2'' and i_3'', respectively;

$$I'' = I_2'' + I_3''.$$

Then in this case, since the radius-vectors I_1, I_2', and I_3', are all in one direction, and I_2'' and I_3'' are in another direction, the radius-vectors I_1, I_2 (resultant of I_2' and I_2''), and I_3 (resultant of I_3' and I_3''), which represent i_1, i_2, and i_3, respectively, must lie in one plane, or the diagram must be a plane figure.

Now, the conditions that i_2'' and i_3'' have similar wave forms are, for the third harmonic,

$$k_{23}\frac{\sin\varphi_3}{\sin\varphi_1} = k_{33}\frac{\sin\psi_3}{\sin\psi_1},$$

and

$$\frac{k_{23}\cos\varphi_3 - k_{13}\cos\varphi_1}{\sin\varphi_1} = \frac{k_{33}\cos\psi_3 - k_{13}\cos\psi_1}{\sin\psi_1},$$

with similar conditions for other harmonics. If the conditions shown above are satisfied by k's, φ's, and ψ's, then the vector diagram for the three alternating quantities will become a plane figure. The above consideration may be extended to any number of alternating quantities.

These conditions are found in a series circuit with resistance and inductance or resistance and capacity (but not resistance, inductance and capacity) and in the arrangements of circuits known as the "three-voltmeter" method and "three-ammeter" method discussed later. So far the writer has found no other cases.

Graphical Method of Addition and Subtraction of Non-harmonic Alternating Quantities.—We have seen that when three or more alternating quantities are to be represented by radius-vectors simultaneously, an idea of a vector diagram in three or more dimensions is necessary. But, even in such cases, when any two of them are taken separately, they can be represented by radius-vectors in a plane figure. Hence, summation and subtraction of any number of alternating quantities may be performed graphically, representing them by radius-vectors, by first taking any two and finding the resultant by a plane vector diagram, combining the resultant with a third in a separate plane diagram, combining the new resultant with a fourth, etc.

Let us take three alternating quantities i_1, i_2, and i_3, whose effective values are I_1, I_2, and I_3, respectively, as shown before. Then the effective value I' of the sum of i_1 and i_2 is

$$I' = \sqrt{I_1^2 + I_2^2 + 2I_1I_2 \cos (\widehat{I_1I_2})},$$

where $(\widehat{I_1I_2})$ represents the equivalent phase difference between i_1 and i_2.

Now, if $(\widehat{I'I_3})$ is the equivalent phase difference between I' and I_3, its value is given by

$$\cos (\widehat{I'I_3}) = \frac{I_1 \cos (\widehat{I_1I_3}) + I_2 \cos (\widehat{I_2I_3})}{I'}, \tag{3}$$

where $(\widehat{I_1I_3})$ and $(\widehat{I_2I_3})$ are the equivalent phase differences between I_1 and I_3, and between I_2 and I_3, respectively.

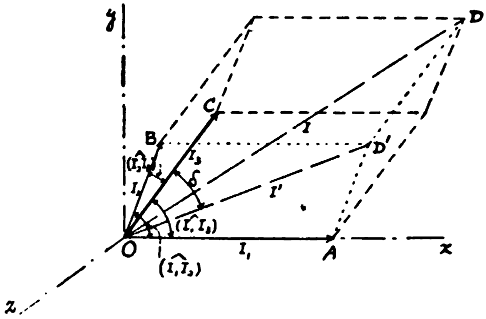


Fig. 3.

On the other hand, if i_1, i_2, and i_3, are represented by radius-vectors OA, OB, and OC, respectively (Fig. 3), the resultant of i_1 and i_2, that is i', will be represented by the radius-vector OD' which is the diagonal of a parallelogram with OA and OB at its two sides. If the angle between OD' and OC is denoted by δ, then we must have

$$OD' \cos \delta = OA \cdot \cos \angle AOC + OB \cdot \cos \angle BOC.$$

Hence

$$\cos \delta = \frac{OA \cdot \cos \angle AOC + OB \cos \angle BOC}{OD'}$$

$$= \frac{I_1 \cos (\widehat{I_1I_3}) + I_2 \cos (\widehat{I_2I_3})}{I'}.$$

Comparing this result with the equation for cos $(\widehat{I'I_3})$ obtained before, it is seen that the angle δ represents the equivalent phase difference between i' and i_3.

A similar consideration can be extended to the case in which any number of alternating quantities are treated. Suppose, for example, we have another alternating quantity i_4 whose effective value is I_4; then the equivalent phase difference between I' and I_4 is given by

$$\cos(\widehat{I'I_4}) = \frac{I_1 \cos(\widehat{I_1I_4}) + I_2 \cos(\widehat{I_2I_4})}{I'} .$$

If i'' is the resultant of i' and i_3, its effective value I'' is

$$I'' = \sqrt{I'^2 + I_3^2 + 2I'I_3 \cos(\widehat{I'I_3})} ,$$

and the equivalent phase difference between I'' and I_4 is given by

$$\cos(\widehat{I''I_4}) = \frac{I' \cos(\widehat{I'I_4}) + I_3 \cos(\widehat{I_3I_4})}{I''} ,$$

$$= \frac{I_1 \cos(\widehat{I_1I_4}) + I_2 \cos(\widehat{I_2I_4}) + I_3 \cos(\widehat{I_3I_4})}{I''} .$$

From the above results we can derive a method of adding or subtracting any number of alternating quantities by means of separate vector diagrams, each of which is a plane figure. For instance, suppose we have four alternating quantities represented by A, B, C, and D, and the equivalent phase differences between each pair of them are all known to be θ_{AB}, θ_{AC}, θ_{AD}, θ_{BC}, θ_{BD}, and θ_{CD}. Let M be the resultant of A and B; then M is found as a diagonal of a parallelogram whose two adjacent sides are A and B, the diagram being a plane figure, as shown in Fig. 4. If θ_{MC} is the equivalent phase difference between M and C, we have, according to equation (3),

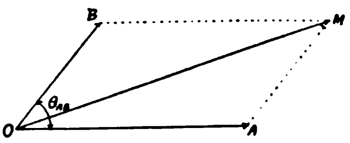


Fig. 4.

$$\cos \theta_{MC} = \frac{A \cos \theta_{AC} + B \cos \theta_{BC}}{M} .$$

The magnitude of θ_{MC} may be found graphically as shown in Fig. 5. Draw *OM* equal in length to *OM* in Fig. 4. Draw *OA* equal in length to *OA* in Fig. 4, making an angle $MOA = \theta_{AC}$. Drop a perpendicular *AR* from *A* upon *OM*. From *R* draw a line *RB* equal in length to *OB* in Fig. 4, making an angle $MRB = \theta_{BC}$; and drop a perpendicular *BS* from *B* upon *OM*. Strike a semicircle upon *OM*, and take a point *K* upon it so that $OK = OS$. Then the angle $KOM = \theta_{MC}$. For

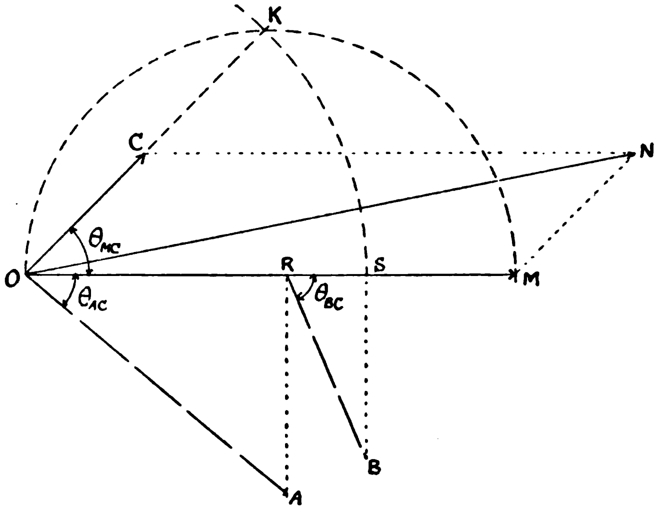


Fig. 5.

$$\cos\angle KOM = \frac{OK}{OM} = \frac{OS}{OM} = \frac{OA\cdot\cos\angle AOR + RB\cdot\cos\angle BRS}{OM},$$

$$= \frac{A\cos\theta_{AC} + B\cos\theta_{BC}}{M} = \cos\theta_{MC};$$

therefore,

$$\angle KOM = \theta_{MC}.$$

Thus θ_{MC} having been known, the resultant *N* of *M* and *C* is found to be represented by the radius-vector *ON* which is the diagonal of a parallelogram with *OC* and *OM* as its two sides, as in Fig. 5.

If in the above procedure, the point *S* happens to fall on the portion of the line *MO*, produced the other side of *O*, the point *K* should be taken on the semicircle drawn on the lower side of *OM*, and OC must be put on the produced portion of KO, so that the angle θ_{MC} will become greater than 90°.

If θ_{ND} is the equivalent phase difference between the resultant N and the fourth quantity D, its magnitude may be found graphically in a similar manner, as shown in Fig. 6. In this case the point T, the foot of the last perpendicular, happens to fall outside of NO, so that the point L is taken on the lower semicircle, and OD is put on the produced portion of the line LO, as seen in the figure. It is also easily shown that the angle DON thus obtained is equal to θ_{ND}. Thus θ_{ND} having been known the next step is only to construct the parallelogram $ODPN$ whose two sides ON and OD represent the resultant N and the fourth quantity D, respectively. Then the diagonal OP will be the radius-vector representing the resultant of N and D, which is therefore the resultant of the given four alternating quantities A, B, C and D.

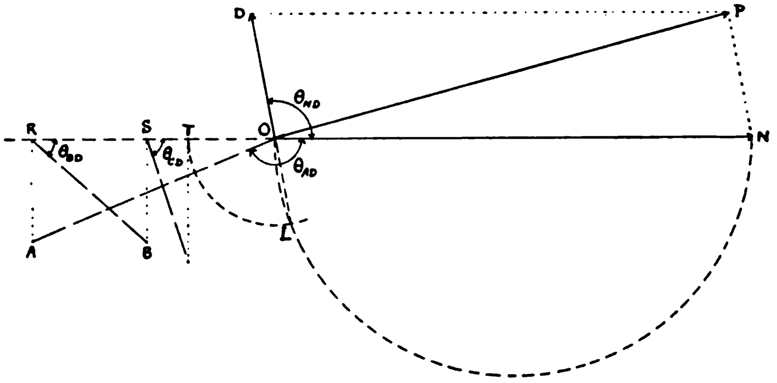


Fig. 6.

If some of these quantities are to be subtracted, the radius-vectors representing them should be drawn in the opposite direction in each diagram; thus, for instance, if the third quantity C is to be subtracted, Fig. 5 and Fig. 6 will be changed to Fig. 7 and Fig. 8, respectively.

Any number of radius-vectors representing non-harmonic alternating quantities can be treated in a similar manner.

Non-harmonic Alternating Electromotive Force and Current in a Circuit. — If a non-harmonic alternating electromotive force is applied to a circuit the wave form of current will generally be different from that of the electromotive force ; the only case in which these two waves can have similar forms is when the circuit contains nothing but non-inductive resistances.

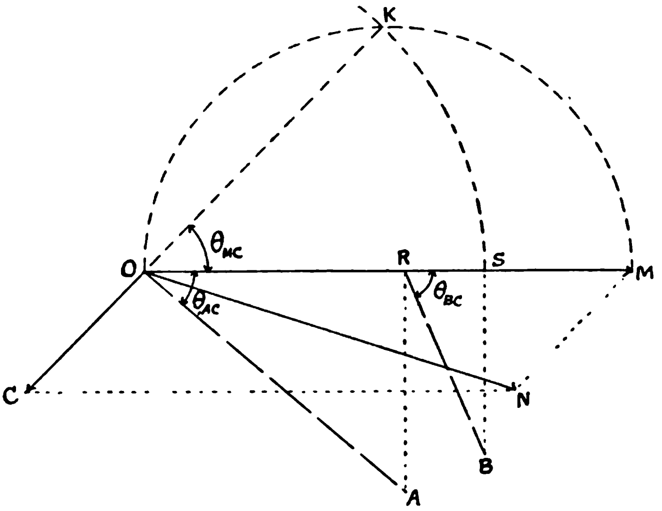


Fig. 7.

If the circuit contains inductance the percentage amplitude of any harmonic in the current wave is always less than that of the corresponding harmonic in the electromotive force wave. If the circuit contains capacity but no inductance, any harmonic in the

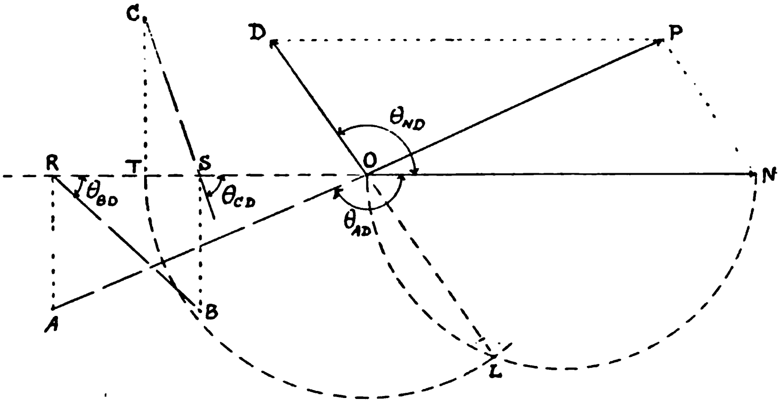


Fig. 8.

current wave has a greater percentage amplitude than the corresponding harmonic in the electromotive force wave.

If the circuit contains both inductance and capacity connected in series, the percentage amplitude of a harmonic in the current wave may be less or greater than that of the corresponding harmonic in the electromotive force wave. If L and C are the inductance and the capacity, respectively, and ω is equal to 2π times the frequency of the fundamental wave, then the harmonics of lower order than $(1/\omega^2LC)$th have greater percentage amplitudes in the current than in the electromotive force wave, but with those of higher order than $(1/\omega^2LC)$th, the percentage amplitudes are less in the current than in the electromotive force wave.

If the current in the circuit is expressed by

$$i = I_m\{\sin \omega t + k_3 \sin (3\omega t + a_3) + k_5 \sin (5\omega t + a_5) + \cdots\}$$

whose effective value is I, the effective value of the electromotive force is:

(1) for a circuit containing resistance R and inductance L in series,

$$E = I\sqrt{R^2 + (\omega L\rho)^2};$$

(2) for a circuit containing resistance R and capacity C in series,

$$E = I\sqrt{R^2 + \frac{1}{(\omega C\lambda)^2}};$$

(3) for a circuit containing resistance R, inductance L, and capacity C, in series,

$$E = I\sqrt{R^2 + (\omega L\rho)^2 + \frac{1}{(\omega C\lambda)^2} - \frac{2L}{C}},$$

where ρ and λ are

$$\rho = \sqrt{\frac{1 + (3k_3)^2 + (5k_5)^2 + \cdots}{1 + k_3^2 + k_5^2 + \cdots}},$$

$$\lambda = \sqrt{\frac{1 + k_3^2 + k_5^2 + \cdots}{1 + \left(\frac{k_3}{3}\right)^2 + \left(\frac{k_5}{5}\right)^2 + \cdots}},$$

which may be called "correction factors" for inductance and capacity, respectively.

The equivalent phase difference Φ between the electromotive force and the current is:

(1) for a circuit containing R and L in series,

$$\Phi = \cos^{-1} \frac{R}{\sqrt{R^2 + (\omega L\rho)^2}};$$

(2) for a circuit containing R and C in series,

$$\Phi = \cos^{-1} \frac{R}{\sqrt{R^2 + \frac{1}{(\omega C\lambda)^2}}};$$

(3) for a circuit containing R, L, and C, in series,

$$\Phi = \cos^{-1} \frac{R}{\sqrt{R^2 + (\omega L\rho)^2 + \frac{1}{(\omega C\lambda)^2} - \frac{2L}{C}}},$$

where, in each case, $\cos \Phi$ is equal to the power-factor of the circuit.

The equivalent phase difference between the current and the resistance voltage IR is zero in each case; and the equivalent phase difference between the current and the inductance voltage $I(\omega L\rho)$, and also that between the current and the capacity voltage $I/(\omega C\lambda)$, are equal to 90 degrees in each case; but the equivalent phase difference between the inductance voltage and the capacity voltage is equal to $\cos^{-1}(-\lambda/\rho)$, which is not equal to 180 degrees. The vector diagram showing the relation of the resistance voltage, the inductance voltage, and the capacity voltage of a circuit must, therefore, become a space figure. When, of the three kinds of quantities, resistances, inductances and capacities, only two kinds are present in a series circuit, the vector diagram becomes a plane figure. In ordinary cases, however, ρ and λ are nearly equal to unity, so that $\cos^{-1}(-\lambda/\rho)$ is nearly equal to 180 degrees.

Verification of Three-voltmeter Method and Three-ammeter Method of Power Measurement.— As an example of the vector representation of non-harmonic alternating quantities, we shall take the case of the so-called three-voltmeter method and three-ammeter method of power measurement, and show that these methods of power measurements are true for any nature of circuit and for alternating currents of any wave form. In both of these methods of power measurement, a certain known non-inductive resistance R is used in connection with the load circuit, the power of which is to be measured.

(*a*) Three-voltmeter method.

In this method, a resistance R is connected in series with the load circuit; if E_1, E_2, and E, are the effective values of the potential differences across the load circuit, across the resistance R, and across the whole circuit, respectively, the formula for the power in the load circuit is

$$W = \frac{(E^2 - E_1^2 - E_2^2)}{2R}.$$

Let I be the effective value of current in the circuit, and Φ be the equivalent phase difference between I and E_1. Since E_2 is the voltage across the non-inductive resistance R, its wave form is similar to that of I, so that the equivalent phase difference between I and E_2 is zero. Therefore, although we have three independent alternating quantities, viz., E_1, E_2, and I, yet they can be represented by radius-vectors in a plane vector diagram.

Since the power-factor of the load circuit is equal to cos Φ, the power in that circuit is

$$W = E_1 I \cos \Phi.$$

But since

$$I = \frac{E_2}{R},$$

and it is seen from the vector diagram that

$$\cos \Phi = \frac{E^2 - E_1^2 - E_2^2}{2E_1E_2},$$

we have

$$W = \frac{E^2 - E_1^2 - E_2^2}{2R}.$$

Thus, this method of power measurement can be used for any circuit without any restriction, whatever the nature of the load circuit and whatever the wave forms of E_1 and I.

(*b*) Three-ammeter method.

In this method, a resistance R is connected in parallel with the load circuit; if I_1, I_2, and I, are the effective values of currents in the load circuit, in the resistance R, and in the main line, respectively, the formula for the power in the load circuit is

$$W = \tfrac{1}{2}R(I^2 - I_1^2 - I_2^2).$$

Let E be the effective value of the electromotive force applied to the circuit, and Φ be the equivalent phase difference between E and I_1. Then, by the same reasoning as in the previous case, the radius-vectors representing I_1, I_2, I, and E, are in a plane diagram.

The power in the load circuit is again expressed by

$$W = EI_1 \cos \Phi.$$

But we have $E = I_2R$, and from the diagram we have

$$\cos \Phi = \frac{I^2 - I_1^2 - I_2^2}{2I_1I_2};$$

therefore

$$W = \tfrac{1}{2}R(I^2 - I_1^2 - I_2^2).$$

Thus, the three-ammeter method of power measurement can also be used for any circuit, whatever the nature of the load circuit, and whatever the wave forms of electromotive force and current.

Power-factor and Reactive-factor. The power-factor of a circuit is defined as "the ratio of the electric power in watts to the apparent power in volt-amperes,"[1] or if p is the power-factor, $p =$ watts ÷ volt-amperes.

If both electromotive force and current waves are sinusoidal, the power-factor can also be expressed by either energy voltage ÷ total voltage or energy current ÷ total current, and is equal to the cosine of the phase difference between the electromotive force and the current.

In the case of non-harmonic alternating currents, however, the power is equal to the product of the effective values of the electromotive force and the current, multiplied by the cosine of the equivalent phase difference Φ between them. Thus the power-factor p is equal to $\cos \Phi$, as in equation (1).

Let the electromotive force and the current be

$$e = E_m\{\sin \omega t + h_3 \sin (3\omega t + \theta_3) + \cdots\},$$

and

$$i = I_m\{\sin (\omega t - \varphi_1) + k_3 \sin (3\omega t + \theta_3 - \varphi_3) + \cdots\},$$

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[1] Standardization Rules, A. I. E. E., Art. 54.

whose effective values are E and I, respectively; and suppose the current i to be resolved into two components, an energy current i_w and a wattless current i_l. The energy current is that part of the current i which has its fundamental and all higher harmonics in the same phase as the fundamental and corresponding harmonics of the electromotive force, and the wattless component is the remaining part of the current i, and has its fundamental and all higher harmonics in quadrature with the fundamental and harmonics, respectively, of the electromotive force wave. Therefore, these two components can be expressed by

$$i_w = I_m \cos \varphi_1 \left\{ \sin \omega t + k_3 \frac{\cos \varphi_3}{\cos \varphi_1} \sin (3\omega t + \theta_3) + \cdots \right\},$$

and

$$i_l = - I_m \sin \varphi_1 \left\{ \cos \omega t + k_3 \frac{\sin \varphi_3}{\sin \varphi_1} \cos (3\omega t + \theta_3) + \cdots \right\},$$

whose effective values are I_w and I_l, respectively.

The equivalent phase difference between the electromotive force e and the wattless current i_l is 90 degrees, but that between e and the energy current i_w is generally not equal to zero, being equal to Φ' and is given by

$$\cos \Phi' = \frac{\cos \varphi_1 + h_3 k_3 \cos \varphi_3 + \cdots}{(1 + h_3^2 + \cdots)^{\frac{1}{2}} (\cos^2 \varphi_1 + k_3^2 \cos^2 \varphi_3 + \cdots)^{\frac{1}{2}}}.$$

Similarly, if the electromotive force e is resolved into an energy voltage e_w, which has its fundamental and all higher harmonics in the same phase as the fundamental and corresponding harmonics in the current, and a wattless voltage e_l, which has its fundamental and all higher harmonics in quadrature to the fundamental and harmonics, respectively, in the current wave, these component voltages are

$$e_w = E_m \cos \varphi_1 \left\{ \sin (\omega t - \varphi_1) + h_3 \frac{\cos \varphi_3}{\cos \varphi_1} \sin (3\omega t + \theta_3 - \varphi_3) + \cdots \right\},$$

and

$$e_l = E_m \sin \varphi_1 \left\{ \cos (\omega t - \varphi_1) + h_3 \frac{\sin \varphi_3}{\sin \varphi_1} \cos (3\omega t + \theta_3 - \varphi_3) + \cdots \right\},$$

whose effective values are E_w and E_l, respectively.

The equivalent phase difference between the current i and the wattless voltage e_t is 90 degrees; the phase difference between i and the energy voltage e_w is, however, generally not equal to zero, but is Φ'', which is given by

$$\cos \Phi'' = \frac{\cos \varphi_1 + h_3 k_3 \cos \varphi_3 + \cdots}{(\cos^2 \varphi_1 + h_3^2 \cos^2 \varphi_3 + \cdots)^{\frac{1}{2}}(1 + k_3^2 + \cdots)^{\frac{1}{2}}}.$$

The power W may then be expressed by either $EI_w \cos \Phi'$ or $E_w I \cos \Phi''$; hence

$$W = EIp = EI_w \cos \Phi' = E_w I \cos \Phi'',$$

therefore

$$p = \frac{I_w}{I} \cos \Phi' = \frac{E_w}{E} \cos \Phi''.$$

For circuits carrying non-harmonic alternating currents, therefore, the power-factor is generally equal to neither $\frac{\text{energy current}}{\text{total current}}$ nor $\frac{\text{energy voltage}}{\text{total current}}$, except in special cases, being equal to the former when $\Phi' = 0$ and equal to the latter when $\Phi'' = 0$.

The reactive-factor of a circuit is defined as "the ratio of the wattless volt-amperes (*i. e.*, the product of the wattless component of current by voltage, or wattless component of voltage by current) to the total volt-amperes."[1] If the electromotive force and the current are of sinusoidal wave form the wattless volt-amperes are equal to the total volt-amperes multiplied by the sine of the phase difference φ between the electromotive force and the current, so that the reactive-factor q is equal to $\sin \varphi$, and also be expressed by either $\frac{\text{wattless current}}{\text{total current}}$ or $\frac{\text{wattless voltage}}{\text{total voltage}}$.

But when the electromotive force and the current are non-harmonic the product of the wattless component of current by voltage, $I_t E$, is generally not equal to the product of the wattless component of voltage by current, IE_t. Hence the above definition given to the term "wattless volt-amperes" does not show completely what it is. The writer ventures to suggest the following definition for this term, or "wattless power" as it is sometimes called:

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[1] Standardization Rules, A. I. E. E., Art. 55.

The "wattless power" in a circuit is the power which would be given by the actual electromotive force in the circuit and a fictitious current having a fundamental and higher harmonics that are equal in amplitude to the fundamental and corresponding harmonics of the actual current but are in quadrature thereto, respectively, in phase; or, by the actual current in the circuit and a fictitious electromotive force having a fundamental and higher harmonics that are equal in amplitude to the fundamental and corresponding harmonics of the actual electromotive force but are in quadrature thereto, respectively, in phase.

If e and i are the electromotive force and the current in a circuit, as given above, the wattless power is the power given either by the electromotive force e and the fictitious current i', where

$$i' = I_m\{\cos(\omega t - \varphi_1) + k_3 \cos(3\omega t + \theta_3 - \varphi_3) + \cdots\},$$

or by the current i and the fictitious electromotive force e', where

$$e' = E_m\{\cos \omega t + h_3 \cos(3\omega t + \theta_3) + \cdots\}.$$

The equivalent phase difference between e and i' is equal to that between e' and i, and will be expressed by

$$\Psi = \cos^{-1} \frac{\sin\varphi_1 + h_3 k_3 \sin\varphi_3 + \cdots}{(1 + h_3^2 + \cdots)^{\frac{1}{2}}(1 + k_3^2 + \cdots)^{\frac{1}{2}}}. \tag{4}$$

Therefore, the wattless power W_l is

$$W_l = IE \cos \Psi,$$

and the reactive-factor q is

$$q = \cos \Psi.$$

The expression for reactive-factor might have been obtained by writing $\sin\varphi_1$, $\sin\varphi_3$, etc., for $\cos\varphi_1$, $\cos\varphi_3$, etc., in equation (1) for power-factor.

The wattless power W_l is generally equal to neither I_lE nor IE_l, and consequently the reactive-factor q is generally equal to neither

$$\frac{I_l}{I} = \frac{\text{wattless current}}{\text{total current}},$$

nor

$$\frac{E_l}{E} = \frac{\text{wattless voltage}}{\text{total voltage}}.$$

The value of $p^2 + q^2$ is always less than unity for non-harmonic alternating currents, except when the circuit has no reactance, in which case $p = 1$ and $q = 0$; but it becomes equal to unity for sinusoidal currents.

CONCLUSIONS.

1. The relative phase positions of two non-harmonic alternating quantities can be expressed by the term "equivalent phase difference," which is equal to the phase difference between two equivalent sinusoidal alternating quantities that give the same value of mean product of instantaneous values as the given non-harmonic alternating quantities.

2. Any non-harmonic alternating quantities can be represented by radius-vectors, if their relative phase positions are expressed by equivalent phase differences between each pair.

3. Generally, a vector diagram in n dimensions is required to show n independent non-harmonic alternating quantities simultaneously; but in special cases they can be shown in a figure of less dimensions.

4. Summation or subtraction of any number of non-harmonic alternating quantities can be performed graphically, by representing them by radius-vectors and combining first two of them in a plane diagram, combining their resultant with a third, etc.

5. The power-factor, or the ratio of the electric power in watts to the apparent power in volt-amperes in a circuit carrying non-harmonic alternating currents, cannot be expressed by the ratio of energy current to total current, nor by the ratio of energy voltage to total voltage, except in some special cases.

The reactive-factor of a circuit is the ratio of the "wattless power" in watts (as already defined) to the apparent power in volt-amperes. With simple harmonic alternating currents it can be expressed by the ratio of wattless current to total current, or by the ratio of wattless voltage to total voltage; but with non-harmonic alternating currents it can be expressed by neither of them, except in some special cases.

The sum of squares of power-factor and reactive-factor of a circuit is always less than unity with non-harmonic alternating currents. With simple harmonic alternating currents, it is equal to 1.

CORNELL UNIVERSITY, ITHACA, N. Y.

A THERMO-HYSTERETIC FREQUENCY METER AND THE APPLICATION OF DIFFERENTIAL THERMO-JUNCTIONS TO A.C.-D.C. COMPARISON.

By C. E. Hiatt.

THE great necessity of knowing the frequency of alternating and oscillating currents has resulted in the invention of numerous devices for frequency determination, but very few, if any, have proven wholly satisfactory. A careful examination of the scientific and technical literature has resulted in finding that only one attempt has been made to utilize as a measure of frequency the rate of heat production when cyclic reversals of magnetic flux are produced in iron, say, by an alternating magnetic field. Meyer[1] investigated the elongation of a steel wire stretched in the axis of a solenoid carrying an alternating current. Careful shielding was necessary and even then the calibration was difficult on account of eddy currents and Joule heat from the coil. The power consumption was also quite large. Guye and Herzfeld[2] might have applied the bolometer method they were using in the study of kinetic hysteresis to the determination of frequency, but they were apparently considering only the problem at hand.

It is the object of this paper to describe first an instrument which is free from the defects mentioned above, but which depends for its action on the rate at which heat is generated by the cyclic reversals of magnetic flux in a few milligrams of iron, say, when placed in a solenoid carrying the current whose frequency is desired. The necessity of empirical calibration has, however, not been eliminated.

A fine iron wire a few millimeters in length was soldered to a copper constantan junction electrically opposed to a similar one thermally and electrically equivalent but not containing iron. This is shown in the heavy line detail drawing of Fig. 1. The junction

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[1] Meyer, Elec. Rev., London, Feb. 19, 1897.

[2] Guye and Herzfeld, C. R., 136, p. 957, 1903.

J_2 contains iron while J_1 does not. So far this equivalence has been obtained only after repeated trials, after which both junctions were placed in a solenoid carrying a constant alternating current. A variation in the frequency of this current would cause a proportional variation in the temperature of the junction containing the iron, thus producing a thermo E.M.F. readily observed by a galvanometer. The use of the differential thermo-junction does away with the necessity of excessive shielding and eliminates the troubles which might arise from Joule heat generated in the coil. The slight disturbing effect which may be present as the result of eddy currents will be considered later.

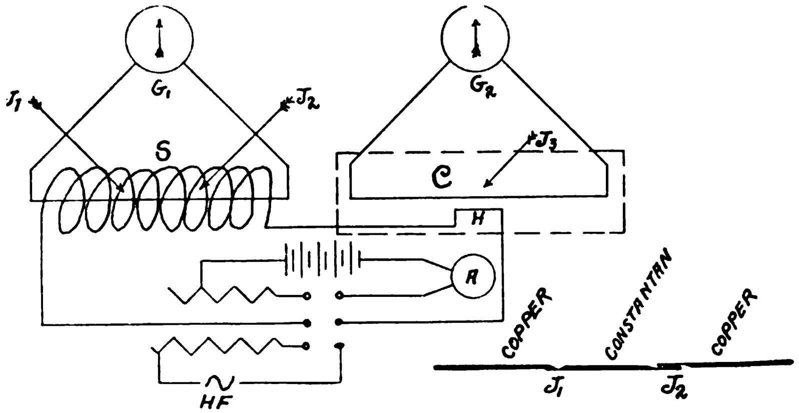

Fig. 1.

Rapidity of action is secured by making the heat capacity of the junctions small and their radiating power high. This, fortunately, is compatible with the practical requirement that the energy consumption of the measuring instrument be small. So small, in fact, is the energy necessary for the magnetization and demagnetization of such minute bits of metal that it seems wholly practical to use several differential thermo-junctions in series, thus permitting the use of a less sensitive galvanometer. For the sake of simplicity and because the galvanometer available for their investigation was sufficiently sensitive, only one set of differential junctions has been used at a time.

The arrangement of the apparatus is shown in Fig. 1. J_1 and J_2 are the opposed copper constantan junctions, J_2 containing the iron.

The wires used were no. 27 B. & S. gauge constantan and no. 32 B. & S. gauge copper, and were connected to a Leeds and Northrup galvanometer G_1, of the type "H.S.," resistance 129 ohms, sensibility 446 megohms. The length of the constantan between J_1 and J_2 was about 2.5 cm. The solenoid S was 4.4 cm. long and consisted of 662 turns of no. 24 B. & S. gauge copper wire having a resistance of 3.6 ohms. The alternating current through the solenoid was kept at a constant and known value by means of the regulating resistance and frequent comparison with a standard direct current as measured by the Weston ammeter, A. This was accomplished with the help of the comparator C, shown enclosed by the hatched line rectangle in Fig. 1. The comparator consisted of a strip of bismuth about 3.5 cm. long soldered end on to a similar one of antimony, forming a junction J_3 just over the heater H. The strips for 5 or 6 mm. on each side of the junction were made thin in order to keep down the heat capacity and permit speed in working. The heater was of manganin with a resistance of 1.8 ohms and extremely small self-inductance and capacity. The heater and junction were mounted on an insulator and wholly enclosed by a cast-iron dome weighing 40 pounds or more supported by a cast-iron plate over an inch in thickness. This arrangement shielded the instrument perfectly, in fact, much better than was necessary. S was also shielded to make sure that no unequal heating could exist from any cause other than radiation from the solenoid. In order to insure a thoroughly steady condition about the junctions, a direct current equal to the alternating current to be used in any determination was passed through the solenoid for some time before taking observations. This was found unnecessary with the balanced junctions and was generally done to examine the permanency of the balancing. The heater wire H was always in series with the solenoid as shown in the figure. Two wires for this purpose and the two from the junction J_3 to the galvanometer G_2 (also Leeds and Northrup, type "P," resistance 121 ohms, sensibility 106 megohms) were run through a cork in the top of the dome and sealed in with "universal wax." The performance of this simple A.C. instrument was most satisfactory as regards constancy and sensibility, but it possessed to a moderate degree the defect of all hot-wire instruments of requiring a few seconds to reach its final deflection. It readily permitted all currents used to be kept constant to within one tenth of one per cent.

The high frequency alternator, H.F. was of the inductor type with 16 teeth in the rotating disc and 5 teeth of equal cross-section on each of the poles at opposite ends of a diameter. The teeth were arranged in the arc of a circle concentric with the rotating disc. This disc was of cast iron, 5.5 in. in diameter and .5 in. thick. Between it and the stationary laminated pole pieces there was about .35 in. of clearance. Around each polar tooth there were fifty turns of no. 28 B. & S. gauge double silk-covered copper wire all connected in series. The disc was rotated by means of a pulley and belt connection to a three fourths horse-power motor run by storage cells. Two storage cells were used on the field. The inductor was, in general, similar to one made by Duddell,[1] but possessed the additional feature that the numerous pole pieces kept the reluctance of the magnetic circuit practically constant, a feature which cannot exist when "two chisel-shaped magnet poles" are placed on either side of a rotating notched disc. For suggesting this type of machine and for assistance in its design and construction I am indebted to Dr. R. H. Hough.

To find the particular grade of iron which would give the most heat per hysteretic cycle a number of similar test samples of equal weights were made up. Quite a collection of steel and iron sheets and wires having a wide range of chemical and physical properties were available. Two different samples would be selected and soldered to each of two equal and opposed junctions. When placed in the same alternating field as J_1 and J_2 in Fig. 1, the direction of the galvanometer deflection gave positive evidence as to which became the warmer. Of course it was necessary to have the pair of opposed junctions well balanced at the beginning. The reliability of this balancing could be tested by sending sufficient direct current through the solenoid to make it actually hot.

The speed of the alternator was measured with a Starrett speed indicator and stop watch, the shaft of the indicator being positively but flexibly connected to the alternator shaft by a short piece of high grade rubber tubing. The number of revolutions per second of the inductor's disc multiplied by the number of teeth it contains gives the frequency of the alternating current through the solenoid. In the following tables column *F* denotes this frequency, *D* corresponding deflections in centimeters of G_1 with the scale at .5 m. distance. Tables I., II. and III. are for 0.15, 0.10 and 0.05 amperes respectively, using a piece of the best iron obtainable, 0.8 cm. long, 0.02 cm. thick and 0.08 cm. wide.

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[1] Duddell, Proceedings of the Physical Society of London, April, 1905, Vol. XIX., p. 431.

I.		II.		III.	
F	*D*	*F*	*D*	*F*	*D*
400	1.70	333	0.70	440	0.40
471	2.20	400	0.93	666	0.61
513	2.61	500	1.26	770	0.70
615	3.11	588	1.55	909	0.80
727	4.06	678	1.80	1,066	0.98
808	4.80	833	2.40	1,230	1.12
899	5.69	941	2.91	1,290	1.20
1,066	7.63	1,116	3.97	1,350	1.31
1,290	9.85	1,230	4.40		
1,404	11.21	1,333	4.91		

These data are plotted in curves I., II. and III., Fig. 2.

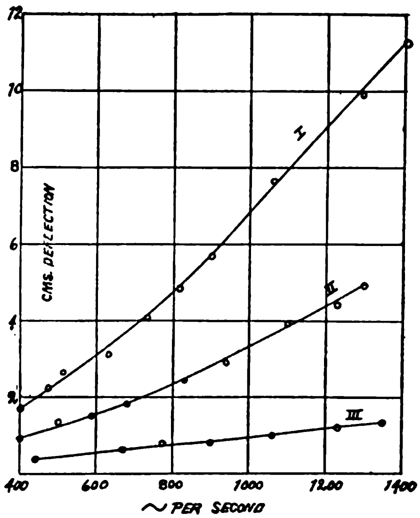

Fig. 2.

The data for Tables IV. and V. were obtained with a piece of tempered Swedish steel of the highest grade and of the same size as the iron specimen used for Tables I., II. and III. These tables are for 0.10 and 0.05 amperes respectively. To facilitate comparison curve II. for iron is repeated in Fig. 3.

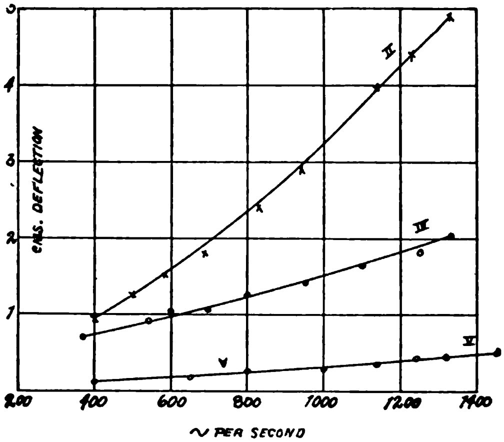

Fig. 3.

IV.		V.		IV.		V.	
F	*D*	*F*	*D*	*F*	*D*	*F*	*D*
374	0.71	408	0.10	952	1.41	1,143	0.33
540	0.92	645	0.18	1,143	1.65	1,210	0.39
597	1.01	727	0.20	1,250	1.82	1,240	0.42
656	1.06	800	0.24	1,333	2.08	1,324	0.45
798	1.28	1,000	0.29			1,450	0.50

A piece of music wire 0.027 cm. in diameter and about 0.8 cm. long gave the results for tables VI., VII. and VIII., all taken on different days and plotted, as *Δ*, *x* and *o* respectively in curve VI., Fig. 4. The current through the solenoid was 0.15 ampere.

Two nickel wires 0.014 cm. in diameter and 0.9 cm. long were twisted together and substituted for the iron and steel used in the preceding cases. The data for tables IX. and X. were taken using 0.15 ampere and 0.10 ampere respectively through the solenoid. Data were also obtained using 0.05 ampere but the deflections were quite small, though the variation with increase of frequency seemed

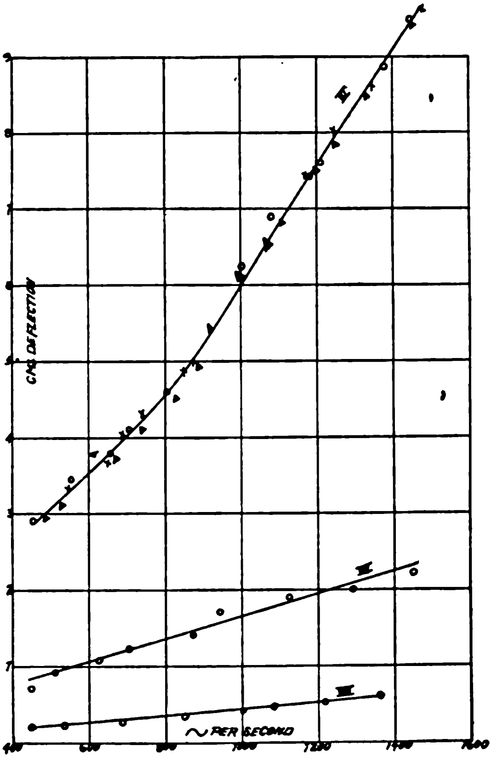

Fig. 4.

VI.		VII.		VIII.	
F	*D*	*F*	*D*	*F*	*D*
488	2.93	548	3.32	467	2.90
526	3.12	645	3.67	554	3.46
666	3.70	690	4.01	656	3.80
741	4.11	741	4.32	702	4.13
833	4.50	851	4.85	800	4.61
889	4.91	879	5.00	1,000	6.28
920	5.39	1,000	6.10	1,080	6.92
1,000	6.10	1,066	6.52	1,180	7.43
1,066	6.53	1,176	7.41	1,380	8.90
1,111	6.80	1,250	8.02	1,450	9.50
1,194	7.50	1,356	8.59		
1,250	7.85	1,481	9.60		
1,333	8.48				
1,450	9.48				

quite regular. Tables IX. and X. are plotted in curves VII. and VIII., Fig. 4.

The extremely high sensibility of this type of machine is readily seen from curves I., Fig. 2 and VI., Fig. 4. Taking curve I., for example, we have over 10 cm. deflection for 1,300 cycles per second. A decrease or increase in this frequency of .05 per cent. if it persists for a few seconds, is easily detectable, assuming the galvanometer at one meter distance. The number of revolutions of the inductor's shaft can of course be obtained with the same accuracy. By using a galvanometer of higher sensibility and more junctions of higher E.M.F. it seems quite probable that these sensibilities can be materially increased.

The flatness of curves III., IV. and V. indicate the desirability of keeping the magnetizing force as small as possible if a frequency meter having a uniform scale and a wide range is desired. It is quite possible that some magnetic alloy, or a combination of nickel and steel such as is used in Foley's[1] modification of the Marconi magnetic detector, can be found which will answer for even strong magnetizations. With the idea of low magnetization in mind an instrument has been constructed on the principle of the Boy's radio-micrometer. The bismuth and antimony strips at the lower end of the silver loop are soldered to a delicate vertical iron needle flush with their lower ends and extending upward about half their length. A rather long solenoid of a few turns of heavy silver wire extends for a few millimeters beyond either end of the bismuth and antimony strips. The small alternating current to magnetize the iron is sent through this solenoid. The instrument has proven sensitive to frequency, but no reliability can at present be attached to its indications. The chief troubles seem to be due to poorly-balanced junctions and to convection currents around the delicately suspended system.

The iron and steel curves for which the higher magnetizing forces are used indicate an increase in hysteretic loss per cycle as the frequency increases. Lloyd[2] in his discussion of the various methods used in the study of kinetic hysteresis shows the necessity of keeping the wave form constant. But even the gradual change in wave form which may have taken place in the current curve of the inductor used in this work will fail to account for the bending in curves obtained with the higher magnetizations. Furthermore, the maximum bending seems to occur always at frequencies between 600 and 1,000. It is also to be noted that the usual objection of insufficient regard for eddy currents can be urged less strongly in this case, inasmuch as both junctions are metallic and in the same alternating field, and while the induction in the iron is higher than in the other junction, yet for the high frequency alternations the impedance corresponding to a variation of flux with time would also be high for this same junction, thus giving a possible equality of eddy currents in the two opposed junctions for one frequency at least.

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[1] Foley, PHYS. REV., 1904, Vol. XVIII., p. 349.

[2] Lloyd, Bulletin of the Bureau of Standards, Vol. V., No. 3, pp. 381–411.

The superposition of rapidly alternating fields with oscillating fields of very high frequency has been tried with results so far in close agreement with those of Piola.[1] The writer expects to finish his work along this line, using widely different frequencies and magnetizations and report in the near future. Some typical results will now be submitted. A solenoid of 20 turns no. 18 B. & S. insulated copper wire 4.4 cm. long and 1.4 cm. in diameter was placed in the core of the fine wire solenoid previously described so that the two were concentric. The arrangement shown in Fig. 1 was disturbed only by the addition of this auxiliary solenoid and a source of oscillating current. With 0.28 ampere of oscillating current (frequency not definitely known, but very high) a deflection of 2.5 cm. was obtained from G_1. With 0.15 ampere alternating current, frequency 1,490, through the fine wire solenoid there was a deflection of 11.9 cm. With the two currents through their respective solenoids simultaneously and the frequencies unchanged, the deflection was 13.4 cm. For these results soft iron was used in the differential junction. With steel in the junction instead of iron, the sum of the deflections due to the two currents taken one at a time equalled the deflection when they were both acting as in the case above.

On account of the very satisfactory performance of the simple comparator shown in Fig. 1, the writer has devised a more elaborate and convenient instrument, using the principle of differential thermo-junctions. Fig. 5 is a diagrammatic view of the connections. J_1 and J_2 are delicate, similar and opposed junctions of antimony and bismuth. H_1 and H_2 are similar heaters of a single short length of manganin wire of 0.9 ohm resistance. Slow motion screws permit delicate variations of distance between the junctions and their respective heaters. The double-throw switch S_1 has two poles above and three below, S_2 is four-pole and double-throw. The two heaters and their corresponding junctions have a heavy shield which is not shown in the diagram.

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[1] Piola, Elettricista, Rome, 5, pp. 4-6, Jan. 1, 1906.

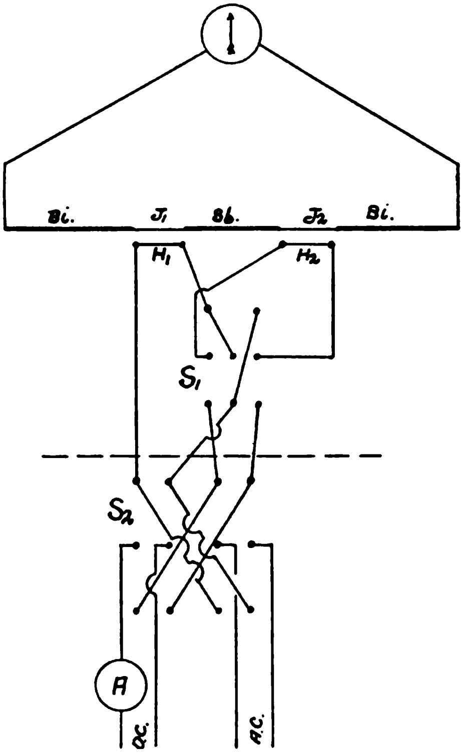

Fig. 5.

To compare an alternating current with a direct the very simple procedure is to put H_1 and H_2 in series by means of the switch S_1 thrown up. The current to be measured or a stronger one is then sent through the heaters by throwing S_2 down. The distance of H

and H_2 from the junctions are then adjusted until the galvanometer shows no deflection. Now since the same current through H_1 and H_2 causes no deflection of the galvanometer, equal currents through H_1 and H_2 will cause no deflection. Then by simply throwing S_1 down the alternating current is sent through H_1 and a direct current through H_2. When these two currents are equal the galvanometer shows no deflection. This equality is secured by adjusting the strength of the direct current. Then the current value is read from the standard ammeter A, or by means of a potentiometer and shunt if higher accuracy is desired.

The switch S_2 is shown only for the purpose of describing a reliability test. After obtaining a balance between a direct current of .5 ampere through H_2 and an alternating current through H_1 and noting that a variation of less than .1 per cent. in the measured current could be readily detected by the galvanometer, S_2 was thrown up in order to interchange the currents in the two heaters. Not the slightest motion of the galvanometer mirror could be observed. This test was for 60 cycles and gave a result which might have been predicted, for the capacities and self-inductance of the heaters are extremely small. In ordinary work not involving excessively high frequencies, S_2 may be dispensed with and, excepting the ammeter A, only the apparatus above the dotted line in Fig. 5 need be used.

The measurement of alternating voltages is quite as easy as that of currents when a non-inductive resistance is available. Two 150-volt Weston A.C.–D.C. voltmeters with a certified accuracy of .2 per cent. have been checked against each other and found to agree within .1 per cent. Currents as low as .5 ampere have been measured with an accuracy of .2 per cent., using the 0.9-ohm heaters.

This high sensibility combined with low resistance possesses obvious advantages over the relatively high resistance devices depending for sensibility upon the elongation of heated wires. From the published constants and sensibilities of our instrument makers it appears at once that to measure a given large alternating current with this new comparator the resistance of the manganin shunt can be decreased at least ten fold. Or, with a given amount of manganin with which to build a low resistance shunt, the current which can be measured has been increased by another order.

With the heater wires so short and so far apart, there is not the possibility of one circuit inductively influencing the other as in the case of instruments using a differential system of two long parallel wires stretched close together. Other serious objections which Thornton[1] has made to the use of parallel, stretched wire systems for comparators only need be mentioned as having no applicability to the differential thermo-junction method of A.C.–D.C. comparison.

It may finally be stated that the new instrument has the following useful features:

1. It is a zero instrument and completely under experimental control, involving as it does no dependence on calibration or instrumental constants.
2. It is extremely sensitive and as perfectly dead beat as the galvanometer with which it is used.
3. It has practically no self-inductance or capacity and is consequently independent of wave form and frequency, probably within very wide limits.
4. The two circuits are widely separated so that the inductive action of one upon the other is entirely negligible.
5. The resistance for high sensibility being quite low, it offers an accurate and inexpensive means for the measurement of very large currents for which the cost of low resistance shunts is an important consideration.
6. By the use of high resistance heaters its lower range can probably be extended to a milli-ampere at least.
7. It is quite simple and once assembled it is sturdy and easy to manipulate.
8. Its cost is trifling compared with that of the Kelvin balance or Siemens dynamometer for large currents.

The writer has under construction a differential thermo-galvanometer which this work has suggested, two opposed junctions in series being connected to the silver loop of the Boy's radio-micrometer. If necessary an insulating wall of mica can stand between the two junctions since the instrument is to be of the no deflection type.

I wish to thank Dr. A. W. Goodspeed for his continual interest and encouragement. The permission he has given me to draw freely upon the time of our mechanicians, Messrs. J. Sidle and F. Kalmbach, has removed many difficulties. I am also greatly indebted to Mr. J. Martin. The skill and appreciation with which he has attacked the mechanical details of the comparator helps to account for its addition to the instrumental equipment of the laboratory.

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[1] Thornton, Electrician, 55, p. 796, Sept. 1, 1905.

THE RANDAL MORGAN LABORATORY OF PHYSICS,
UNIVERSITY OF PENNSYLVANIA.

CRYSTAL AND SOLID CONTACT RECTIFIERS.

By Alan E. Flowers.

Contents.

1. Search for and Selection of a Rectifier of large capacity.
2. Characteristics of the Galena Rectifier: (*a*) *Volt Ampere Characteristics*, (*b*) *Effect of the material of the Point*, (*c*) *Effect of the shape of the Point and of the Ratio of Contact Areas*, (*d*) *Effect of Mechanical Pressure and Mutilation of Surface*, (*e*) *Effect of Variation of Frequency*, (*f*) *Effect of Electrical Discharges*, (*g*) *Effect of Heat.*
3. Artificial Production of Solid Contact Rectifiers by Chemical and Electrochemical Treatment.
4. Applications.
5. Discussion of the Results and Conclusions.

In the Physical Review for March, 1909, there appeared an account of tests made by G. W. Pierce on a number of rectifiers, consisting of points resting on crystals. The crystals used were anastase, brookite and molybdenite. Of these the molybdenite was by far the most sensitive and possessed the largest current carrying capacity. In the accounts given of these experiments, and in the accounts published elsewhere, no definite and final conclusions were drawn as to the cause of the phenomenon, though various possible causes were investigated. Wishing to determine if possible the cause of this action a series of experiments was carried out and a description of these experiments and their results follows.

1. Search for and Selection of a Rectifier of Large Capacity.

It seemed that the first and most important thing to do was to find a rectifier of large current carrying capacity, as the rectifiers previously described were capable of delivering, at most, but a few milliamperes of rectified current.

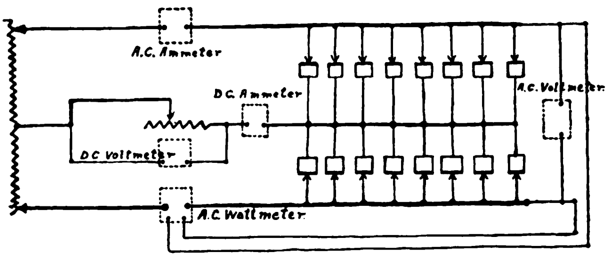
Believing that high resistance was not necessary, the crystals having high conductivity were selected for study. The sulphides appear to be particularly high in conductivity and iron, zinc and lead sulphides were tried for current carrying capacity and rectifying properties.

Of these, lead sulphide, galena, has by far the highest conductivity, and fortunately it showed in several samples not only large current carrying capacity, but very appreciable rectification. Single crystals showing sometimes 150 to 200 milliamperes of rectified direct current with a 3-volt (effective) alternating current supply. The crystals studied appeared to rectify about half the alternating current, though here it is well to emphasize the fact that different samples and different places on the same sample give widely varying results.

The samples of molybdenite examined required roughly a 21-volt effective alternating current supply to give 20 to 30 milliamperes of rectified direct current, and in such cases the alternating current ammeter indicated sometimes 500 or more milliamperes.

Believing that still larger capacities might be desired, two rectifiers were made by setting eight to ten samples of rectifying crystals in a little metal trough containing melted lead.[1] The lead on hardening made an absolutely certain contact with the body of the crystal and the metal trough constituted one electrode.

Pointed copper wires, in parallel, attached to a common return were set so that each touched a rectifying spot on one of the crystals. (It is not essential to have but *one* point in contact with any one crystal, for if the rectifying surface be large enough, as many additional points may be used as there is room for on the rectifying surface.) See Fig. 1.



RECTIFIER CONNECTIONS

Fig. 1.

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[1] Later I found that a similar method, using solder, had been used by L. W. Austin. Bulletin Bureau of Standards, Vol. 5, No. 1, p. 133, 1908.

These two rectifiers were connected in series-opposition across the secondary of a small transformer. One terminal of the direct current circuit was connected to the mid-point of a transformer and the other to the interconnection between the two rectifiers.

This rectifying set was left on the circuit all day without showing much change in its performance. A set of readings taken is shown in Table I.

From these results it was concluded that galena was the most promising crystal to study in order to find out the characteristics and cause of the rectification.

TABLE I.

Galena Crystal Rectifiers on Alternating Current. Frequency 60 Cycles.
Observer A. E. F. Rectifiers A_{1-8} and B_{1-9}. Date of Test, April 10, 1909.

Time.	Alternating Current Input.			Direct Current Output.			
	Total Volts.	Total Amperes.	Total Watts.	Volts.	Amperes.	Watts.	Efficiency.
3:12 P. M.	3.75	.555	1.75	.56	.493	.276	15.7
10:12 P. M.	3.82	.560	1.75	.56	.483	.270	15.4

Note that B_7 was disconnected.
See Fig. 1 for diagram of connections.

2. CHARACTERISTICS OF THE GALENA RECTIFIER.

In order to analyze the action of the crystal it seemed best to try the effects on a direct E.M.F.

(*a*) *Volt Ampere Characteristics.*— A large number of observations were taken on different crystals. Of these the set of readings given in Table II. and plotted Fig. 2 are characteristic.

Discussion of Volt Ampere Characteristic.— The values given in Table II. and shown in Fig. 2 are fairly representative. A large number of other runs were made which showed similar results. In some cases 4.2 volts would give a current of .850 or more amperes from crystal to copper and only .025 amperes in the opposite direction.

The most important thing about these characteristics is that they show a point at which breakdown occurs. This point occurs at a fairly definite *current* for any *given set* of conditions of *setting.* In general, however, each crystal and each setting will have its own breakdown point. This point may be 15 or more volts with 30 to 40 milliamperes flowing from point to crystal on fresh rectifying spots and may be as low as 5 volts and 90 milliamperes after several breakdowns. In general the higher breakdown voltages are reached when the current is small.

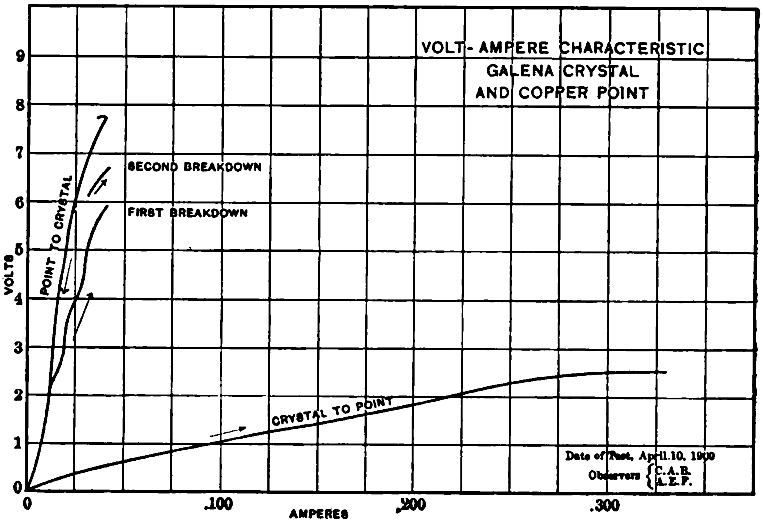


Fig. 2.

This breakdown point may occur at a higher voltage (and the same current) if the circuit be broken at the instant of partial failure and the crystal allowed to recover; while if the current is allowed to continue to flow after breakdown, partial or permanent destruction of the unidirectional conductivity or rectifying property will result.

The rectifying property is however not destroyed by the passage of considerable currents in the direction of crystal to point. In the case just cited 850 milliamperes did not destroy the rectifying property of the crystal.

Excessive current in *either* direction will break down and partially or permanently destroy the rectification, and breakdown occurs at a lower voltage when the current is reversed at each value of impressed voltage than it does when the voltage is simply increased in the direction of least current, *i. e.*, with the point as anode.

TABLE II.

Volt Ampere Characteristic of Galena Rectifier. Rectifier C, Copper Point Anode. Observers A. E. F. and C. A. B. Date of Test, April 10, 1909.

Time.	Volts.	Amperes.
2:35 P. M.	.824	.005
	1.492	.010
	2.230	.013
	2.500	.016
	2.704	.018
	3.060	.019
	3.288	.020
	3.804	.023
	3.900	.024
	4.008	.025
	4.292	.028
	4.472	.029
	4.780	.030
	4.960	.031
	5.180	.032
	5.308	.033
	5.468	.035
	5.940	.039
At this point the crystal resistance seemed to break down, allowing several hundred milliamperes to pass. The voltage was reduced till resealing occurred and then raised.		
	6.104	.031
	6.36	.034
	6.44	.035
	6.58	.037
	6.66	.039
	6.66	—
Breakdown occurred again and the voltage was reduced till resealing occurred and then raised again. The following readings were taken to prove that the passage of current in the previous runs had not spoiled the rectifying properties of the crystal.		
	1.92	.005
	5.80	.020
	6.36	.023
	7.17	.028
	7.48	.030
	7.64	.030
	7.78	.032
	7.93	.033
	7.98	.033
	0	0
Current was off for a short time while arranging for higher voltage.		
2:35 P. M.	6.98	.026
	7.58	.028
	7.86	.030
	8.36	.034
	8.57	.039
	8.96	.039 — Breakdown occurred at this point.
	6.64	.055
	4.16	— ?
Breakdown occurred again. Resealing began at this voltage.		
	7.72	.035
	7.76	.039
	7·72	.039
	7.38	.035
	6.90	.030
	5.69	.026
	5.39	.020
	3.36	.015
	1.83	.010
	.72	.005
Copper Point Kathode.		
	.16	.009
	.272	.018
	.32	.020
	.416	.030
	.60	.049
	.72	.052
	.944	.090
	1.12	.110
	1.28	.130
	1.408	.143
	1.68	.179
	1.792	.196
	1.872	.203
	2.065	.229
	2.25	.241
	2.275	.257
	2.41	.276
	2.52	.310
	2.56	.329

Breakdown seems to be due to the heat produced. When viewed under a high-power microscope the point can be seen to sink into the body of the crystal, the surface bending and cracking like thin ice over mud, and little globules of molten material ooze through the crevices.

The point leaves a hole in the surface visible to the naked eye and breakdown carried to this extent is permanent.

Even before the final breakdown is reached one can perceive partial breakdown occurring and immediate resealing, resulting often in a decreased current. This is shown admirably in the varying curvature of the volt-ampere characteristic from point to crystal in Fig. 2.

The fact that the reactions are taking place at the surface or in a surface film is shown strikingly by measuring the potential difference between the copper point and the surface of the crystal near the point. It is found that practically the whole E.M.F. impressed on the rectifier is required for the potential difference between the copper point and the crystal. For example, a drop of 5.95 volts was measured between the copper point and the nearest spot on the crystal surface when the total voltage impressed on the rectifier was 5.98 volts.

(*b*) *Effect of the Material of the Point.* — Having found a crystal that was capable of rectifying large currents the next thing done was to try to determine whether the effects observed were in any way dependent on the material of the point used. Volt-ampere characteristics were taken with points of copper, steel, platinum, lead, zinc, aluminum, brass, solder and graphite.

The characteristics were similar to the ones illustrated in Fig. 2 and Table II., and apparently the material of the point made no difference in the rectification. Small differences, however, such for instance as those which might be caused by variation in heat conductivity of the points, might easily be masked by the considerable variations in the volt-ampere characteristics of different crystals.

(*c*) *Effect of the Shape of the Point and the Ratio of Contact Areas.* — The rectification did not seem to be affected in any way by change of shape of point. The steel needle, point end, seemed to show the same results as the eye end and sharp and blunt ends of copper wire points seemed to give equally good results. Even the substitution of a globule of mercury about 3 mm. in diameter or of melted lead dropped on the crystal surface and allowed to harden in place so that it made intimate contact with a large area of the surface gave good rectification. It is unusual to find a large surface showing uniformly good rectification, and if any *part* of the globule touches a spot that does not rectify the effect of the rectifying surface is masked or lost.

For comparison a copper point was set on the rectifying surface near the globule of lead; the other contact to the crystal being made by a lead setting such as has already been described. The results of these tests are given in Table III.

TABLE III.

Comparison of Point Contact and Large Area of Contact on Rectification.
Observer A. E. F. Date of Test April 30, 1909.

Volts. Crystal to Point.	Amperes.	Resistance, Ohms.	Ratio of Resistance.	Remarks.
— 4.00	.003	1333		
+ 4.00	.190	21.05	.01577	Copper point under very light pressure.
— 4.00	.012	333		
+ 4.00	.192	20.8	.0625	
— 1.00	.023	43.5		Connection made to globule of lead.
+ 1.00	.186	5.38	.01236	
— 1.00	.049	20.2		Readings taken after several reversals.
+ 1.00	.216	4.63	.229	
— 2.00	.100	20		Current decreasing.
+ 2.00	.600	3.33	.1663	Current increasing.
— 3.00	.210	14.28		Current increasing.
+ 3.00	.980	3.06	.214	Current increasing. Breakdown finally.
— 1.00	.044	22.7		Contact made with crystal by a globule of mercury.
+ 1.00	.118	8.57	.373	
— 1.00	.039	25.6		Mercury globule contact on rectifying surface and copper wire point on non-rectifying crystal for the other connection.
+ 1.00	.088	11.36	.443	

The use of a large area of contact to the rectifying surface of the crystal does not destroy the rectification.

When the lead or mercury globule was used to make connection to the rectifying surface the substitution of a copper wire point in contact with the non-rectifying surface of the crystal instead of the usual lead setting had no very great effect on the rectification.

Finally a crystal was selected which showed good rectifying properties. This was mounted in a lead setting and the crystal cut down to a point. This pointed crystal was then set in contact with a large block of metal and also in contact with another crystal and it was found that rectification was produced as before.

The results of this test are given in Table IV.

Table IV.

Effect of Cutting the Rectifying Crystal Down to a Point. Contact Pressure 200 gms. Observer A. E. F. Date of Test April 25, 1909.

P. D. Volts.	Current, Amperes.	Resistance Ohms.		Ratio of Resistances.	Remarks.
		To Crystal.	From Crystal.		
−4.00	.010	400			Crystal *E*, touching a steel hack saw blade.
+4.00	.066		60.6	.151	
−4.00	.010	400	100		Crystal *E*, touching a zinc plate.
+4.00	.040			.25	
−4.00	.020	200			Crystal *E*, touching lead plate.
+4.00	.105		38.1	.191	
−4.00	.011	363.5			Crystal *E*, touching brass plate.
+4.00	.091		44.0	.1209	
+4.00	.004	1,000			Crystal *E*, touching a rectifying galena surface.
−4.00	.004		1,000	1	
−4.00	.012	333		.0333	Crystal *E*, touching a non-rectifying galena surface.
+4.00	.360+		11.1		
At this voltage the current continues to increase till breakdown occurs.					
+4.00	.074		54.1		Crystal *E*, touching aluminum plate.
−4.00	.011	363.5		.1486	
+4.00	.070		57.2	.1715	Crystal *E*, touching brass plate.
−4.00	.012	333			
−6.35	.030	212		—	
−6.18	.074	83.5	—		
At this point breakdown is beginning.					
−5.3	.590				
Breakdown is complete.					

(*d*) *Effect of Mechanical Pressure and Mutilation of Surface.*— When a good rectifying surface has been found the copper point may be pressed against the rectifying surface with very considerable force without impairing the rectification and with a great increase in conductivity (in both directions). However, if the pressure is high and the voltage is carried to the breakdown point the breakdown of the spot touched is likely to be permanent. The point sinks into the crystal, or perhaps a hole is melted in the crystal and the point sinks into this hole.

It was found that a moderate pressure (about 200 grams) was sufficient to give steady contact and in most cases this was the pressure used.

A crystal having a rectifying surface was often found to have other rectifying surfaces underneath and parallel when the layers were split off, but scratching or scarring a rectifying surface usually spoiled more or less completely its rectifying properties.

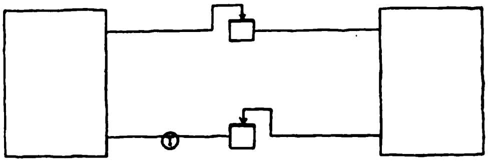
(*e*) *Effects of Frequency.*— No attempt has as yet been made to determine a relation between per cent. rectification and frequency; however, the following significant tests bearing in a general quantitative way on this were carried out. A rectifier consisting of a copper point and galena crystal was connected in series with a fairly sensitive galvanometer and a telephone receiver. Speaking into the receiver caused large deflections of the galvanometer and loud continued vowel sounds would throw the galvanometer off scale. The sensitiveness of this galvanometer is such that $.0286 \times 10^{-6}$ ampere gives 1 mm. deflection, so that the rectified current must have exceeded 7 microamperes.

The frequency of telephone currents is of the order of 1,000 cycles per second.

Another arrangement of the rectifier was tried as a receiver for the electric waves produced by a static machine. The diagram of the connections is given in Fig. 3.

This arrangement gave galvanometer deflections of 3 to 4 cm. corresponding to a current of the order of a microampere with very high frequencies. Similar effects were obtained when using an induction coil as a source for the electric waves.

When using the galena crystal rectifier with a Tesla oscillator large galvanometer deflections were obtained, but here a different type of rectifying effect also occurs between the point and any large conducting surface, such as the lead setting, a piece of brass, etc. That the type of rectification obtained with the Tesla oscillator is entirely different from the type of rectification here studied is made more clear by the results of the tests on the effect of electrical discharges through gases. This type of rectification appears to be similar to the effects observed by



RECTIFIER AS WIRELESS RECEIVER
Fig. 3.

Tamm, Drude's Annalen, VI., p. 259, 1901,
Precht, Wiedemann's Annalen, XLIX., p. 150, 1903,
Himstedt, Wiedemann's Annalen, LXVIII., p. 294, 1899,

which show differences in the minimum voltage for discharge and in the conductivity of the gas between a point and a metal plate. The effects may be due to rectification at the point of connection to the oscillator where both brush discharge and sparking may be occurring and not at all at the point and crystal.

(*f*) *Effect of Electrical Discharges.* — A crystal, D_5, was mounted and connected to a storage battery with meters to measure the voltage impressed on the rectifier and the current flowing from the battery.

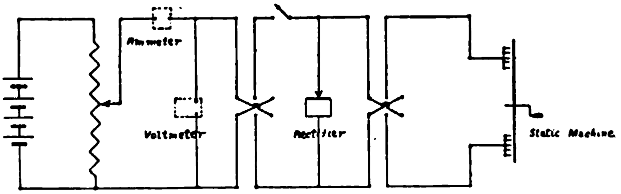


Fig. 4.

A static machine was then so connected that the static discharge could be sent in either direction through the rectifier as illustrated in Fig. 4.

It was found that the static discharge could be passed continuously in either direction through the rectifier, whether the rectifier were connected to or disconnected from the battery, without affecting the conductivity of the rectifier or its rectifying properties. However, it was found that if the static discharge produced a spark anywhere in its path the effect for any connection of battery or static discharge is to increase the conductivity in either direction and to spoil the rectifying property.

(*g*) *Effect of Heat.* — The determination of the effect of heat is complicated by the fact that the current that flows when the determination is being made may cause local heating and so intensify and mask the general effect. There may be also electro-chemical effects.

In general, however, breakdown occurs at a lower voltage with an increase of temperature and the conductivity rises with increase of temperature, but the increase is greater for the direction of least conductivity so that the per cent. of rectification decreases with increase of temperature.

The rectification disappears at a temperature of about 270° C., but is partially regained on cooling, though the parts of the crystal which carried no current are apparently not affected by heating to 270 or 280° C. under oil and then cooling. The effect of the passage of current through the rectifier while it was cooling is given in Table V.

Table V.

Effect of Continued Passage of Current on Rectifier while Cooling.

Volts.	Amperes.	Resistance.		Ratio of Resistances.
Cooling without current.				
—1	.039	25.68		
+1	.074		13.52	.527
Cooling with current flowing from point to crystal.				
—1	.014	71.5		
+1	.051		19.61	.2745
Cooling with current flowing from crystal to point.				
+1	.013		77	.539
—1	.007	142.9		
+1	.020		50	
—1	.008	125		.400

It is worth while noting also that even at 280° C. there is a tendency for the current, if from point to crystal to decrease, and if from crystal to point to increase, if the current is allowed to flow.

It is especially significant that the rectifying property is not entirely destroyed by the continued passage of current in either direction while cooling under oil from this temperature though the rectification is best when the current is allowed to flow from point to crystal.

Heating the rectifier in air in a closed vessel gave similar results up to 270° C., but at slightly higher temperature, 300–320, the difference in conductivity was regained. Here it often appeared that the difference in conductivity was not obtained till an appreciable time had elapsed. At 345° C. the difference in conductivity again decreased, and on cooling there was a continued decrease in conductivity in both directions till the current flowing at 1 volt was less than a milliampere. Parts of the crystal not carrying current were not affected.

After the passage of current in either direction the rectifier if disconnected quickly and connected to a galvanometer, showed a feeble E.M.F. always in the same direction and this direction was always opposite to that required for the difference in conductivity. This appeared to be due to a thermo-electric E.M.F. in the same direction as that which was produced when the *crystal* was gently heated by a flame.

3. ARTIFICIAL PRODUCTION OF SOLID CONTACT RECTIFIERS BY CHEMICAL AND ELECTRO-CHEMICAL TREATMENT.

An attempt was made to produce rectification in samples of galena that did not rectify naturally, by subjecting the crystal to the action of various chemicals. Water and the conducting solutions and acids tried (*i. e.*, nitric acid, alcohol, potassium bichromate), if put on the crystal while the crystal was carrying current, usually increased the conductivity, caused breakdown and spoiled rectification. Although such results did not occur when the chemical treatment was not coincident with passage of current. The non-conducting acids and solutions tried (*i. e.*, conc. H_2SO_4, CS_2 and oils) seemed to have little effect.

The copper point was then covered with burning sulphur, the flame blown out and the remaining sulphur cooled quickly so that it was left in the amorphous state. A point so treated when set on a galena crystal surface that previously showed no rectifying properties would at first show little conductivity, but the passage of current first in one direction and then in the other produced in a few minutes a very fair degree of rectification *but in the opposite direction to that found in native crystals.*

A set of values is given in Table VI.

Table VI.

Rectifier Produced Artificially by Electro-chemical Treatment of Copper Point with Amorphous Sulphur.

Potential Difference, Crystal to Point.	Current.	Resistance.		Ratio of Resistances.
		Point to Crystal.	Crystal to Point.	
−1.00	.172	5.88	—	.209
+1.00	.036	—	27.80	

The treatment of a crystal that previously possessed no rectifying properties with amorphous sulphur and then with current, produced rectifying properties similar in all respects to the effect obtained when the copper point was treated.

The effect of the electro-chemical treatment with amorphous sulphur of the point set on metallic lead or the treatment of the lead surface itself was much more marked. Table VII. gives the results obtained with this rectifier.

It is worthy of note that this contact rectifier showed itself far superior in rectifying power to any of the crystal rectifiers so far tried, its resistance in one direction being .3 to .4 per cent. of the resistance in the other direction. Its breakdown point was reasonably high and it showed at one point the partial breakdown and immediate resealing or self-recovery found with the crystal rectifier.

The copper point of a rectifier produced in this way (by electro-chemical sulphur treatment while the point was in contact with a piece of lead) could be removed and set on a block of brass or other metal and be made to exhibit rectifying properties, but the rectifying film had to be produced first in contact with lead.

TABLE VII.

Copper-point Lead-block Contact Rectifier Produced Artificially by Sulphur Treatment.

Potential Difference, Lead to Copper. Volts.	Current, Amperes.	Resistance.		Ratio of Resistances.
		Copper to Lead. Ohms.	Lead to Copper. Ohms.	
−1.00	.002	500		
+ .62	.312		1.986	.00397
−1.00	.002	500		
+ .60	.404		1.485	.00297
−2.00	.003			
−3.00	.004			
−4.00	.022			
−5.00	.004			
−6.00	.008			
−7.00	.009			
−8.00	.010			
Breakdown.				

Attempts to produce the rectifying film by electro-chemical treatment of a copper point with sulphur while in contact with brass, failed to produce definite results, though good results were obtained by setting the copper point, after being treated with sulphur, on a small block of graphite.

4. APPLICATIONS.

Rectifiers of this type, when perfected, might be used in measuring telephone currents, and sound intensities, in detecting feeble electric waves and in charging small storage batteries.

5. DISCUSSION OF THE RESULTS AND CONCLUSIONS.

It is worth while noting that other workers in this field have emphasized the thermo-electric potential difference of the two materials of the contact rectifier or have emphasized the difference in heat and electric conductivity of the two materials.

The rectification cannot be due to thermo-electric forces, since they have neither the magnitude nor the direction necessary. The rectification cannot be due to difference in conductivity of the two materials in contact, because graphite, which has a higher specific resistance than the lead sulphide, could be used for a contact making point. There are only two possible explanations: One is that the film, when once formed, has unidirectional conductivity. This seems improbable in solids. The other explanation is that the film is produced in an infinitesimally short length of time by the passage of a minute quantity of electricity.

The tendency of the resistance to rise when current is flowing to the crystal, the direction of highest resistance, and to fall when the current is flowing from the crystal, seems to give color to this view. The sluggishness of change of resistance sometimes observed at high temperatures and with some crystals also tends to strengthen the probability that the rectifying film is re-formed electro-chemically with each reversal.

If the film is re-formed at each reversal, then there must be a definite quantity of electricity, however small, required for the production of the film, and one should find that the rectification is less perfect for very small currents or for very high frequencies. The facilities for determining this effect are not now at my disposal, but I trust to see this point settled in the near future.

The results of the tests described seem to justify the following conclusions:

1. The rectifying action is produced at the surface, or in a surface film, and is not due to the point employed.

2. The rectification is not an effect of the sharpness of the point or the ratio of contact areas.

3. The rectifying film will break down at a moderate voltage (6 to 15 volts) which is higher the smaller the current.

4. The rectifying film can be destroyed by the action of the electric current.

5. The feeble evanescent E.M.F. existing after the passage of current in either direction through the rectifier is insufficient to account for the rectification, and its direction is opposite to that which would be required for rectification.

6. There is apparently no difficulty in rectifying currents of very high frequency.

7. The rectification disappears at high temperature, *i. e.*, about 270° C., but is partially regained on cooling.

8. The rectifying film can be produced artificially by *electro-chemical* treatment with amorphous sulphur.

9. The rectification is probably due to the electro-chemical formation of a resisting film at each reversal.

It is a pleasure to acknowledge here the assistance rendered in various ways by a number of my colleagues.

UNIVERSITY OF MISSOURI,
June 22, 1909.

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ABSORPTION OF GASES BY THE ANODE IN A GLOW CURRENT.

By V. L. Chrisler.

THE first quantitative measurements on the absorption of gases under a glow current appear to have been made by Mey[1] who used a sodium-potassium alloy as cathode for purifying helium. Riecke[2] also passed a current for several hours through an atmosphere of nitrogen and measured the decrease in gas pressure. Neither of these observers found any electrolytic relation between the rate of absorption and the electric current. Skinner,[3] however, found later that most fresh metals when used as cathode with a glow current in helium or argon gave off hydrogen for a few minutes at the rate required by Faraday's law for electrolytes, while in hydrogen the pressure remained practically constant. He explained the absence of an increase in pressure in hydrogen by assuming that the cathode gave off gas at the same rate in hydrogen as in helium but that the anode absorbed it for a time at an equal rate. From experiments performed in a nitrogen atmosphere[4] he concluded that it also was absorbed by the anode for a time at the same rate as hydrogen.

The object of the present investigation was to obtain by a direct test the rate of absorption of these gases by the anode. The plan followed was to use a cathode which had previously been sufficiently freed of its content of gas so that it contributed nothing appreciable to the change in pressure. Then the effect of absorption at the anode could be directly obtained from the decrease in the pressure of the gas in the system.[5]

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[1] K. Mey, Ann. d. Phys., Vol. 11, p. 127.

[2] E. Riecke, Ann. d. Phys., Vol. 15, p. 1003.

[3] C. A. Skinner, Phys. Rev., Vol. 21, p. 1; Phys. Zeitschr., 6, p. 610; Phil. Mag., Vol. 12, p. 481.

[4] C. A. Skinner, Phys. Rev., Vol. 21, p. 169.

[5] This plan had already been tried in this laboratory by Mr. Kiesselbach, who used several of the common metals with a limited degree of success. He found that they absorbed hydrogen from one to two minutes at the rate required by Faraday's law.

After first testing silver as anode, which was found to absorb hydrogen from two to four minutes at about the calculated rate, the alkali metals, sodium, potasssium, and a sodium-potassium alloy, were tried with marked success in a hydrogen and also in a nitrogen atmosphere. Finally mercury was tested, giving positive results in hydrogen but no absorption in nitrogen. In a few isolated cases out of a large number of tests helium was also absorbed by a sodium and by a mercury anode at double the rate (by volume) that hydrogen is under the same current. In most of the tests, however, practically no absorption of helium was obtained.

APPARATUS.

A battery of small storage cells served for producing the glow current, which was regulated by a cadmium iodide and amyl alcohol resistance, and measured by a Weston milliammeter. The gases used were hydrogen, nitrogen and helium. Hydrogen, obtained from aluminium in a solution of potassium hydrate, was, after careful drying by phosphorus pentoxide, stored in a flask which had stood evacuated for a time sufficient to free it of occluded gases. Nitrogen was obtained by passing air through phosphorus vapor, through a wash bottle containing a potassium hydrate solution, and finally through a tube containing phosphorus pentoxide. After drying, it was also stored in a flask freed of occluded gases. The helium used was obtained in sealed glass bulbs from Messrs. Thomas Tyrer & Co., London, from which it was transferred to another provided with a stop-cock, through which connection with the system was afforded.

In the vacuum system all ground joints and stop-cocks were lubricated with a mixture free of hydrocarbons. Phosphorus pentoxide was used as a drier. Evacuation was produced by a Barr and Stroud mercury pump. The pressure was measured by a McLeod gauge — one millimeter on the scale representing .01 mm. mercury pressure. The probable error in any measurement was not greater than .005 mm. although a maximum error of .01 mm. might be made.

EXPERIMENTAL RESULTS.

Silver as Anode. — The form of tube used for testing a silver anode in hydrogen is shown in Fig. 1. It was of glass provided with the main electrodes *A* (the anode of silver) and *C* (the cathode of nickel), and an auxiliary electrode *B* of aluminium wire. The cathode was chosen of nickel as this metal was found to give off as cathode less gas than the others. Both electrodes *A* and *C* were made removable and were always carefully polished before mounting for a test. These preparations having been made the tube was evacuated and helium, to a pressure of about two millimeters of mercury, admitted. A very strong glow current was then passed simultaneously from *A* and *C* as cathodes to *B* as anode until the increase in gas pressure ceased. This cleaning current was made sufficiently intense to drive off the outer surface of the silver thus leaving a fresh surface which had been unexposed to other than the helium atmosphere. The success of all tests depended upon this treatment.

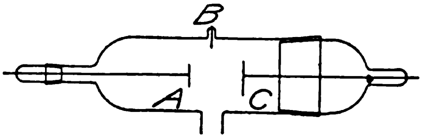

Fig. 1.

Having cleaned the electrodes in this manner the gas was quickly pumped out and hydrogen admitted. There was with all the metals tested a small gradual decrease in pressure of this gas for a time after admission arising from the absorption by the clean electrode and the metal deposited on the walls of the tube.[1] The nickel electrode, however, did not appear from the tests to absorb any of this gas. When the pressure became constant the electric current was started, with the nickel as cathode and the silver as anode. The change in gas pressure with time, under a current of about one milliampere, for three different tests is given in Table I. and plotted in Fig. 4. In the plot the straight line represents the decrement in gas pressure calculated from the electrochemical equivalent of hydrogen (104×10^{-7} gr. per coulomb), its density at room temperature and 76 cm. pressure (83×10^{-6}), and the volume of the tube (382 c.c.). It is to be noticed that for two minutes after the current started the decrease in pressure approximated closely the calculated value, from which it is to be concluded that with each atom of hydrogen absorbed is associated a negative charge of the same magnitude as that carried in solutions. When the surface of the metal becomes saturated with the gas it is to be expected that the absorption will stop, or, as suggested by Skinner (*l. c.*), that uncharged gas will leave the metal as fast as the charged gas enters it. In the case of silver this condition was reached in a short time and absorption ceased abruptly.

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[1] See W. Heald, PHYS. REV., Vol. 24, p. 269.

Soda Glass as Anode. — The investigation was next directed to sodium with the hope of obtaining absorption of longer duration. It was introduced into the tube first by electrolysis of the glass according to Warburg's[1] method. The tube used (shown in Fig. 2) was made of soda glass and furnished with a nickel electrode C which in the tests served as cathode, and an auxiliary electrode B. The end of the tube was immersed in a cup containing sodium amalgam through which the tube was heated by a Bunsen flame to a temperature such that the current could be passed through the glass and from it as anode through the gas to the cathode. Before the test the cathode was freed of gas by passing a current from B as anode. To obtain first a deposit of sodium inside the tube a glow current was passed through the glass and a helium atmosphere for a time before testing for the absorption of hydrogen. Having obtained the desired deposit the helium was removed and hydrogen admitted. The results from the different tests were very inconsistent, and, since the temperature was sufficient to deposit sodium (by vaporization) over the inner walls of the tube and on the other electrodes, the suggestion was that the natural absorption (or evolution of gas) by this deposit masked the effect of absorption by the anode.

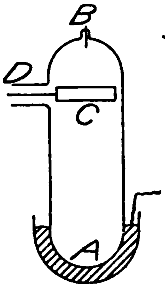

Fig. 2.

Soda Glass as Cathode. — An interesting thing was noticed in connection with the tests of the conducting glass. When it was used as cathode in helium the gas pressure increased at a rate about equal to that calculated on the assumption that hydrogen was the carrier of the current from the cathode. The spectrum, however, indicated that the gas freed from the glass was nitrogen. Representative observations of the increase in gas pressure with time are given in Table II. Like those obtained by Skinner[1] with a carbon electrode these results indicate that a negative electron is associated with each atom of nitrogen leaving the glass cathode. This is consistent with the fact that the chemical valence of nitrogen is in some cases unity. On reversing the current the gas which had been freed was reabsorbed (presumably by the glass), and on two occasions practically all of the helium atmosphere was absorbed with it.

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[1] E. Warburg, Ann. d. Phys., Vol. 40, p. 1.

Sodium Anode in Hydrogen. — To test the absorption of sodium at lower temperatures than the electrolytic method of introduction allowed, metallic sodium was introduced directly into the tube, the form of which is shown in Fig. 3. The sodium was first placed in the bulb *D* and the end of this side tube sealed. During continuous operation of the evacuating pump the sodium was then

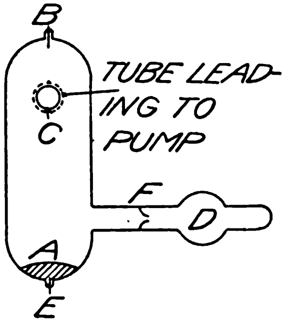

Fig. 3.

TABLE I.

Silver Anode in Hydrogen. Volume 382 *c.c.*

Duration Current (Min.).	Current 1 ma. Initial Pressure 2.04 mm.		Current 1 ma. Initial Pressure 1.7 mm.		Current .9 ma. Initial Pressure .94 mm.	
	Pressure Decrement.	Calculated Decrement.	Pressure Decrement.	Calculated Decrement.	Pressure Decrement.	Calculated Decrement.
1	.02	.015	.015	.015	.01	.0135
2	.035	.03	.025	.03	.02	.026
3	.035	.045	.025	.045	.03	.040
4	.04	.06	.02	.06	.04	.054
5	.045	.075			.03	.067
6	.05	.09				

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[1] C. A. Skinner, Phys. Rev., Vol. 21, p. 168.

TABLE II.

Glass Cathode in Helium.

Duration Current (Min.).	Current 1 ma. Volume 277 c.c. Initial Pressure 2.21 mm.		Current 1 ma. Volume 304 c.c. Initial Pressure 2.245 mm.	
	Pressure Increment.	Calculated Increment.	Pressure Increment.	Calculated Increment.
1	.02	.0207	.025	.019
2	.04	.041	.045	.038
3	.06	.062	.065	.057
4	.07	.083	.075	.076
5	.08	.104	.085	.096
6			.09	.113
7			.095	.132

TABLE III.

Sodium Anode in Hydrogen.

Duration Current (Min.).	Current 1 ma. Volume 263 c.c. Initial Pressure 1.84 mm.		Current .5 ma. Volume 263 c.c. Initial Pressure 2.165 mm.		Current 1.5 ma. Volume 268 c.c. Initial Pressure 1.6 mm.	
	Pressure Decrement.	Calculated Decrement.	Pressure Decrement.	Calculated Decrement.	Pressure Decrement.	Calculated Decrement.
1	.025	.022	.02	.011	.03	.032
2	.05	.043	.03	.021	.055	.064
3	.07	.065	.04	.032	.08	.096
4	.095	.085	.045	.043	.11	.128
5	.115	.109	.05	.053	.13	.160
6	.14	.131	.06	.065	.16	.192
7	.16	.151	.07	.075	.18	.224
8	.18	.174	.08	.086	.21	.256
9	.205	.196	.085	.096	.23	.288
10	.225	.218	.095	.107	.25	.320
11	.25	.240	.01	.118	.27	.352
12	.27	.262	.11	.128	.29	.384
13	.29	.283	.11	.139	.31	.416
14	.31	.305			.33	.448
15	.33	.327			.34	.480
16	.345	.349				
17	.36	.371				
18	.38	.392				
19	.39	.414				
20	.41	.436				
21	.43	.458				
22	.45	.479				
23	.46	.501				
24	.48	.523				
25	.49	.545				
26	.50	.567				

melted and allowed to flow through the trap *F* into the bottom of the tube at *A* where electric connection with it was made by a platinum wire sealed in at *E*. Preliminary to testing for the absorption of hydrogen the sodium and nickel *C* were freed of gas as before by using them as cathodes with a very strong glow current in helium.

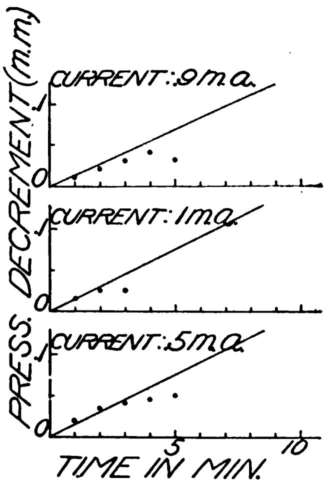

Fig. 4.

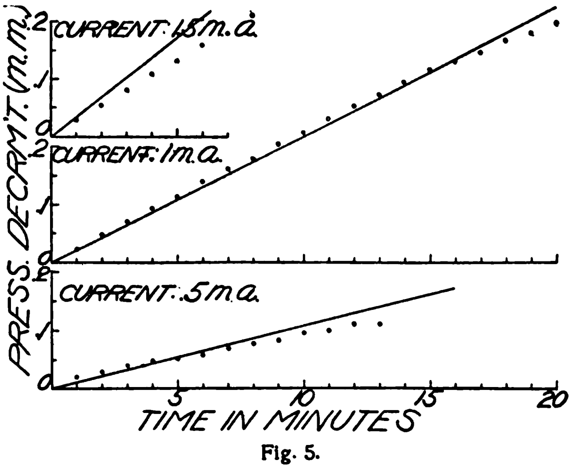

Fig. 5.

In Table III. and Fig. 5 is given, for the sodium as anode and nickel as cathode, the decrease in pressure of a hydrogen atmosphere with duration of current. Since absorption continued much longer than with a silver anode the initial pressure of the gas was taken at the end of the first minute to eliminate the effect of a change in temperature and the excess of initial absorption caused by starting the current. With .5 milliampere the absorption agreed with the calculated value for about ten minutes but stopped completely at the end of twelve minutes. With 1 milliampere it followed the calculated value for 15 minutes and continued slowly even after twenty-six minutes. With 1.5 milliamperes absorption according to the calculated rate lasted for only two minutes although it continued to absorb slowly for fifteen. The short duration in the last case is ascribed to a less favorable condition of the metal surface. It was noticed that during the absorption the surface of

Table IV.

Potassium Anode in Hydrogen. Volume 276 c.c.

Duration Current (Min.).	Current 1 ma. Initial Pressure 1.67 mm.		Current .5 ma. Initial Pressure 1.985 mm.	
	Pressure Decrement.	Calculated Decrement.	Pressure Decrement.	Calculated Decrement.
1	.02	.021	.01	.0104
2	.04	.042	.02	.021
3	.06	.062	.035	.031
4	.075	.083	.045	.042
5	.095	.104	.055	.052
6	.115	.125	.065	.062
7	.135	.146	.08	.073
8	.155	.166	.09	.083
9	.170	.187	.10	.094
10	.19	.208	.11	.104
11	.21	.229	.125	.114
12	.22	.250	.135	.125
13			.145	.135
14			.155	.146
15			.165	.156
16			.175	.166
17			.185	.177
18			.200	.187
19			.21	.198
20			.22	.208
21			.225	.218
22			.235	.229
23			.235	.248

the sodium was blackened, indicating that sodium hydride was being formed. By using the sodium as cathode in a helium atmosphere it was found that this surface was almost entirely cleaned off under a strong current, and the disappearance accompanied by an increase in gas pressure slightly greater than the amount of hydrogen previously absorbed.

Potassium Anode in Hydrogen. — The form of tube used with the potassium, and the treatment of the metal, were the same as with sodium. The results of tests made under similar conditions are given in Table IV. and plotted in Fig. 6. Of these the .5 milliampere gave the best results. The absorption followed the calculated value for twenty-two minutes and then suddenly stopped. With 1 milliampere absorption lasted for twelve minutes slightly below the calculated value. The test with 1.5 milliamperes gave no absorption, arising undoubtedly from an unfavorable condition of the surface of the metal.

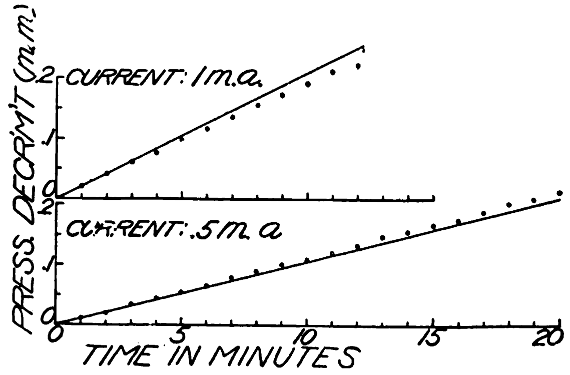

Fig. 6.

Sodium-Potassium Anode in Hydrogen. — Sodium and potassium alloyed in atomic ratio was tested in the same form of tube and treated in the same manner as the two preceding metals. The results are given in Table V. and plotted in Fig. 7. With .5 milliampere the absorption followed closely the calcutated value for eight minutes, then suddenly stopped. With 1 milliampere it lasted about eleven minutes at a rate slightly below that calculated, while with 1.5 milliamperes agreement with the calculated value only lasted for about three minutes.

TABLE V.

Sodium-Potassium Anode in Hydrogen. Volume 273 c.c.

Duration Current (Min.).	Current 1 ma. Initial Pressure 2.375 mm.		Current .5 ma. Initial Pressure 2.24 mm.		Current 1.5 ma. Initial Pressure 1.455 mm.	
	Pressure Decrement.	Calculated Decrement.	Pressure Decrement.	Calculated Decrement.	Pressure Decrement.	Calculated Decrement.
1	.02	.021	.015	.0105	.03	.0315
2	.045	.042	.025	.021	.06	.063
3	.06	.063	.035	.032	.09	.094
4	.08	.084	.04	.042	.11	.126
5	.10	.105	.05	.053	.13	.158
6	.12	.126	.06	.063	.145	.221
7	.14	.147	.07	.074		
8	.16	.168	.08	.084		
9	.18	.189	.08	.095		
10	.20	.21				
11	.22	.231				
12	.23	.252				
13	.24	.273				
14	.26	.294				
15	.275	.315				

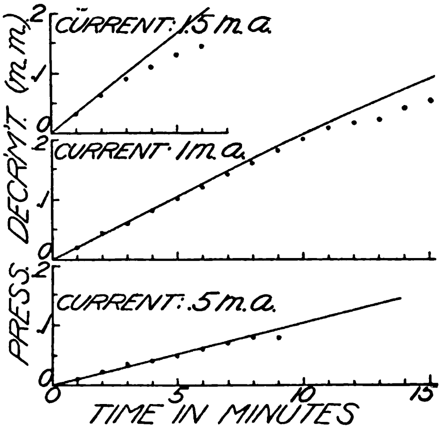

Fig. 7.

Mercury Anode in Hydrogen.—The form of tube used and the treatment of this metal was the same as with the alkali metals. The amount of hydrogen absorbed by the mercury before the current started was found to be much greater than in any other cases. In Table VI. is given a series of observations showing the gradual

Table VI.

The Absorption of Hydrogen by Mercury without Current. Volume 290 *c.c.*

Time (Min.).	Gas Pressure (mm.).	Pressure Decrement.
0	2.41	.0
1	2.4	.01
2	2.39	.02
3	2.38	.03
4	2.38	.03
5	2.365	.045
6	2.37	.04
7	2.365	.045

change in pressure of hydrogen after simply being admitted into the discharge tube. The results of its absorption by mercury as anode are given in Table VII. and Fig. 8. All tests show first an excessive initial absorption but later agree with the calculated rate. With 1.5 and 2 milliamperes agreement is found between the two minute and the eight and ten minute points, while with 1 milliampere it is between the three and fifteen minute points. The excessive initial

Table VII.

Mercury Anode in Hydrogen.

Duration Current (Min.).	Current 1 ma. Volume 295 c.c. Initial Pressure 2.03 mm.		Current 1.5 ma. Volume 290 c.c. Initial Pressure 2 83 mm.		Current 2 ma. Volume 290 c.c. Initial Pressure 3.18 mm.	
	Pressure Decrement.	Calculated Decrement.	Pressure Decrement.	Calculated Decrement.	Pressure Decrement.	Calculated Decrement.
1	.03	.0194	.08	.0297	.11	.0396
2	.055	.039	.15	.059	.18	.079
3	.085	.058	.18	.089	.22	.119
4	.11	.078	.22	.119	.27	.158
5	.135	.097	.25	.149	.31	.198
6	.155	.116	.28	.178	.35	.237
7	.175	.136	.32	.208	.39	.277
8	.195	.155	.34	.238	.42	.317
9	.215	.175	.37	.267	.44	.356
10	.235	.194	.395	.297	.455	.396
11	.25	.214				
12	.27	.233				
13	.285	.252				
14	.30	.272				
15	.315	.291				
16	.33	.311				

absorption which was so marked with mercury took place to a slight degree also with the alkali metals. With them, however, it occurred within the first minute after the current started so that the effect was eliminated by beginning observations after this time had elapsed. This excessive absorption during the first few minutes of current indicates that the metal will absorb more uncharged gas in a conducting state than when non-conducting. It is probably related to the fact noted by the writer[1] in an earlier paper that the photo-electric activity of a metal when placed in an atmosphere of hydrogen increased far more rapidly when the gas was conductive than when not.

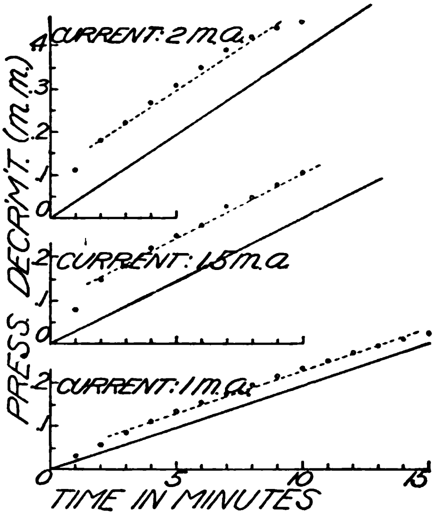

Fig. 8.

Sodium Anode in Nitrogen. — The tests in hydrogen were followed by similar ones in nitrogen — the form of tube and treatment of the metal being the same. The results of the absorption of nitrogen by a sodium anode are given in Table VIII. and plotted in Fig. 9. The calculated value assumes that each atom absorbed carries the same charge as hydrogen. Absorption was not of as long duration as in the case of hydrogen. With .5 milliampere it agreed with that calculated for the first five minutes, then practically stopped; with 1 milliampere it agreed for the first four minutes and

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[1] V. L. Chrisler, Phys. Rev., Vol. 27, p. 267.

Table VIII.

Sodium Anode in Nitrogen. Volume 295 c.c.

Duration Current (Min.).	Current 1 ma. Initial Pressure 3.65 mm.		Current .5 ma. Initial Pressure 2.41 mm.	
	Pressure Decrement.	Calculated Decrement.	Pressure Decrement.	Calculated Decrement.
1	.02	.0194	.01	.010
2	.04	.039	.025	.019
3	.06	.058	.03	.029
4	.08	.078	.04	.039
5	.09	.097	.05	.049
6	.105	.116	.05	.059
7	.12	.136	.055	.068
8	.13	.146		

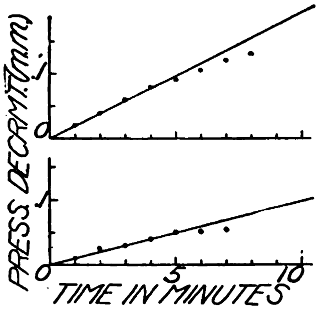

Fig. 9.

finally stopped at the end of eight minutes. A test with 1.5 milliamperes gave no absorption, probably owing to unfavorable surface conditions or possibly because the current density was too great.

Potassium Anode in Nitrogen. — The results showing the absorption of nitrogen by a potassium anode are given in Table IX. and Fig. 10. The duration of absorption was about the same as with sodium. The .5 milliampere gave the best results, the absorption agreeing closely with that calculated for five minutes and stopping entirely at the end of six minutes. With 1 milliampere the absorption followed the calculated value for four minutes, while 1.5 milliamperes agreed with that calculated for only two minutes.

Sodium-Potassium Anode in Nitrogen. — Results of absorption of nitrogen by this alloy are found in Table X. and Fig. 11. It lasted

TABLE IX.

Potassium Anode in Nitrogen. Volume 295 c.c.

Duration Current (Min.).	Current 1 ma. Initial Pressure 2.53 mm.		Current .5 ma. Initial Pressure 2.83 mm.		Current 1.5 ma. Initial Pressure 2.595 mm.	
	Pressure Decrement.	Calculated Decrement.	Pressure Decrement.	Calculated Decrement.	Pressure Decrement.	Calculated Decrement.
1	.02	.0194	.01	.0097	.03	.029
2	.04	.039	.02	.019	.06	.058
3	.06	.058	.03	.029	.075	.087
4	.08	.078	.04	.039	.09	.116
5	.09	.097	.05	.049	.10	.146
6	.095	.116	.055	.058		
7			.055	.068		

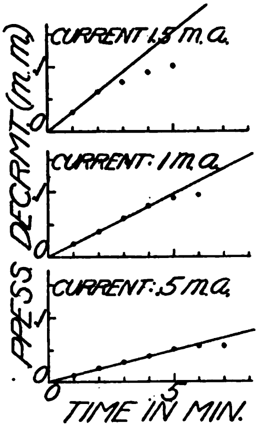

Fig. 10.

longer than with either sodium or potassium. With 1 milliampere the absorption was, for the first two minutes, greater than calculated, probably for the same reason that mercury absorbed an excessive amount of hydrogen for the first two or three minutes. Between the two and 15 minute points the rate agreed quite closely with that calculated. A current of .5 milliampere gave absorption which agreed fairly well with the calculated value for ten minutes while with 1.5 milliamperes it agreed for three minutes.

Glass, Sodium and Mercury Anodes in Helium. — It was noticed when depositing sodium by electrolysis of glass, using an atmosphere of helium, that the helium was sometimes absorbed. In sev-

Table X.

Sodium-Potassium Anode in Nitrogen. Volume 295 c.c.

Duration Current (Min.).	Current 1 ma. Initial Pressure 2.73 mm.		Current .5 ma. Initial Pressure 2.48 mm.		Current 1 5 ma. Initial Pressure 2.02 mm.	
	Pressure Decrement.	Calculated Decrement.	Pressure Decrement.	Calculated Decrement.	Pressure Decrement.	Calculated Decrement.
1	.03	.0194	.01	.0097	.03	.0291
2	.07	.039	.02	.019	.06	.058
3	.09	.058	.04	.029	.085	.087
4	.11	.078	.05	.039	.11	.116
5	.13	.097	.06	.049	.13	.146
6	.15	.116	.07	.058	.14	.175
7	.17	.136	.08	.068		
8	.185	.155	.09	.078		
9	.205	.175	.10	.087		
10	.225	.194	.11	.097		
11	.24	.213	.115	.107		
12	.26	.233	.115	.116		
13	.275	.252				
14	.29	.272				
15	.315	.291				
16	.325	.311				
17	.345	.330				
18	.355	.349				

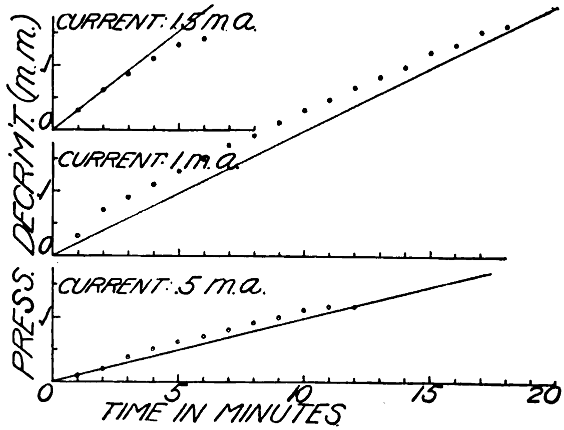

Fig. 11.

eral cases it was in only relatively small quantities, but in two others practically all of it was absorbed. The maximum rate of absorption in these two cases, which was attained a few minutes after the current started, lasted for several minutes in approximate agreement with that obtained later with sodium and mercury anodes. Results obtained from these two metals are given in Table XI. and Fig. 12.

TABLE XI.

Duration Current (Min.).	Mercury Anode in Helium. Current 1 ma. Volume 295 c.c. Initial Pressure 2.845 mm.		Sodium Anode in Helium. Current 1 ma. Volume 297 c.c. Initial Pressure 1.05 mm.	
	Pressure Decrement.	Calculated Decrement.	Pressure Decrement.	Calculated Decrement.
1	.035	.0388	.04	.0386
2	.075	.078	.08	.077
3	.105	.116	.11	.116
4	.125	.155	.14	.154
5	.155	.194	.16	.193
6	.165	.233	.17	.232
7	.175	.273	.18	.270
8	.195	.310	.20	.308
9	.205	.349	.21	.347

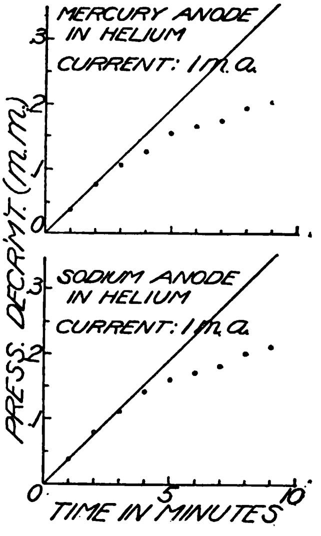

Fig. 12.

In the case of the sodium the helium used had been stored for several months in the bulb and hence might have been impure. That used with mercury, however, was fresh, having been transferred from the sealed glass bulb, in which it was obtained only a few days before the test. In both cases the results agree with each other very closely, giving a decrement in pressure just twice that which would have occurred had the same current passed through an atmosphere of hydrogen instead of helium. The calculated value was obtained by assuming that helium is monatomic and that with each atom absorbed there was associated the same charge as with hydrogen and nitrogen. Close agreement is found for the first three minutes after the current was started. Several attempts were made later to reproduce these results, but without success, probably owing to the fact that proper surface conditions were not obtained. The rate of absorption in the successful experiments as well as the failures to obtain any absorption indicate that the effects did not arise from the presence of the common gases as an impurity in the helium, because with these absorption always took place and at a distinctly different rate.

To sum up, it has been shown by the foregoing tests that hydrogen is absorbed by sodium, potassium, a sodium-potassium alloy and mercury when used as anode in a glow current, at such a rate as to indicate that with each atom absorbed is associated a negative charge of the same magnitude as the positive carried by it in solutions. The same has also been shown with nitrogen, using sodium, potassium and a sodium-potassium alloy as anode; and in two cases, with helium, using sodium and mercury as anodes. The fact that nitrogen carries the same charge as hydrogen is consistent with the chemical valence of nitrogen, being in some cases unity. Rutherford[1] has concluded from his experiments that the α particle from radium is a helium atom carrying a positive charge of *twice* the magnitude of that carried by a hydrogen atom in solution, while the results given here indicate on the other hand that with the helium atom is associated a negative charge of the *same* magnitude as that carried by hydrogen.

I wish here to express my appreciation and thanks to Dr. C. A. Skinner, of the University of Nebraska, for his help and inspiration to me in this work.

THE BRACE LABORATORY OF PHYSICS,
UNIVERSITY OF NEBRASKA, LINCOLN,
June, 1909.

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[1] E. Rutherford, Phil. Mag., p. 281, February, 1909.

CRYSTAL RECTIFIERS FOR ELECTRIC CURRENTS AND ELECTRIC OSCILLATIONS.

PART III. IRON PYRITES.

BY GEORGE W. PIERCE.

Reference to the Previous Work. — In the previous installments[1] of this paper the writer has given an account of some experiments on the rectification of small alternating currents and oscillations by certain conducting crystal substances (carborundum, anatase, brookite and molybdenite) in contact with solid metallic electrodes. During the progress of these experiments L. W. Austin[2] has independently made similar experiments on rectifiers consisting of silicon, carbon and tellurium in contact with steel or aluminum.

These experiments are of considerable theoretical and practical importance. The practical importance arises in the fact that the rectifiers have met with highly successful application as detectors for the electric waves of wireless telegraphy.

The present note describes an experiment on the rectification of alternating currents by iron pyrites in contact with copper. Iron pyrites is one of the substances originally investigated for unilateral conductivity by Ferdinand Braun[3] in 1874. In an account of his experiments Braun describes the phenomenon as follows: "With a series of natural metallic sulphides — copper pyrites, iron pyrites, galena, and copper antimony sulphide — I obtained in general the phenomenon that the current strength was different for different directions of the current; that this difference increased with increase of the current, and that, on keeping the circuit closed, the current for that direction in which the resistance was smaller increased, while for the opposite direction it decreased."[1] The use of iron pyrites in Braun's original experiments was mentioned in the historical note in Part I. of the writer's account.

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[1] G. W. Pierce, "Crystal Rectifiers for Electric Currents," Part I., PHYS. REVIEW, Vol. 25, pp. 31-60, 1907; Part II., ibid., Vol. 28, pp. 153-187, 1909, and Proc. Am. Acad., Vol. 44, pp. 317-349, 1909. See also Pierce, "A Simple Method of Measuring the Intensity of Sound," Proc. Am. Acad., Vol. 43, pp. 377-395, 1908.

[2] L. W. Austin, "Some Contact Rectifiers of Electric Currents," Bull. of the Bureau of Standards, Vol. 5, pp. 133-147, 1908.

[3] Ferdinand Braun, Pogg. Ann., Vol. 153, p. 556, 1874.

While in Braun's experiments, so far as can be judged from his data, the current in one direction was at most twice as much as the current in the opposite direction under the same voltage, the phenomenon described by the writer in the present series of papers is of entirely a different order of magnitude from that investigated by Braun. In the present experiments, with various crystal rectifiers, the current in one direction is often several hundred times as great (in one case 4,000 times as great) as that in the opposite direction under the same applied voltage.

The Specimens of Iron Pyrites. — Recently, Mr. Melville Eastham, of the firm of Clapp and Eastham of Boston, has had the kindness to present the writer with some specimens of iron pyrites that rectify much more strikingly than the specimens studied by Braun, and compare favorably with the results obtained by the writer with carborundum, anatase, brookite and molybdenite. One of Mr. Eastham's selected specimens was used in the present experiment.

The fragment of iron pyrites was held in a clamp holder (see Figs. 2 and 3 of Part II.), and a copper rod was brought into adjustable contact with an exposed portion of the pyrites. With a careful adjustment of the contact at particularly favorable spots of the pyrite the device is a very sensitive detector for electric waves.

Current-voltage Curves for Iron Pyrites. — The rectifying action of the pyrites is shown by the current-voltage curves of Fig. 1, in which the current through the rectifier is plotted against applied steady voltage. The upper curve gives the current when the voltage is applied in one direction; the lower the corresponding current when the voltage is reversed. As was the case with all the solid rectifiers thus far investigated, the writer finds that for some adjustments of the contact the greater current is from the copper to the crystal and for other adjustments (*e. g.*, with a different pressure or at a different point of the crystal) the greater current is from the crystal to the copper. With a fixed adjustment, however, and at a constant temperature the current under a given applied voltage is quite constant.

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[1] Translated from Braun, l. c., p. 558. The change of intensity of the current with its duration, as described in the paragraph from Braun's account, does not appear in the present writer's experiments, except in cases in which the current is large enough to hea the contact.

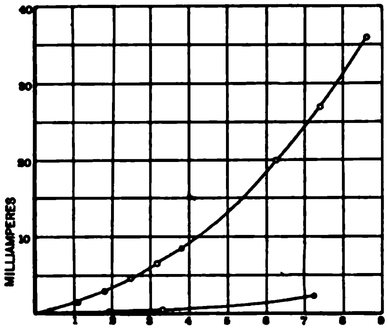

Fig. 1.

The Oscillogram for Iron Pyrites and the Oscillographic Apparatus. — Immediately after the current-voltage measurements plotted in Fig. 1 were made and with the adjustment of the contact of the rectifier undisturbed an oscillogram was taken of the current through the rectifier with a 60-cycle applied alternating voltage. This oscillogram is shown in Fig. 2. The oscillographic apparatus, which is described in detail in Part II. where several other similar oscillograms are reproduced, makes use of a Braun's cathode tube. Some small changes were made in this apparatus since the experiments of Part II. The most significant of these changes was a diminution of the resistance of the coils used for deflecting the cathode beam. In the present experiment, these coils had a resistance of 43.2 ohms, while in the previous experiments the resistance of these coils was 436 ohms. The inductance of the coils in the present experiments was also less than that of the coils previously employed. The value of this inductance is determined below. The diminution of the inductance and resistance of the deflecting

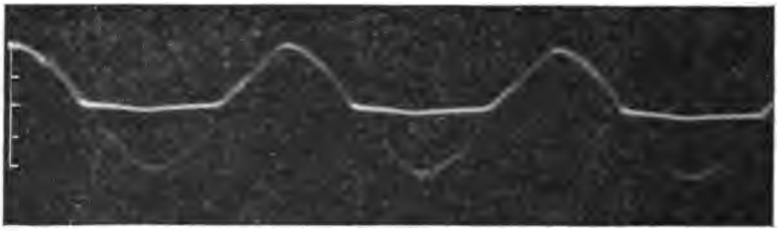

Fig. 2.

coils, although resulting in a decreased sensitiveness of the oscillographic apparatus, carries with it a double advantage. The first advantage is that it improves the rectification by a given rectifier; for with a large series impedance in the rectifier circuit and with a given alternating voltage applied to the circuit the drop in voltage about the rectifier is small in the direction in which the current is large, on account of the large drop in the deflecting coils, while in the direction in which the current is small the drop in the rectifier is large because of the small drop in the deflecting coils. This has the effect of reducing the ratio that the large current bears to the small current. A second advantage of the diminution of the resistance and inductance of the deflecting coils of the oscillographic apparatus is that it reduces the masking effect that large series inductance and resistance have on any slight change of phase relations that might accompany the use of the rectifier.

Description of the Three Exposures. — In taking the present oscillogram, as in taking the oscillograms of Part II., three exposures were made on the film: First, an exposure of 20 seconds was made with the circuit open. This gave the axis of no current, as a light horizontal straight line through the picture. Second, with an applied alternating voltage of 5.78 R.M.S. volts and with the rectifier in the circuit an exposure of two minutes was made. This exposure gave the rectified cycle, consisting of an upper loop for a half period and a nearly straight line (with a slight downward depression) for the next half period. Third, the rectifier was replaced by an ohmic resistance (the "equivalent resistance") selected to make the maximum point of the excursion of the spot, as seen on the luminescent screen of the cathode tube, agree with the maximum point of the excursion obtained with the rectifier in circuit. This exposure (of one minute) gave the approximately sine-curve, which in Part II. was called the "voltage-phase cycle."

Data of the Oscillographic Apparatus. — The data of the oscillographic circuit in the present experiment were:

Maximum rectified current = 21.7 milliamperes,
Applied alternating voltage = 5.78 R.M.S. volts,
Equivalent resistance = 320 ohms,
Resistance of deflecting coils = 43.2 ohms.

The deflections of the luminescent spot were proportional to the current, and the iron of the deflecting coils was so feebly magnetized that it showed no hysteresis. As in Part II., the inductance of the coils was calculated from the amplitude of the voltage-phase cycle as follows: The maximum applied voltage was $\sqrt{2} \times 5.78 = 8.16$ volts. Equating the maximum current of the voltage-phase cycle to the maximum E.M.F. divided by the impedance, we have

$$2.17 \times 10^{-3} = \frac{8.16}{\sqrt{363^2 + L^2\omega^2}};$$

whence

$$L\omega = 98,$$

and the angle of lag of the current of the voltage-phase cycle behind the external E.M.F. is

$$\varphi = \tan^{-1}\frac{98}{363} = 15.1^\circ.$$

Coördinates. — The time coördinate is the abscissa of the oscillogram and is drawn as usual from left to right. The current values, which are the ordinates of the curve, are given by the scale at the left of the picture, one division of the scale being 10 milliamperes.

Discussion of the Oscillogram for Iron Pyrites. — Since a detailed discussion of the oscillograms taken with various crystals rectifiers has been given in Part II.,[1] the oscillogram of iron pyrites will be treated only briefly. Except for the slight "building up" segment of the photographic curve at rising from the zero axis, the upper loop of the curve is the same as that obtained with the substituted resistance, chosen to give an equal deflection. This means that the pyrites introduce no phase changes in this part of the cycle other than those introduced by a resistance. If now we examine the lower loop of the rectified cycle we find that its maximum leads the negative maximum of the voltage-phase cycle. The two-curves are also of very unequal amplitudes in their respective lower loops. Suppose we should have put into the circuit enough resistance to make their amplitudes the same in the negative loop, would the phase of their maxima have also been the same? This question can be answered by a simple calculation. The amplitude of the negative loop of the cycle is one twelfth that of the voltage-phase cycle. Therefore, if R_1 is the resistance required to reduce the voltage-phase cycle to the amplitude of the negative maximum of the rectified cycle, we have

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[1] See discussion of "building up" segment of Part II.

$$\frac{\sqrt{R_1^2 + 98^2}}{\sqrt{363^2 + 98^2}} = 12,$$

whence $R_1 = 4570$ ohms. The corresponding phase angle would be

$$\varphi_1 = 1.2^\circ,$$

and the difference between φ and φ_1 would be 13.9°. The measured value of the lead of the negative maximum of the rectified cycle ahead of the voltage-phase cycle is 13.6°, which agrees with the calculated value within the limit of error of measuring the original photograph. Whence we may conclude that in the negative loop also the current through the rectifier has the same phase as it would have if replaced by a resistance producing the same amplitude.

Comparison of the Iron Pyrites Rectifier with Molybdenite. — In the present experiment a larger rectified current was obtained with the iron pyrites rectifier at practically perfect rectification than was obtained with the molybdenite rectifier. The apparent advantage of the pyrites rectifier arises in the employment of the deflecting coils of smaller impedance in the present experiment. The manner in which this affects the perfection of rectification was discussed above. A comparison of the current-voltage curves of Fig. 1 with the corresponding curves for molybdenite (Fig. 5, Part II.) shows that the molybdenite in the adjustment at which it was there used is a better rectifier than the iron pyrites at the adjustment at which it was used in the present experiment. The writer has tried several different adjustments of the two rectifiers with the result that the molybdenite was the more sensitive as a rectifier for alternating currents. It is entirely possible, however, that a comparison of other specimens of the two minerals may lead to the discovery of specimens of iron pyrites superior to the molybdenite. I should say that the two minerals have probably about equal capabilities as detectors for electric waves, although up to the present I have not found the pyrites detector quite so easily adjusted or quite so rugged as the molybdenite detector.

We are apparently dealing with the same kind of a phenomenon in the case of all the crystalline substances used as detectors for electric waves. If we may accept the evidence obtained in the experiments on molydenite, described in Part II., this action does not take place through the intermediation of thermoelectricity. The fact that all the crystal rectifiers have also large thermoelectromotive forces against the common metals may be due to the fact that rectification and thermoelectricity have a common basis. For example, if we suppose that the surface of separation of two conductors has such a character as to permit the passage of electrons more easily in one direction than in the opposite direction, the boundary will act as a rectifier for alternating currents; it will also give rise to a thermoelectromotive force when heat is applied to the boundary, provided the velocity of the electrons is increased with an increase of temperature. This is merely given as a suggestion of one way in which the phenomena may be correlated. Further experiments are in progress.

The expenses of this research were in part defrayed by a liberal grant from the Bache fund of the National Academy, for which the author wishes to express his sincere thanks.

JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY, CAMBRIDGE, MASS.,
April 14, 1909.

THE SOFTENING OF COLD, GLASS-HARD STEEL, IN TWENTY-FOUR YEARS.

By Laura C. Brant.

1. *Introductory.* — The electrical resistances and thermo-electric qualities of the following batch of about twenty glass-hard Stubb's steel rods (1 broken) were measured by Professor Barus[1] in June, 1885, only a few days after tempering, again in July, 1888, and finally in September, 1897. They were tempered glass-hard by heating with the electrical current and sudden quenching in cold water, by means of a specially designed machine, in a manner which insured marked uniformity in the results. The apparatus, the mass constants, homogeneity of the rods, etc., are fully given in the papers quoted. For the case of a given type of material like the steel examined, the electrical constants may be systematically computed. This facilitates many of the reductions. Thus the thermo-electric power y, in microvolts relatively to soft silver, bears to the specific resistance s_0 (microhms, c.c., 0° Centigrade) when both qualities vary with temper, the relation

$$y = 15.176 - 0.4123 s_0.$$

Moreover, the temperature coefficient a, corresponding to the specific resistance s_0, for corresponding variations is

$$(s_0 + .78)(a - .0001435) = .0682.$$

Steel is peculiarly favorable for observations of this kind, inasmuch as s_0 increases from soft to hard more than *threefold.* Hence the resistance variations due to temper in steel are enormous as compared with the corresponding variations due to drawn strains, etc., in the standards, so that this source of error is quite negligible. Naturally, the same standards (1 ohm and .1 ohm) were used throughout the work, these having been preserved together with the steel wires.

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[1] Phil. Mag. (5), XXVI., p. 397, 1888; *ibid.* (5), XLIV., p. 486, 1897.

The following are the mean results for the electrical resistance as found in the earlier work (l. c.):

No. of Rods.	Diam.	Resistance, r_t, at t° C., per cm.						Spec. Resistance, s_0, at 0° C.				
		1885	t	1888	t	1897	t	1885	1888	δs_0	1897	δs_0
	cm.											
Ten	.0806	9,042	27	8,232	28	7,717	22	45.0	40.7	−4.30	38.2	−6.73
Five	.044	30,536	25	27,892	29	26,112	22	44.8	40.6	−4.24	38.2	−6.64
Four	.1265	3,463	27	3,048	29	2,837	22	41.9	36.5	−5.49	34.2	−7.77

Comparing these results with the effect produced by annealing at higher temperatures (66°, 100°, 185°, etc.), Professor Barus concluded that atmospheric temperature acting for a period of 180 months had produced about the same effect as 66° acting for a period of 180 minutes. One may observe, moreover, that the thin rods fall off less and from higher initial tempers (resistances), while the thick rods fall off more and from smaller values.

2. *Data for May, 1909.*—The following results were obtained recently for the same steel rods and standards, by stepping off equal potentials from the ends of the rods and of the standards on the Wheatstone bridge. This makes it unnecessary to solder the ends, or of otherwise interfering with the rods, as the resistances of all connections, if fixed, are eliminated by the method. In the following table the individual resistances in microhms per linear cm. of length, at t°, the corresponding specific resistances at 0° Centigrade, as well as the mean values of these data are given in detail.

3. *Inferences.*—The results for specific resistance are graphically represented for the different diameters of steel wire in the figure.

Thus in the twenty-four years of observation the thinner rods have softened to the extent of losing nearly 20 per cent. (19.5 to 19.7) of the original specific resistance, while the thick rods have lost nearly 21 per cent. In both instances this diminution has taken place along the characteristic exponential curves discussed elsewhere. The total decrement from glass-hard to soft steel may be reckoned from 60 to 70 per cent., showing that a considerable part (about one third) of the maximum interval possible has been passed spontaneously at atmospheric temperatures. In fact the thick rods have nearly reached the practical annealing limit of 100°, acting for

Resistance per Linear cm., r_t, and Specific Resistance s_0, of Cold Glass-hard Steel, May, 1909. Effective Length of Rods about 23 cm.

No.	Diameter. $\times 10^3$	Resistance. r_t	Temperature. t	Specific Resistance. s_0 at 0° C.
	cm.	microhm.	° C.	
1	80.6	7,607	21.2	37.3
2		7,565	21.2	37.1
3		7,135	21.2	35.0
4		7,103	21.2	34.8
5		7,124	21.2	34.9
6		7,594	21.2	37.3
7		7,533	21.2	36.9
8		7,604	21.2	37.3
9		7,062	21.2	34.6
10		7,357	21.2	36.1
Mean..........	80.6	7,368	21.2	36.13
11	44	24,693	21.2	36.0
12		24,536	21.2	35.8
13		24,719	21.2	36.1
14		24,235	21.2	35.4
15		25,160	21.2	36.8
Mean...........	44	24,668	21.2	36.03
16	126.5	2,683	1.2	32.0
17		2,676	21.2	31.9
19		2,945	21.2	35.3
20		2,702	21.2	32.3
Mean...........	126.5	2,751	21.2	32.87

three hours, and they will probably quite reach this limit after forty years, *i. e.*, in 1925. In the thin rods the initial advantage of relatively greater hardness is sustained; in other words, the absolute amount of the decay has in both cases been about the same and equivalent to 8.8 to 9.1 microhms per c.c., respectively, during the twenty-four years.

The fact that the secular softening after about one quarter of a century already approaches the three hour annealing effect of 100°, suggests the interesting question whether in the lapse of sufficient time glass-hard steel would actually reach the soft and plastic states spontaneously. If the mean rates for the last twelve years be taken in case of the above steels, the thin rods should be soft in 170 years and the thick rods in 250 years, results of course which are inferior limits of the interval in question. It does not seem improbable, however, that if n be the number of hard molecules per c.c. of steel, the rate $-dn/dt$, where t is the time of exposure to a given temperature, will be some function of n. The change therefore will take place at a diminishing rate through infinite time, but in a way that the eventual complete softening of hard steel after a period great in comparison with 250 years is altogether probable.

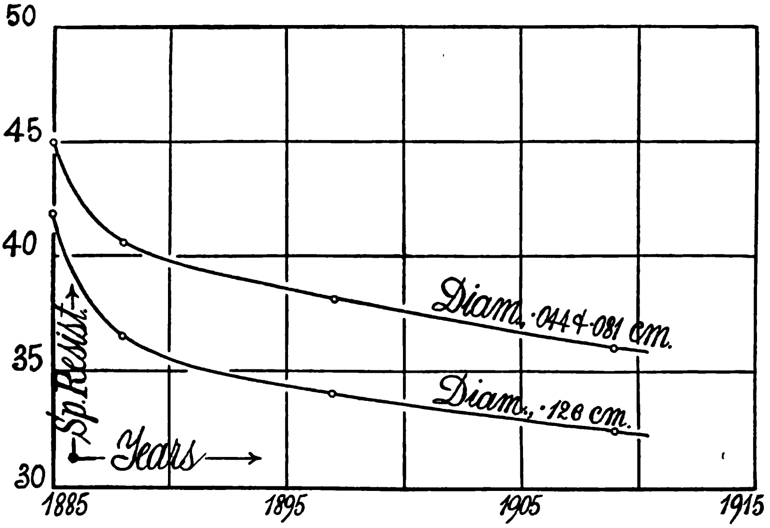

Fig. 1.

Brown University,
Providence, R. I.

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ON THE ELECTRIC CHARGES ACQUIRED BY INSULATED POTASSIUM SALTS AND OTHER RADIOACTIVE SUBSTANCES IN HIGH VACUA.[1]

BY J. C. McLENNAN.

SINCE the discovery in 1907 by N. R. Campbell and A. Wood[2] that potassium salts possess in a very definite though relatively small degree the property of radioactivity, the radiations from these salts have been examined by a number of investigators including among others Campbell,[3] McLennan and Kennedy[4] and Levin and Ruer.[5]

Amongst other properties of the rays examined by these investigators was their absorption by different thicknesses of various substances. From the results of such measurements the view came to be held that the rays from potassium salts were heterogeneous and consisted of several types varying in penetrating power from the beta rays of uranium downwards. The average intensity of the rays, moreover, was found to be about 1/1,000 of that of the radiation from uranium salts.

In a particular set of experiments which were carried out by Campbell the rays were passed between the plates of a large zinc grid and on emergence their intensity was ascertained from the conductivity they imparted to a mass of gas in an ionization chamber. This grid was so arranged that alternate plates were metallically connected and consequently when the two sets of plates were joined to the two terminals of a battery an electric field was established in the intervals between the plates. Campbell found when a field of 8,000 volts was applied to this condenser that the intensity of the emerging rays was diminished by approximately 14 per cent. This led him to the conclusion that the rays consisted of streams of electrically charged particles, and from additional experiments on the direction of the deviation of the deflected beams he concluded that the charge carried by the rays was a negative one.

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[1] Read before the Royal Society of Canada, May 26, 1909.

[2] Proc. Camb. Phil. Soc., Nos. 14-15, 1907.

[3] Proc. Camb. Phil. Soc., No. 14, 211, 1907.

[4] Phil. Mag., Sept., 1908.

[5] Phys. Zeit., 9 Jahr., No. 8, Seite 248.

It followed therefore from these experiments that the rays in all probability belonged to the beta type of radiation.

In some absorption experiments made by the writer special care was taken to examine the rays from potassium salts for the presence of a radiation of the alpha type, but no evidence of the presence of such rays was obtained from the form of the experimental absorption-ionization curves, and the conclusion was drawn that if any such radiation was present, it must have been emitted with a velocity considerably less than that possessed by the alpha rays from other well known radioactive substances.

More recently E. Henriot[1] made an examination of the rays emitted by potassium salts, and while the results of his experiments agree in the main with those of the earlier investigators, they differ from them in that they point to a complete homogeneity in the radiation. For on measuring the ionization produced by the rays after passing through various layers of tinfoil, he found on plotting the logarithms of the ionization current for the rays from potassium sulphate and potassium chloride against the thicknesses of tinfoil used that the points all lay on a straight line.

Henriot, in his paper on the subject, also claims that in the course of his measurements he found some slight evidence of the presence of slightly penetrating rays of the alpha type in the radiation from the potassium salts examined.

Quite recently an examination was made by Strutt[2] of the amount of helium present in a number of saline minerals, and the results of this examination which are given in Table I. make it clear that the potassium-bearing minerals and especially sylvine are rather remarkable for the comparatively great amount of helium which they have been shown to contain. In discussing the origin of the gas Strutt considered that it was altogether improbable that the minute traces of uranium and radium present in sylvine could account for so much helium, and in view of Campbell and Wood's discovery of the radioactivity of potassium salts he was led to regard potassium itself as the source of the helium in the saline minerals examined.

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[1] Henriot, Comptes Rendus, 148, pp. 910–912, 1909.

[2] Strutt, Proc. Roy. Soc., Series A, Vol. 81, No. A547, p. 278.

TABLE I.

Mineral.	Composition.	Helium c.mm. per 100 Grammes.
Rock salt	$NaCl$	0.0233
Sylvine	KCl	0.55
Carnallite	$KMgCl_2 6H_2O$	0.151
Kieserite	$MgSO_4 H_2O$	0.0179

In view of the identity established by Rutherford between the alpha particles and the atoms of helium, it would follow from the view taken by Strutt — that the origin of helium in the saline minerals is the potassium which they contain — that rays of the alpha rays should be present in the radiation from potassium salts.

With the exception of Henriot no one, however, seems to have observed any indication of the presence of such rays. Usually however these rays have been detected and identified either by their fluorescent and ionizing action or by the deflections which they undergo in electric or magnetic fields. But for these methods to be applicable the alpha particles must have velocities which will take them away a few millimeters at least from the substances which emit them.

It is quite possible then that potassium and even other substances may be emitting alpha particles with exceedingly low velocities, and that these rays have hitherto escaped detection largely through the inadequacy of the means adopted to bring them into evidence.

In looking for a means of demonstrating the possible existence of these rays it seemed to the writer worth while to make an attempt to investigate the radiations emitted by the potassium salts, by examining them for the acquisition of an electric charge when placed on insulating supports within a highly exhausted vessel.

Amongst others M. and Mme. Curie,[1] Paschen,[2] Strutt,[3] Aschkinas[4] and Makower[5] have applied this method with success to the investigation of different types of radiation. In Paschen's experiments it was found when a lead cylinder containing a small quantity of a radium salt was insulated in a vessel from which the air was removed that the lead cylinder acquired a positive charge through the action of the rays which were emitted. This effect was thought by Paschen to prove that the gamma rays from radium consisted of streams of rapidly moving negatively charged particles but this was afterwards shown by Eve[6] to be due to the excitation and consequent emission of a secondary radiation of the beta type in the lead by the passage through it of the gamma rays.

The method too was applied by Strutt (*loc. cit.*) in his interesting experiment popularly known as the radium clock, and more recently it was applied by Aschkinas to demonstrate the existence of the delta rays from deposits of polonium, to which radiation attention was first drawn by the experiments of Sir J. J. Thomson,[7] Logeman,[8] Ewers,[9] and others.

Makower has also recently used it to measure the beta radiation from radium and from these measurements to deduce the number of beta particles emitted per gram of that substance.

By applying this method to the radiation from potassium salts the writer has observed in certain cases that potassium salts when insulated in high vacua, acquire a positive charge, but up to the present it has not been found possible from the characteristics of the acquired charge to decide whether this charge arises from the radiation being wholly of the beta type or whether it is due to the emission of rays of both the alpha and beta types with the beta type of radiation in excess.

[1] M. & Mme. Curie, Comp. Rend., CXXX., p. 647, 1900.

[2] Paschen, Wied. Ann., 14, 1, pp. 164–171, 1904.

[3] Strutt, Phil. Mag., Nov., 1903.

[4] Aschkinas, Phys. Zeit., 8, p. 773, Oct. 24, 1907.

[5] Makower, Phil. Mag., Jan., 1909.

[6] Eve, Nature, Sept. 8, 1904.

[7] Thomson, Camb. Phil. Soc. Proc., 13, pp. 49–54, Feb. 18, 1905.

[8] Logeman, Proc. Roy. Soc., Series A, Vol. 78, No. A523, p. 212.

[9] Ewen, Phys. Zeit., March, 1906, pp. 148–152.

Among other phenomena observed when applying the method are those associated with the Volta effect. If a metallic rod or vessel connected to an electrometer be placed within and insulated from a second metallic vessel, it will be found when both the containing vessel and the insulated vessel are joined to earth, that generally a potential difference will exist between them. This potential difference will of course be more marked when the two vessels are made of different metals. It will also, however, be observable ordinarily when the two vessels are made of metals supposedly the same, but in this case the existence of the effect only goes to show that some slight difference exists in the composition or condition of the two pieces of metal used in the construction of the two vessels.

If now, with the arrangement just described, where one vessel is placed within and insulated from a second, the earth connection of the electrometer be removed it will be found that the insulated body or vessel more or less rapidly acquires an electrical charge, the sign of the charge being determined by the direction of the potential gradient. This charge arises from the air or gas between the two vessels possessing a conductivity which is imparted to it either by radiations emitted from the walls of the containing and contained vessels or from the passage through the gas of the penetrating radiation which is known to be present at the surface of the earth. The conductivity possessed by the air will tend to diminish the potential difference set up between the two bodies when they were both joined to earth, and the charge acquired by the electrometer under the action of the conductivity when earth connection to the inner vessel is removed, will give a measure of the magnitude of the Volta effect.

This then is one type of charge which is nearly always acquired by such an insulated system as that described. It has, however, certain definite characteristics which make it readily detectable. For example, the rapidity with which this charge is acquired depends largely on the degree of conductivity possessed by the intervening gas, and as this conductivity can be increased at will by bringing more or less near to the vessel a small quantity of radium or other radioactive body, it is possible to make the insulated system practically take up at once a charge which will suffice to annul the Volta effect.

If, further, now, the inner vessel or body in the arrangement described possesses in addition a radiating surface which emits alpha or beta rays, the charging action of these rays can generally be brought into evidence by a reduction of the pressure of the gas between the two bodies. This reduction of the pressure will produce a diminution in the conductivity of the gas but it will not affect the rate of emission of charged particles from the radiating surface. Consequently as the pressure of the gas is reduced the insulated system should, under the action of its charged radiation, acquire a charge, and so set up a potential difference between the inner and outer vessels.

The actual current through a gas, however, at any particular pressure depends so long as the saturation current has not been attained, upon the potential difference producing the current, and so it happens that at each pressure a state of equilibrium is brought about, by virtue of which the insulated system is maintained at such a potential as will produce a current through the gas of such magnitude as to exactly counterbalance the gain of charge through the emission of the radiation.

But as the pressure of the gas is lowered the equilibrium potential of the insulated system becomes gradually greater and greater, and the sign of the charge on the free system corresponding to this gradually increasing potential difference will be the opposite of that of the emitted radiation which is in excess.

Further, the extent of the equilibrium potential corresponding to any particular pressure will give a measure of the magnitude of this excess radiation.

When applying the method to the investigation of any particular radiation, the earth connection to the insulated system should first be broken and time allowed for the free system to come into equilibrium under the action of the conduction current arising from the Volta effect. When this equilibrium has been reached the scale reading corresponding to the position then assumed by the movable system may therefore be taken as the initial reading in considering the charging action due to the radiation itself. Two lines of procedure are then open. If the radiation from the insulated system be a strong one the movable system of the electrometer will take up the equilibrium position practically at once for any particular pressure and so the pressure may be lowered by stages and the corresponding scale readings observed. But if the radiation be a weak one it may take the movable system a long time to acquire the equilibrium potential corresponding to any particular pressure and under these circumstances it is best, if the object of the investigation is to ascertain the character of the excess emitted radiation, to reduce the pressure as rapidly and as low as possible, and then, while maintaining the low pressure, to observe whether a movement occurs in the movable system of the electrometer.

If such a motion occurs, the direction of the motion will indicate the sign of the charge acquired and the rate of movement will give a measure of the magnitude of the intensity of the charge producing radiation.

II. Charging Action of the Radiation from Polonium.

Some preliminary attempts were made to obtain a charging effect with some potassium salts in a high vacuum but it was soon found that such charging action was exceedingly small, and it was thought best to carry out a few parallel experiments with some of the better known radioactive substances, in order to gain some information regarding the pressures at which a charging action would be exhibited by various types of rays from active substances placed in a number of differently shaped vessels.

The first experiment was made with polonium deposited on a strip of copper. The area of this deposit was about 6 sq. cm. The copper strip which carried it was supported by a piece of amber insulation at the center of an-air tight brass cylinder 20 cm. long, and about 5 cm. in diameter. This cylinder was connected to a McLeod gauge and also to a Gaede mercury exhausting air pump. The insulated copper strip bearing the polonium was also connected to the free quadrants of a Dolzaleck electrometer, which gave about 200 mm. divisions deflection per volt. With this arrangement it was found repeatedly when the earth connection to the free quadrants was broken, that the needle moved slightly in the positive direction and came to rest about three centimeters from the zero reading. This deflection was taken as a measure of the Volta effect.

As the pressure was lowered no further change occurred in the electrometer reading until a pressure of approximately .4 mm. was reached. At this pressure the quadrants always commenced to gain a positive charge and as the pressure was still further lowered the needle at once moved out and took up a definite position corresponding to each pressure. When the pressure was taken below .01 mm. the charging action became very marked and produced deflections beyond the limits of the scale.

A set of readings taken with this polonium-coated copper strip in the neighborhood of the initial rise due to the radiation is given in Fig. 1, and a curve drawn from the readings is shown in the same

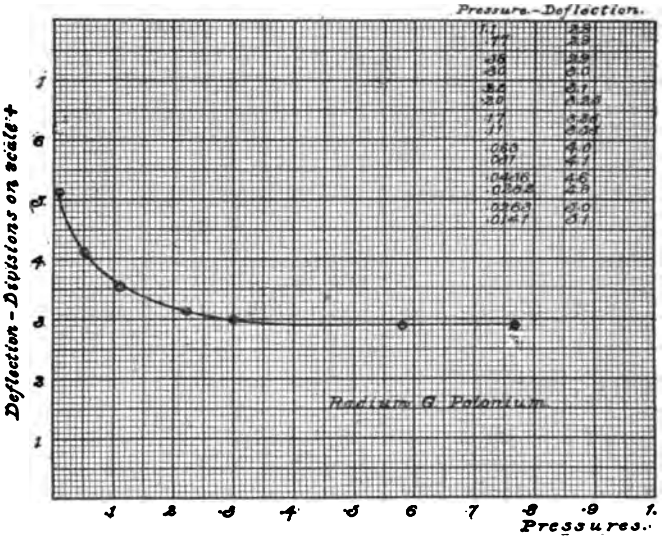

Fig. 1.

figure. This curve is typical of the different ones obtained with this active product.

It shows quite clearly that this substance omits an excess of negatively charged particles, and it also exhibits very definitely the pressure at which this excess comes into evidence.

III. Charging Action of Secondary Rays Excited in Aluminium by the Gamma Rays from Radium.

In this experiment one milligram of radium bromide was enclosed in a small sealed glass tube, whose walls were about 2 mm. thick. This glass tube was placed inside a tube of aluminium, which was then closed at the top and bottom. This tube which was about 7 cm. in length and had walls about 2 mm. in thickness was then insulated as before and suspended in the brass cylinder used in the last experiment. In this experiment the charging action was very much more marked than in the experiments with polonium. It also exhibited certain characteristics which were not observed in the measurements with that substance.

A set of results which illustrate its behavior is shown in Fig. 2.

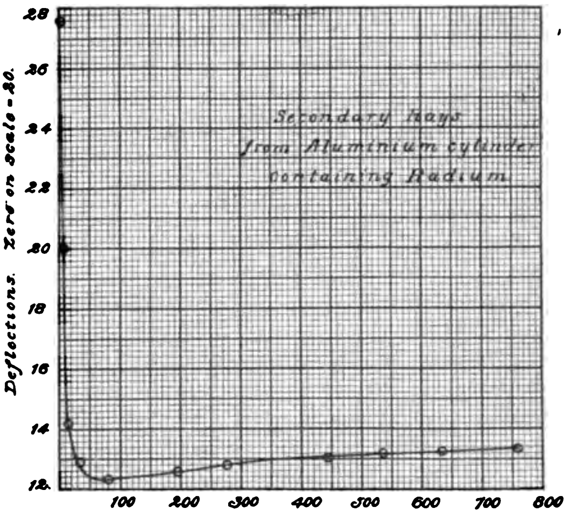

Fig. 2.

When the outside vessel and the free system were joined to earth the zero reading on the scale was 20, and as soon as the earth connection to the quadrants was removed the needle moved out at once in the negative direction and took up a position at 13.3. When the pressure was lowered the negative charge on the quadrants gradually increased at first but ultimately at a pressure of about 80 mm. this acquisition of a negative charge ceased and when the pressure was reduced, still further the charging action was reversed and for lower pressures the free system more and more rapidly gained a positive charge. In this experiment the Volta effect would make the aluminium positive to the outside brass cylinder and consequently this would explain the acquisition of an initial negative charge when the earth connection to the free quadrants was broken. It is probable, however, that the difference between 20 and 13.3 did not represent the exact magnitude of the Volta effect for in this case the radiations present would include secondary rays from the walls of the brass outside cylinder as well as others from the walls of the inner aluminium tube. The reading 13.3 would then represent the equilibrium potential acquired by the free system through the agency of (1) the Volta effect, (2) the conduction current, (3) the secondary rays from the aluminium tube, (4) the secondary rays from the brass outside vessel and also (5) any tertiary rays from the two opposing surfaces which might be present. The initial increase in the negative charge acquired by the aluminium tube in the range of pressures extending to 80 mm. was probably due to modifications produced by the lowering of the pressure in the amount of secondary rays coming to the aluminium tube from the walls of the brass outside vessel. The charging action of these would initially mask the charging action of the various excited secondary rays issuing from the surface of the aluminium tube. But ultimately, as the pressure was lowered a point would be reached when these secondary rays from the outside vessel would exert their maximum

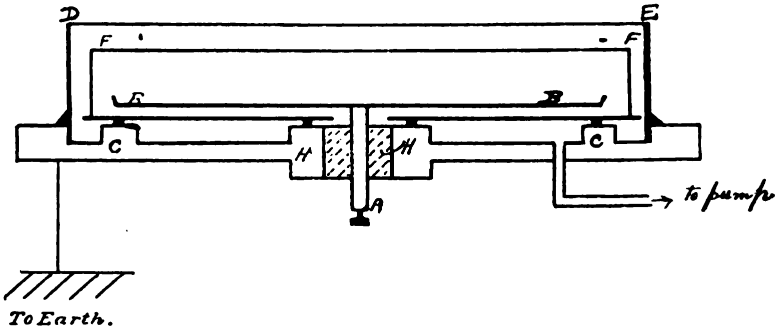

Fig. 3.

effect. For lower pressures the conduction current through the gas would become less and less. This would bring the secondary rays from the aluminium tube more into evidence and so account for the rapid increase in the positive charge acquired by the free system, which the curve shows took place at the lowest pressures. From this experiment it will be seen that the charging action of the secondary rays produced in aluminium by the gamma rays from radium was brought into evidence at a very much higher pressure, 80 mm., than a similar action by the rays from polonium.

IV. ON THE CHARGES ACQUIRED BY URANIUM SALTS AT LOW PRESSURES.

In this experiment the form of the vessel used in making the examination is shown in Fig. 3. A flat tray of brass *BB* was supported by an ebonite plug, on the brass plate *CC* shown in the figure. A shallow brass cover *FF* was placed over the tray *BB* and rested on the lower plate. Over the whole a second cover *DE* was placed which fitted snugly into a groove provided in the lower plate *CC*. In making the experiment the salt to be examined was placed on the tray *BB*, the covers were then placed in position, and finally all the joints made air tight with sealing wax. The salt used in making the experiment was a sample of uranium nitrate. The measuring instrument used was again a quadrant electrometer, but in this case it possessed a sensibility of 600 divisions per volt. In carrying out the experiment it was found that when the earth connection to the free quadrants was broken, the needle moved out in the positive direction on the scale for a distance which represented approximately one fortieth of a volt, and came to rest in this position. This deflection was taken to represent the deviation arising from the Volta effect. The chamber was then slowly evacuated, but as the exhaustion proceeded no additional deflection of the needle was observed. An attempt was then made to reduce the pressure as low as possible but although a pressure of .003 mm. was reached no indication was obtained of any additional charge being acquired by the insulated tray which carried the uranium nitrate.

As it is known that both alpha and beta rays are emitted by these salts, this result pointed to the conclusion that the rays carried away from the salt equal amounts of positive and negative electrical charges. In order to test this matter still further a second form of exposing chamber was constructed.

Its design is shown in Fig. 4. The salt was spread out in thin layers some 3 or 4 mm. thick on a set of 12 brass trays *AA* whose diameters were about 9 cm. These were all fastened to a central brass rod *C* which was suspended in the outer brass cylinder by an insulating support of ebonite. This vessel carried a small tray of P_2O_5 in the bottom and it was also provided with a set of plates,

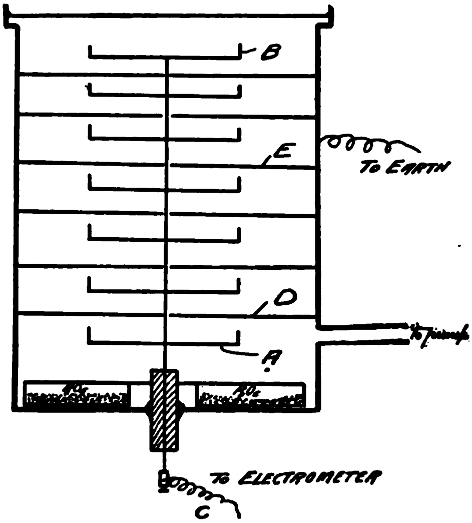

Fig. 4.

DD, so arranged between the trays as to intercept the rays issuing from the salt. The apparatus was provided with a cover as shown in the figure, and all the joints were again made air tight with solder and sealing wax.

With this form of apparatus the Volta effect was again in the positive direction, but the corresponding displacement was greater in this case, and represented approximately one quarter of a volt. After the deflection resulting from the Volta effect had become steady, the air was rapidly exhausted from the receiver and although the pressure was reduced to .02 mm., no indication of any charging action was obtained.

From the fact that no charging action was obtained in either of the experiments with the two forms of apparatus, it seems clear that the alpha and beta rays from the nitrate of uranium at least carry away with them from the salt equal amounts of positive and negative electricity.

In order to test the accuracy of this conclusion an additional set of experiments was made with uranium nitrate when using this second form of apparatus. The salt on each tray was covered with two layers of aluminium leaf each about .007 mm. in thickness. As this thickness was sufficient to cut off all the alpha rays,[1] but only a small proportion of the beta rays it was evident that a charging action should occur, and some experiments were made to see at what pressure it could be observed.

As aluminium in the Volta series is positive to brass, the charge acquired in this case through the annulling of the Volta effect was a negative one. On reducing the pressure it was found that the electrometer needle commenced to move away from the stationary position assumed under the influence of the Volta effect, when a pressure of approximately 8 cm. of mercury was reached, and the direction of displacement was such as to correspond to the acquisition of a positive charge. The sign of the charge acquired, it will be seen, fitted in then with what was to be expected from the absorption of the rays. For the higher pressures the rate of motion of the needle as it moved to take up the deflection corresponding to a selected pressure was slow and on this account the second line of procedure mentioned above was followed. In taking the observations the pressure was reduced as low as possible and then the rate of motion of the needle noted as it moved out to take up the equilibrium position.

Examples of these movements of the needle are shown in Figs. 5 and 6. In the one case the movement corresponds to a pressure of .15 mm. and in the second to a pressure of .08 mm. The rate of movement, as was to be expected, is greater for the lower pressure than it was for the higher one. The fact that positive charges were acquired under these circumstances lends further support to the conclusion drawn from the earlier experiments that equal quantities of opposite kinds of electricity were emitted with the rays from uranium nitrate when freely exposed.

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[1] Phys. Zeit., 8, p. 773, Oct. 24, 1907.

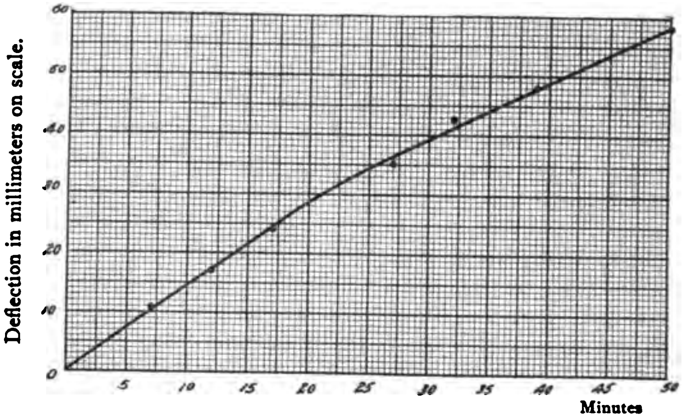

Fig. 5.
Aluminium-covered uranium nitrate. Pressure = .15 mm.
Voltage on needle = 243 positive.

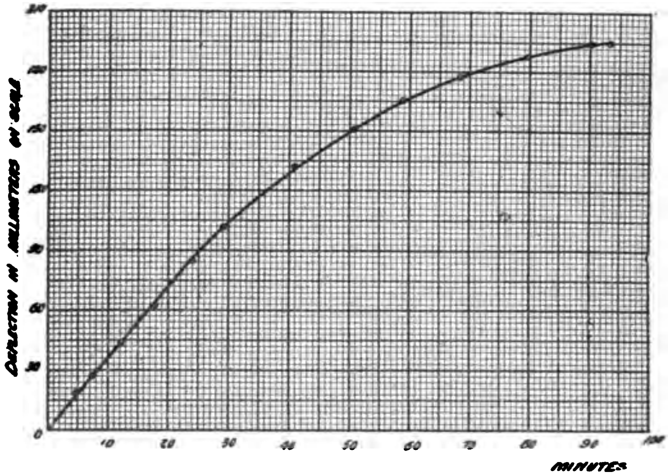

Fig. 6.
Aluminium-covered uranium nitrate. Pressure = .08 mm.

V. Potassium Salts: Electrical Charging Action.

In making the observations with potassium salts pieces of apparatus similar in form and dimensions to the two shown in Figs. 3 and 4 were used in turn. In the first experiment a layer of potassium chloride, about 4 mm. thick, was spread on the tray *BB* in Fig. 3, and then the space surrounding the tray was as highly evacuated as possible.

Some preliminary observations had shown that extreme care had to be taken to see that no electrification was imparted by friction to the insulating support *HH*, during the operation of setting up the apparatus. Even touching it slightly with the finger or allowing its surface to be scraped with any object, was found to leave an electrification behind which gradually disappeared, but which in doing so brought irregularities into the results and often vitiated them.

In taking the readings given below, exceptional care was therefore taken to make certain that all the insulating supports were entirely free from electrification. The electrometer was also exceptionally well set up, the needle being so carefully adjusted and the suspension wire kept so free from torsional strain that when both pairs of quadrants were joined to earth no displacement from the zero took place for hours.

With the apparatus in this condition the test was made for a charging effect with the potassium chloride, first when the needle of the electrometer was positively charged to 240 volts, and secondly when it was negatively charged to the same voltage. The sensibility of the electrometer was such that a potential difference of 1 volt between the quadrants gave a deflection of 600 mm., divisions on the scale. In making the test the pressure was lowered to .003 mm., and maintained at this low pressure throughout the measurements.

The results obtained with the needle positively charged and also when it was negatively charged are shown in Fig. 7. From these curves it will be seen that in both cases the tray containing the salt slowly acquired a positive charge. The rate at which this charge was acquired, moreover, was greater when the needle was positively charged than when it was negatively charged, but this difference was probably due to the small conduction current which must have existed in the air between the needle of the electrometer and the free quadrants.

The effect it will be seen is a small one and the time required to make the readings was long. The readings, however, were quite regular, and with the precautions and care taken there seems no reason for interpreting the results otherwise than as representing a charging action produced by the radiation emitted by the insulated potassium salt. Observations were made under precisely the same conditions with the tray empty, but these failed to show any such charging action as that obtained with the salts. These experiments with the potassium chloride were made, as stated above, with the apparatus shown in Fig. 3, but they were carried out before the apparatus was used in the uranium nitrate experiments, and consequently there was no possibility of a connection between the charging action observed with the potassium salt and any contamination of the apparatus through contact with the uranium nitrate. This charging action with a salt of potassium was also observed with the apparatus shown in Fig. 4. In this set of observations the salt used was the sulphate of potassium. Two sets of measurements were again made. In the one the potential on the needle was positive and equal to 243 volts, and the pressure of the residual air in the apparatus was .08 mm. In the second set the needle was kept at a negative potential of 243 volts, and the pressure of the air was maintained at .06 mm. In these two latter measurements it was

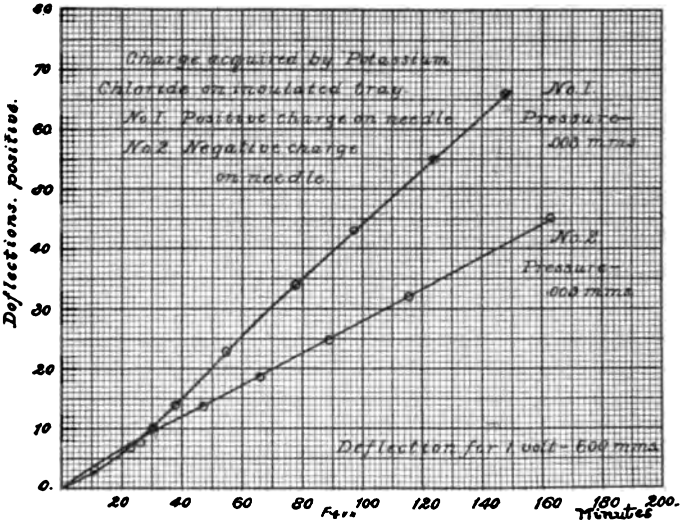

Fig. 7.

not found possible to lower the pressure of the gas below the readings stated, although the Gaede pump used in the exhaustion was kept in action continuously. It is possible that the vessel containing the salt had some minute opening in its walls, but although careful search was made for such an opening none was detected. The readings taken in these observations are plotted in Figs. 8 and 9.

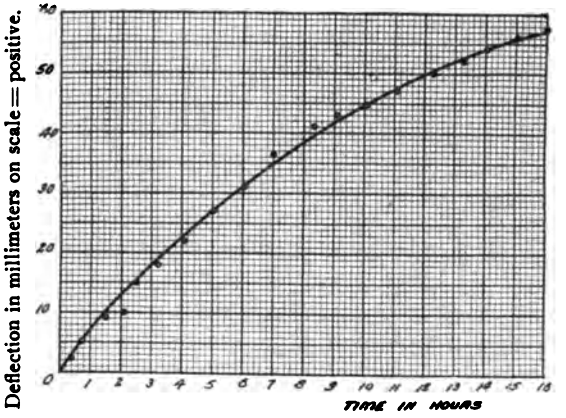

Fig. 8.
Charging action of potassium sulphate. Pressure = .08 mm.
Voltage on needle = 243 positive.

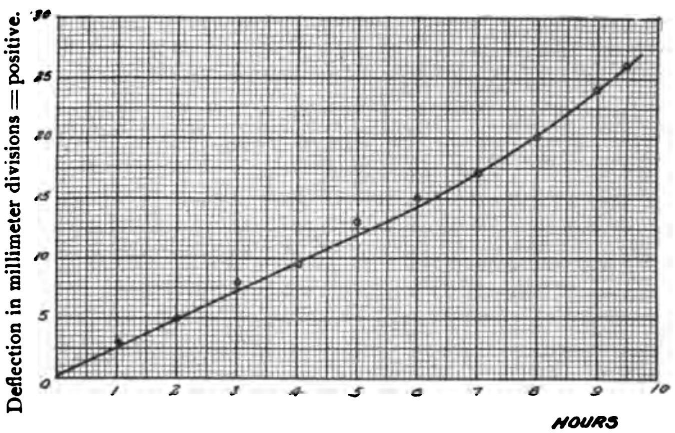

Fig. 9.
Charging action of potassium sulphate. Pressure = .06 mm.
Voltage on needle = 243 negative.

Here again it will be seen the charging action was a positive one in both cases and when the needle was positively charged its motion was somewhat more rapid than when it was negatively charged. In both of these two latter sets of observations the Volta effect produced a deflection in the negative direction when the earth connection to the free quadrants was broken at atmospheric pressure; but in each case after these earth connections were broken the complete apparatus was left undisturbed for some 8 hours in order that the needle might take up its stationary position corresponding to the Volta effect displacement. From time to time during this period observations on the scale readings were made and in this way the time was ascertained when the deflections became stationary. When these stationary states had been reached the exhaustions were then made and when the limiting pressures given above were reached, readings on the charging effect were commenced. These, it will be seen, confirm the results obtained with the potassium chloride in the earlier experiments.

These results show that in high vacua both potassium nitrate and potassium sulphate emit an excess of charged particles of the beta type. The effect observed in all cases was small and it will be noted that it required exceptional conditions to bring it into evidence. From the results obtained so far it has not been found possible to decide whether any radiation of the alpha type is emitted by the potassium salts or not. It is the intention of the writer, however, to examine the charging effect with the potassium salts covered with thin layers of aluminium. The use of such layers while preventing the emission of any alpha rays which might be present should not diminish to any great extent the intensity of the beta radiation.

The measurements made in this investigation demanded considerable patience and care, and I am indebted to Mr. M. E. Liezert and Mr. B. L. Cooke for kind assistance on occasions during the progress of the observations.

Physical Laboratory,
Toronto, May 26, 1909.

THE HEAT OF DILUTION OF ZINC AMALGAMS.

By W. D. Henderson.

WHEN the solution of a salt of a given concentration is reduced to a lower concentration by the addition of water, heat may be absorbed or evolved. Similarly, when an amalgam is diluted by the addition of mercury a heat change occurs. The heat thus absorbed or evolved is called the heat of dilution.

The relation of the heat of dilution to the electromotive force of a simple concentration cell has been pointed out by Carhart,[1] and his conclusions verified experimentally by Mellencamp for the case of a series of salts in solution, the heats of dilution of which had been determined calorimetrically by Thomsen.

The original object of the present research was to extend the investigation, above noted, to the determination of the heats of dilution of zinc amalgams. The method employed was to set up a cell of the type, conc. Zn amal. : sol. $ZnSO_4$: dil. Zn amal., and to measure the factors E.M.F. and dE/dt, and then from these data to compute the heat of dilution by means of the Helmholtz equation,

$$E = \frac{H}{nF} + T\frac{dE}{dt},$$

in which

$E =$ electromotive force in volts,
$T =$ absolute temperature,
$dE/dt =$ change of E.M.F. per degree centigrade,
$n =$ valence of the positive ion,
$F =$ 96,500 coulombs per gram equivalent, and
$H =$ heat of dilution in joules per gram ion.

Electrolyte.

The electrolyte used, both in the preparation of the amalgams and in the concentration cells as finally set up, was a nearly saturated solution of zinc sulphate, made from pure recrystallized salt.

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[1] Trans. Am. El. Chem. Soc., XII., 399, 1907.

Amalgams.

The amalgams were prepared electrolytically out of contact of air. A short rod of pure zinc, carefully wrapped in filter paper, was used as the anode; while pure twice-distilled mercury served as the cathode. In the preparation of the amalgams for each experiment, two cells were set up, one cell containing 20 grams of mercury, the other 40 grams. After thoroughly exhausting by means of an air pump, the cells were filled with hydrogen, after which they were connected up with a silver coulometer, the three being in series, and a current of 0.3 ampere passed through the system for a given time. At the close of the experiment, the mass of silver deposited in the platinum cup of the coulometer was carefully weighed, and from this it was possible to determine very accurately the amount of zinc deposited in each amalgam cell. The electrochemical equivalent taken for silver was 0.0011182 gram per coulomb.

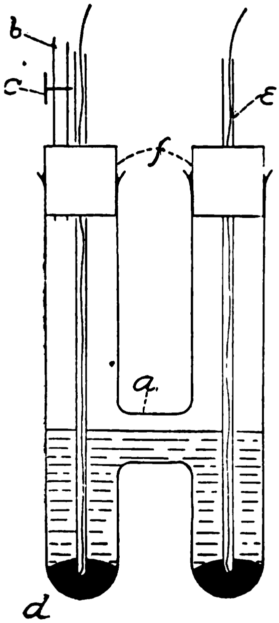

Fig. 1.

Concentration Cell.

The cell used for the determination of E.M.F. and dE/dt was of the H-form shown in Fig. 1.

The cross-arm *a* was made relatively short (about 3 cm.) in order to reduce the resistance as low as possible. Moreover, the diameter of this arm was made rather large, so that when the cell was set up, there remained a space above the liquid in the arm, thus making it possible to exhaust both limbs by means of a single connection at *b*, which was provided with a stop-cock *c*. Contact with the mercury electrodes *d* was made by means of platinum wires sealed into glass leads *e*. Each limb of the cell was closed at *f* with rubber stoppers, thoroughly cleaned and sealed in by means of a thin solution of Le-Page's glue. At the beginning of the work an all-glass cell was employed. It was found, however, that this precaution was not necessary with amalgams of the concentrations employed.

SET UP OF THE CELL.

In setting up the cell, the first step was to test the platinum tips, which communicated with the amalgams, with respect to an initial E.M.F., no tips being employed that were not practically "dead" to each other. This precaution was necessary in order to make it possible to measure the E.M.F. to the sixth decimal place. The required amount of zinc sulphate was then added to the cell, after which the amalgams were transferred from the cells in which they had been prepared. The platinum leads were put in place, and the cell was immediately connected up with a powerful air pump and exhausted. In the tube connecting the air pump and the cell was a three-way stop-cock which allowed communication both with the pump and also with an apparatus for the generation of pure hydrogen. A few strokes of the pump were sufficient to reduce the pressure in the cell to the boiling point of the solution at room temperature. The cell was first exhausted, then filled with hydrogen, this process being repeated a number of times. After being thoroughly washed out with hydrogen, the cell was filled with this gas and then sealed off. It was then placed in a constant temperature bath and readings taken.

E.M.F.

E.M.F. determinations were measured against a Weston Normall cell by means of a Wolff potentiometer reading directly to 0.00001 volt. The resistance of the concentration cell was so low that it was easily possible to interpolate readings to the sixth decimal place.

dE/dt.

The change of electromotive force with change of temperature for a given pair of amalgams was obtained by determining the E.M.F. for two or more different temperatures, and then computing the change of E.M.F. per degree centigrade. Four baths were employed having respectively temperatures of 0°, 20°, 25° and 30°. The ice-bath could be used, however, only for the lower concentrations, for reasons which appear later.

The method employed was, in general, to place the cell first in the 25-degree bath; then to carry it through the entire range of temperatures possible for the given solution and amalgam, allowing the E.M.F. readings to come to a constant value in each case. Usually each cell was taken through several such cycles, the time required being two or three days.

In every case the dE/dt values were positive, that is, an increase of temperature gave on increase of electromotive force.

Concentration of the Amalgams.

In the preparation of each set of amalgams the mercury used in every case was in the ratio of 1 : 2 by mass — that is, 20 grams of mercury in one leg of the cell, and 40 grams in the other, the same mass of zinc being deposited in each. In speaking of the concentration of any pair of amalgams, then, reference is always made to the ratio between the mass of zinc and mercury of the more concentrated electrode. For example, if in 20 grams of mercury (the amount used in every case for the concentrated electrode) there was deposited 0.2 gram of zinc, this amalgam would be referred to as a concentration of 1 per cent.

When the amalgams are prepared electrolytically and in series, as in the present case, and it is desired to plot curves in which the concentration is one of the variables, the above method and notation make for both accuracy and convenience.

Data.

Concentration cells set up as described gave remarkably constant and consistent values, no evidence of oxidation effects being manifested over a period of several days.

The following record illustrates very well the constancy of the values obtained.

Even with a concentration as low as 0.5 per cent., the values of the E.M.F. readings show practically no deviation, for a given temperature, as shown by Table II.

For convenience in referring to the different concentrations, the entire series is divided into three groups — namely, group one comprising the lower concentrations; group two, the higher concentra-

TABLE I.

Cell II-B. Concentration, 0.918 *per cent.*

Date.	Notes.	Temp.	E.M.F.
November 16, 9:00 A. M.	Cell in 25° bath.		
" 4:00 P. M.	"	25°	0.007966
" 4:15 "	Cell in ice bath.		
" 4:30 "	"	0°	7243
" 5:00 "	"	0°	7240
" 5:15 "	Cell in 25° bath.		
" 6:45 "	"	25.03	7970
" 7:00 "	"	25.03	7970
" 7:15 "	Cell in ice bath.		
" 7:30 "	"	0°	7240
" 7:45 "	"	0°	7240
" 8:00 "	"	0°	7240
" 8:15 "	Cell in 25° bath.		
" 17, 10:00 A. M.	"	25.01	7970
" 1:00 P. M.	"	25.02	7970

tions; and group three, those concentrations in the 2 per cent. region.

For the lower concentrations — that is, from 0.5 to about 2 per cent. — the electromotive force came to a constant value within a few hours after the cell was set up and placed in the 25-degree bath. Moreover, an increase of concentration gave a uniform decrease of E.M.F. and a slight though uniform increase of dE/dt values.

TABLE II.

Cell IX-C. Concentration, 0.496 *per cent.*

Date.	Notes.	Temp.	E.M.F.
March 22, 2:00 P. M.	Cell in 25° bath.		
" 6:00 "	"	25.04	0.008383
" 9:00 "	"	25.04	8388
"	Cell successively in baths at 30, 20 and 0 degrees.		
" 23, 10:00 A. M.	Cell in 25° bath.		
" 1:30 P. M.	"	25.05	8385
" 2:30 "	"	25.03	8380
"	Cell in baths at 30 and 0 degrees.		
" 7:00 P. M.	Cell in 25° bath.		
" 24, 8:00 A. M.	"	25.05	8387

In the region of the higher values — from about 2.5 to 3 per cent. — the behavior of the amalgams with respect to E.M.F. and dE/dt was quite similar to that of group one, with the exception of an increase of the slope of the E.M.F. and dE/dt curves, as shown in Fig. 2.

Concentrations lying in the 2 per cent. region gave anomalous results, and require special consideration. Three things are to be noted with respect to this region: First, the electromotive force of a given set of amalgams did not come at once to a final constant value, but seemed to approach constancy by steps; second, the E.M.F. curve makes a sharp change in direction in this region; and third, the dE/dt values undergo a sudden and enormous increase.

Table III.

Cell XIII-C. Concentration, 2.148 *per cent.*

Date.	Notes.	Temp.	E.M.F.
March 30, 4:00 P. M.	Cell in 25° bath.		
" 5:00 "	"	25.05	0.006757
" 9:00 "	"	25.06	6758
"	Cell in 30° bath.		
" 31, 8:00 A. M	Cell in 25° bath.		
" 9:00 "	"	25.04	6833
" 10:00 "	"	25.04	6833
"	Cell in 30° bath.		
" 6:00 P. M.	Cell in 25° bath.		
April 1, 8:00 A. M.	"	25.04	7110
" 10:00 "	"	25.04	7110
"	Cell successively in baths at 30 and 20 degrees.		
" 8:00 P. M.	Cell in 25° bath.		
" 2, 7:45 A. M.	"	25.12	7113
"	Cell in baths at 30 and 20 degrees.		
" 6:00 P. M.	Cell in 25° bath.		
" 3, 8:00 A. M.	"	25.05	7113
" 2:00 P. M.	"	25.05	7110
"	Cell in ice bath. E.M.F. constantly shifting.		
" 3:00 "	Cell in 25° bath.		
" 5:00 "	"	25.05	7115

The first point noted above is well illustrated by Table III., which record may be taken as a type of the behavior of the cells of the 2 per cent. region. This cell (No. XIII–C, concentration 2.148 per cent.) was under test for something over 90 hours, and approached a constant value (E.M.F.) by at least two distinct steps. The cell was first placed in the 25-degree bath, where it gave a constant E.M.F. reading of 0.006757 volt. It was next thoroughly agitated and transferred to the 30-degree bath, at which temperature readings were taken, after which it was brought back to the 25-degree bath, where the E.M.F. registered was 0.006833 volt. On again being agitated and transferred to the 30-degree bath, and then again brought back to the 25-degree bath, the cell came to a final constant value of 0.007110 volt, after which no series of changes to either higher or lower temperatures seemed to affect its constancy.

In computing the heats of dilution H, the final constant E.M.F. values for 25 degrees were substituted for E in the equation,

$$E = \frac{H}{nF} + T\frac{dE}{dt}.$$

A summary of the data obtained from a series of fourteen sets of concentrations with respect to E.M.F., dE/dt and H values is given in Table IV.

TABLE IV.

No.	Conc. of Amal.	E.M.F.	dE/dt	H
1	0.499 per cent.	0.008360	0.0000280	+ 3
2	0.960	7890	281	− 193
3	1.180	7675	309	− 296
4	1.512	7365	320	− 419
5	1.676	7120	307	− 390
6	1.842	7000	300	− 375
7	1.928	6855	277	− 270
8	2.040	6840	263	− 200
9	2.148	7110	318	− 457
10	2.224	6867	400	− 976
11	2.305	6320	900	−3960
12	2.412	5730	1710	−8736
13	2.740	4562	1800	−9480
14	3.070	3410	1830	−9920

The relationship of the four sets of values of Table IV. is beautifully shown by the following curves of Fig. 2:

Discussion of Fig. 2.

The three curves of Fig. 2 are plotted, with respect to abscissas, in terms of concentrations; with respect to ordinates, in terms of E.M.F., *dE/dt* and *H* respectively. Concentrations, to repeat, refer to the ratios of zinc to mercury of the more concentrated amalgam of each cell. The E.M.F. and *dE/dt* curves, plotted above the heavy line, represent observed values and are positive; the *H*-curve represents computed values, which are negative. The entire *H*-curve is not given in the figure, since the range of its values is so great, as compared with those of the other two curves.

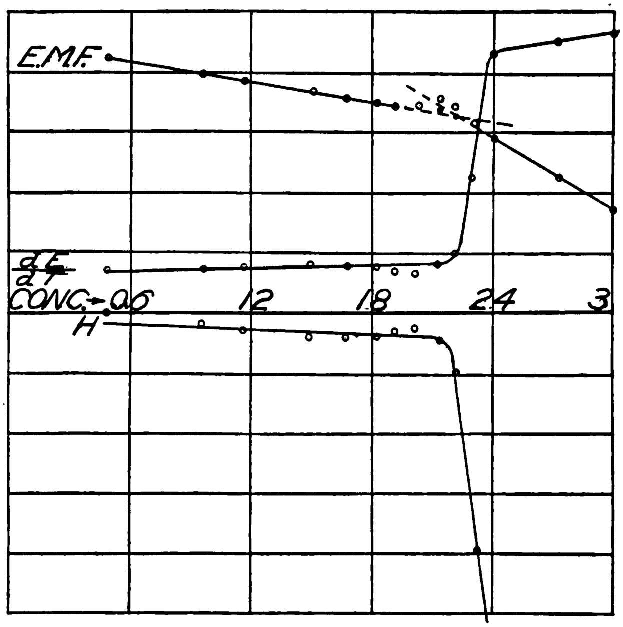

Fig. 2.

The E.M.F. curve is the most significant one of the group. It consists, as shown in Fig. 2, of two straight lines intersecting in the 2 per cent. region. The light circles represent final constant values; the dark points, initial constant values (see Table III.). This curve is worthy of special note for several reasons: First, while it is very uniform for the lower and the higher concentrations, it shows a decided break in the 2 per cent. region. This break in the E.M.F.

curve, together with the simultaneous and remarkable changes indicated by the dE/dt and H-curves, undoubtedly indicates the occurrence of some fundamental change in the constitution of the amalgam, the form of the curves of this region suggesting something analogous to a transition point. Second, the dark points of the E.M.F. curve, which represent *initial* constant values for the given concentrations, lie, it will be observed, very nearly on the projections of the two straight lines. This would imply that in this region the amalgams tend initially to obey the same laws which govern the formation of the straight line portions of the curve. And in conclusion, the final constant values of the 2 per cent. region are higher than the initial values, and the process by which they are reached does not appear to be a reversible one.

SUMMARY.

1. For the case of a concentration cell as herein specified, it was found that an increase of the zinc content of the amalgams composing the electrodes gave a decrease of E.M.F.

2. A similar increase of the zinc gave an increasing value for the dE/dt factor.

3. Assuming that the Helmholtz equation applies throughout the range of the concentrations used, it was computed that the heat of dilution for the entire series of amalgams is negative, and varies from a value that is nearly or quite negligible for a concentration of 0.5 per cent., to about 10,000 joules per gram ion for a concentration of 3 per cent.

4. In the 2 per cent. region a marked change occurs in the constitution of the amalgam.

5. A further and more thorough investigation of the behavior of zinc amalgams in the 2 per cent. region is desirable. Such an investigation is now in progress by the writer of this paper.

In conclusion, it is desired to state that this research was undertaken at the suggestion of Prof. H. S. Carhart, whose counsel throughout the progress of the work has materially aided in bringing the investigation to its present state.

UNIVERSITY OF MICHIGAN,
May, 1909.

THE SECULAR SOFTENING OF HARD STEEL.

By C. Barus.

1. *Introductory.* — The interesting result recently obtained by Miss L. C. Brant, showing persistent and marked softening of a batch of nineteen hard wires, at ordinary temperatures in the lapse of a second period of twelve years, has recalled to my mind a number of older observations to which it may be worth while to refer. The wires in question were tempered 24 years ago, by aid of a special electrical contrivance, devised by myself and used in the old Clifton Laboratory of the Johns Hopkins University, to which I was given access for the purpose through the courtesy of Professor H. A. Rowland. The strong heating currents available made it easy to temper several hundred steel wires glass hard, and to a high degree of uniformity, as the tests showed, throughout their length. In fact the hardness reached was exceptional (specific resistance 45 microhms) and much above the maximum obtained in earlier experiments abroad (specific resistance 40 microhms), by somewhat similar methods. The bearing of this on the phenomenon of temper will appear below, § 5.

The chief result gained in the present paper is this, that the limits of temper corresponding to any temperature, however practically useful and sufficient, are temporary limits only. All hard steel tends to become ultimately soft, at a rate that has been sharply recognized at all temperatures, at least in comparative values.

2. *Temper as Related to the Temperature of the Annealing Bath and the Time of Exposure to it.* — In order to interpret the results for secular softening it will be necessary to make use of the data[1] investigated by Doctor V. Strouhal and myself for the general behavior of tempered steel.

They contain an admirable example of the law subject to which chemical reaction proceeds in case of a solid, in its dependence on time and temperature, particularly so, since every detail of the variation may be traced to a nicety. These results have been systematized in the following table.

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[1] Bulletin 14, U. S. Geological Survey, 1885; or Wied. Ann. der Physik, XI., pp. 930–977, 1880.

Table I.

Change of Specific Resistance, s (microhm, c.cm., 0° C.) of Glass-hard Steel Rods, with the Temperature T and the Time t (minutes) of Tempering.

T	t	Observed. s	Computed. s	T	t	Observed. s	Computed. s
66°	0^m	40.8	40.8	185°	0^m	39.1	39.1
	60	39.4	39.4		10	26.7	26.8
	120	38.8	38.8		30	25.4	25.6
	180	38.3	38.3		60	24.7	24.8
					120	24.3	24.0
100°	0^m	39.0	39.1		180	23.7	23.5
	10	36.4	35.2				
	30	34.5	34.0	330°	0^m	37.7	37.7
	60	33.2	33.1		1	17.2	17.9
	120	31.8	31.9		30	16.8	16.7
	180	31.1	31.3		90	16.7	16.3
				1,000°	—	15.0	

Here T denotes the temperature at which the glass-hard rod is tempered by exposure during a time t minutes, s the resulting specific resistance at zero Centigrade. The computed values of s are discussed at the end of this and the subsequent paragraphs.

From the appearance of these curves (see figure) and from general considerations one would suppose that an equation like $-dn/dt = an$, where n is the number of hard molecules per c.cm. and a a constant for any given temperature T, would express the conditions of decay met with. It would then follow that the sum of the conductivities per c.cm. is

$$\frac{1}{s} = \frac{1}{s_s} - \left(\frac{1}{s_s} - \frac{1}{s_h}\right) e^{-at}$$

where s denotes the specific resistance of the sample, s_h and s_s the initial final hard and soft specific resistances, respectively, and t minutes is the time of exposure to the given temperature T. I made these computations from the observations at the initial parts of the curves using $a/2.3$ for common logarithms,

$T=$	66°	100°	185°	330°,
$10^6 \times a/2.3=$	356	1,926	7,490	29,560 ;

and also for the two ends of the curves by introducing an intermediate constant, or special hard state for each rod. But it is easily seen that the equation is inapplicable. In fact one need merely compare a criterion like the quantity $\log\,[(1/s_0 - 1/s_h)/(1/s_0 - 1/s)]\cdot$ with t, which relation should give a straight line on the assumption, to see that the result is about as curvilinear as the original equation. The only advantage of the a-values (first series) is that they convey some clue as to the character of these variations with temperature, T.

The next supposition as to the law of decay would be $-dn/dt = an^2$ where the hard molecules are supposed to react on each other. This is equally inapplicable, by the same tests.

It is clear therefore, that the law of decay contained in the above results, Table I., is not apt to be of a simple character. I therefore contented myself with the most direct exponential capable of reproducing the data in question, with a reasonable degree of approach. This may be found by beginning with the curves for resistance and deducing the law of decay from it. A comparison of the above criterion and t^n, where n is the fraction, shows that

$$\frac{1}{s} = \frac{1}{s_0} - \left(\frac{1}{s_0} - \frac{1}{s_h}\right)e^{-at^{30/T}} \qquad (1)$$

where T is the temperature at which the rods are tempered and t the time of exposure, gives a fairly good account of the phenomenon as a whole, so far as these observations are concerned. The constants are

$T=$	66°	100°	185°	330°
$10^5 \times a/2.3=$	123	153	996	577

and Table I. summarizes the values of s so computed.

It will be seen that at 66° and 185° the coincidence of observed and computed is satisfactory. At 100° and 330° there are outstanding discrepancies. In the former case (100°) these may be explained away by postulating an initial rod whose hardness is $s = 41$ instead of 39, which would be permissible since in the Baltimore rods even the hardness $s = 46$ is reached. At 330° such a rod would have to be much too soft; but here small differences of s at the limits of error have a disproportionate effect. On the whole the observations are sufficiently reproduced for the present purposes, since variations in the homogeneity and structure of the rods necessarily detract from the sharpness of the observed values. Moreover, everything depends on the degree to which the *initial* rod may have been under-tempered, as will presently appear, § 5. Finally, it is difficult to define the soft state accurately.

3. *Time Required to Completely Soften Hard Steel at Various Temperatures.*— The observations and the computed curves are given in the following figure.

In the same figure the results of secular or spontaneous softening,

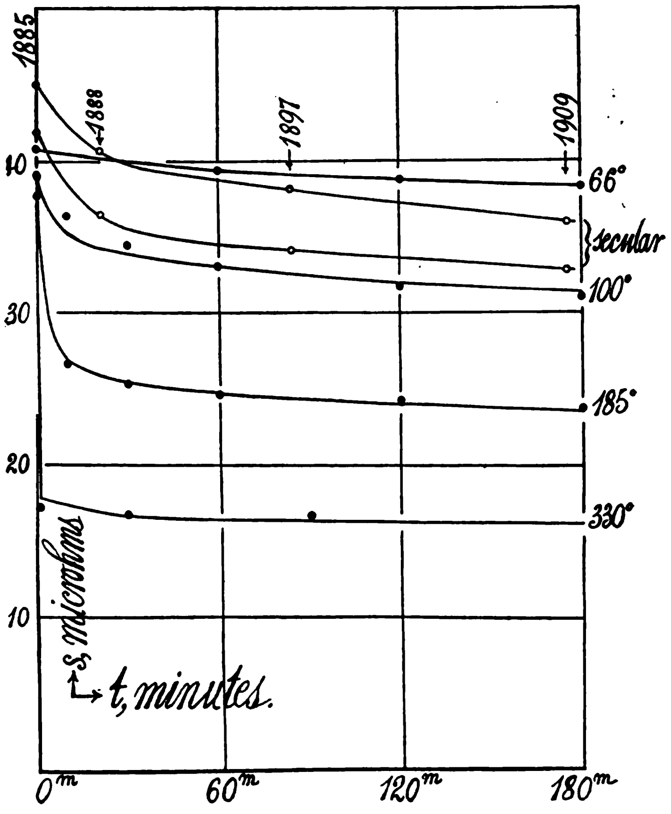

Fig. 1.

for the period of 24 years, may be inscribed, showing how far the decay of hardness has progressed. They will be referred to at greater length in § 4. Clearly, however, the original inference drawn by Doctor Strouhal and myself from observations extending throughout periods of hours, viz., that there is a definite limit of hardness for each temperature, no matter how long the rod may be annealed, is no longer tenable. The limit, when the exposure is prolonged through a period of years, can only be the soft state.

The question thus suggests itself how long it would take the above rods to become as soft as corresponds to temperature at $T = 330°$ where $s = 17.9$. If t_{330} is this time at $T°$

$$t_{330} = \left(\frac{1}{a} \log \frac{1/s_t - 1/s_h}{1/s_t - 1/s} \right)^{T/30}$$

and the results are

$T =$	66°	100°	185°	330°
$t_{330} =$	.45 year	.36 year	.10 year	1 minute

so far as the equation is trustworthy. The times so obtained are smaller than one would anticipate, and without doubt too small. The reason for this is referable to the relatively low value entered for the initial glass-hard state. When observations are conducted through so short a time the degree of glass-hardness from which the tempering proceeds is of marked importance in its bearing on the character of the curves. It will presently appear that the above batch of wires must initially have been below the extreme of hardness.

4. *Temper Varying at Ordinary Temperatures in the Lapse of Time.* — The equation (1) assumed was not a little alluring, inasmuch as it gives at ordinary temperatures $T = 30°$, the simple law of decay $-dn/dt = an$. In fact it seemed not unreasonable that with the avoidance of the initial shock, as it were, experienced on plunging hard steel into a high temperature bath the real conditions of decay would appear. But this expectation is necessarily doomed to disappointment, when rods like the present of exceptionally high initial temper are compared with the preceding. By beginning with softer rods, however, the continuity of the distributive equation (1) need not be broken.

Here, if the above test be made by comparing the logarithmetic quotient and t^n where t is now expressed in years, it will be found that n is at least $\frac{1}{2}$. For the thicker rods n is smaller. In both cases the observation after three years is relatively too high, which means probably that the average temperatures of the storage room during these years was higher than during the later years; but other conditions may have supervened. The observations after 12 and 24 years in the case of the diameters .081 cm. and .044 cm., are practically coincident. I will therefore accept

$$\frac{1}{s} = \frac{1}{s_s} - \left(\frac{1}{s_s} - \frac{1}{s_h}\right) e^{-a\sqrt{t}}$$

for these rods giving $\alpha/2.3 = .0116$ for the former and $\alpha/2.3 = .0115$ for the latter. The thick rods do not conform to the equation. The following table shows the results.

TABLE II.

Spontaneous Softening of Hard Steel. $1/s = 1/s_s - (1/s_s - 1s_h)e^{-a\sqrt{t}}$; *t in Years.*

$t =$	0	3.1	12.2	24.0	
s computed =	44.8	41.1	38.1	36.1	Diameter, .044 cm.
s observed =	44.8	40.6	38.1	36.0	$a/2.3 = .0115$
s computed =	45.0	41.2	38.2	36.0	Diameter, .081 cm.
s observed =	45.0	40.7	38.2	36.1	$a/2.3 = .0116$
s computed =	41.9	37.3	33.9	31.7	Diameter, .126 cm.
s observed =	41.9	36.5	34.2	32.9	$a/2.3 = .0175$

The agreement would have been better if in t^n, n were somewhat smaller than $\frac{1}{2}$. The period which must elapse before these rods soften to the temper corresponding to 330° is $t_{330} = 2,840$ years in the first and second cases (diameters .044 and .081) and $t_{330} = 1,180$ years in the third (unsatisfactory) case (diameter .126).

For these thick rods the exponent $n = \frac{1}{2}$, as the table shows, is clearly too large. It should even be somewhat smaller than $\frac{1}{8}$. Accepting the latter, the agreement of observed and computed hardness is very close, but the age at which these rods would soften to the temper of 330° naturally becomes much larger in comparison. I omit the data.

5. *The Initial State.* — The behavior of the new rods will repay

special consideration. In case of the thick rods it is possible that the steel was of a different kind (lower in carbon) than the thin batches; but it is more probable that the thick rods do not begin with the glass-hard state, a portion of their substance (the inner layers) being softer. If the part of a section, $l - a$, where a is a fraction, is permanently soft and the part a hard at the beginning,

$$\frac{1}{s} = \frac{1}{s_0} - a\left(\frac{1}{s_0} - \frac{1}{s_h'}\right) e^{-\alpha\sqrt{t}}, \tag{2}$$

s_h' being the specific resistance of the ideal hard rod. If

$$t = 0, \qquad \frac{1}{s_h} = \frac{(1 - a)}{s_0} + \frac{a}{s_h'},$$

and therefore

$$\frac{1}{s} = \frac{1}{s_0} - \left(\frac{1}{s_0} - \frac{1}{s_h}\right) e^{-\alpha\sqrt{t}},$$

reproducing the above equation, which has been found inapplicable for the thick rods.

It is interesting to note, however, that if equation (2) be assumed, it is equivalent to postulating an initial rod of less hardness than the observed value. Thus, for instance, for the thick rods, $a = .964$, whence $\alpha/2.3 = .0121$ and $s_h' = 39.2$ replaces $s_h = 41.9$, with a nearly complete coincidence of observed and computed hardness. It is, moreover, interesting to note that α now has a value agreeing more nearly with the thin rods and the period of softening would be $t_{330} = 2,340$ years. The earlier rods of § 2 were thus inadvertently observed along a different part of the complete curve, a part more remote from the origin, and therefore less steep than for the case of the new rods.

In fact, by postulating a softer initial glass-hard rod, about of the same temper as in the cases of §§ 2, 3, it is possible to retain the equation

$$\frac{1}{s} = \frac{1}{s_0} - \left(\frac{1}{s_0} - \frac{1}{s_h}\right) e^{-\alpha t}$$

there inferred, as expressing the case of secular softening at ordinary temperatures. I will omit the data with the remark that the

the time t_{330} of softening would thus also be much reduced, being 343 years for the thin and 293 years for the thick rods, and corresponding to the relatively small values given in § 3 for the initially softer rods discussed. As a matter of fact, however, the present long period of tempering (24 years) shows that an equation with the first power of t is not applicable even when the initial state is disregarded for the logarithmic ratio or criterion continues to bear a curvilinear relation to t.

6. *Conclusion.* — From the results as a whole it follows that the specific resistance s of tempered rods as related to temperature T and time of exposure t may be expressed by an equation of the form

$$\frac{1}{s} = \frac{1}{s_s} - \left(\frac{1}{s_s} - \frac{1}{s_h}\right) \epsilon^{-at^{\alpha/T}}$$

where a and α are constants and s_s and s_h are the specific resistances of the extremes of soft and glass-hard steel. The value of a depends very essentially on the initial degree of hardness obtained. This equation corresponds to a law of decay with two constants α and a relatively to time t,

$$n = n_0 e^{-at^{\alpha/T}},$$

where n is the number of hard molecules after a time t at a temperature T, and where α is a function of temperature. Hence

$$-dn/dt = \frac{a\alpha}{T} n \left(\frac{a}{\log(n_0/n)}\right)^{T/\alpha - 1}.$$

Such a law is unfortunately not suggestive. It has thus very little interest beyond the temporary use made of it. It is far removed from the special case $T = \alpha$, or $-dn/dt = an$, so valuable in other similar discussions. Nevertheless it should be accentuated that the sensitiveness of the resistance method must lead to a severe test of the true law of decay whenever it is found. This is elusive in the above cases of spontaneous softening, because the thin rods experience less decay from greater initial hardness, while the temper of the thick rods shows greater decay from less initial hardness. Such an observation is troublesome to reconcile with any usual decay equation.

The conclusion which apparently follows if experiments are conducted for a period of hours, viz., that there is a limiting degree of hardness for each temperature, is not ultimately correct, though it reproduces the phenomenon in its practical aspects. The only final limit is the soft state, and this may be reached at ordinary temperatures, though it takes enormously longer than at higher temperatures. In a general way one may state that if by any miracle the tools in Joseph's carpenter shop, at the beginning of the Christian era, had been glass-hard steel, they would now be so soft as to need retempering to be efficient.

BROWN UNIVERSITY, PROVIDENCE, R. I.

DISPERSION OF ELECTRIC DOUBLE REFRACTION AND ORDINARY DISPERSION IN LIQUIDS.

BY HAROLD E. McCOMB.

THE discovery and historical development of the work on electric double refraction has been thoroughly discussed by Blackwell.[1] It has been largely confined to a study of carbon bisulphide with, on the whole, very divergent results. For the sake of comparison the following table gives the constant for this liquid as determined by various observers.

TABLE I.

Observer.	Quincke.[2]	Elmén.[3]	Blackwell.	Hagenow.[4]
Wave-length,	590	560	590	590
Constant,	3.27×10^{-7}	4.02×10^{-7}	3.57×10^{-7}	2.85×10^{-7}

(Lemoine[5] gives the value as 3.70×10^{-7} for "red light.")

The wide divergence of these values, which cannot be ascribed to the difference in wave-length used, is difficult to explain. It undoubtedly arises in part from a difference in the carbon bisulphide used but in the writer's opinion it is also to be ascribed in part to inaccurate measurements of some of the experimental factors involved in the determinations. The double refraction in most cases was measured by a Babinet compensator. Elmén and Hagenow used a Brace half-shade system. The potentials were usually furnished by a static machine and measured by an absolute electrometer. Blackwell, however, used a storage battery and should have had little difficulty in measuring the potentials accurately.

The equation used by Elmén for calculating the retardations from the observed rotation of the compensator is incorrect. The observations which he made at low potentials are also questionable since they do not agree with the results obtained by Morse,[1] who used the same method later. Koenig[2] has suggested that the apparent departure from Kerr's law, as found by Elmén, might arise from double refraction in the cover glasses. This could hardly be true however, since the retardation is obtained from settings made with the potentials "on" and "off," which would eliminate any effect of this kind.

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[1] H. L. Blackwell, Proc. Am. Ac. Sci., Vol. 41, p. 647, 1906.

[2] G. Quincke, Wied. Ann., 19, p. 729, 1883.

[3] G. W. Elmén, PHYS. REV., 20, p. 54, 1905.

[4] C. F. Hagenow, PHYS. REV., 27, p. 196, 1908.

[5] J. Lemoine, Compt. Rend., 122, p. 835, 1896.

Blackwell's investigation was primarily to obtain the dispersion, relatively little attention being directed toward obtaining absolute values, while Hagenow sought to establish the absolute value as well as the dispersion.

Havelock[3] has recently developed a general theory for the dispersion of double refraction and shown that it is in agreement with the results of Blackwell and Hagenow. He finds the following relation

$$\frac{B\lambda n_0}{(n_0^2 - 1)^2} = C,$$

in which B is the double refraction constant, λ the wave-length, n_0 is the refractive index, and C is a constant for the substance.

The object of the present investigation was to determine the double refraction constants of various liquids, for various colors, and to compare them with the refractive indices.

Twelve liquids have been investigated, all of which are good insulators, and with the exception of two (chlor-benzol and brom-benzol) none distorted the field of view under the electric strain. There were cases in which the electric force caused the field of view to become blurred, but by successively applying and removing the potential a few times it could always be cleared. All but one of the liquids (ethylene chloride from Eimer and Amend) were obtained from Kahlbaum and only in the case of carbon bisulphide was further purification undertaken. This was usually distilled with $CaCl_2$ and then either filtered through a porous cup into the testing cell or distilled directly into the cell without filtration. This difference in treatment did not appreciably change the optical properties.

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[1] L. B. Morse, PHYS. REV., Vol. 23, p. 252, 1906.

[2] W. Koenig, Ann. d. Phys., 28, p. 413, 1909.

[3] T. H. Havelock, Proc. Royal Soc., Vol. 80, p. 28, 1907; PHYS. REV., Vol. 28, p. 136, 1909.

1. The Optical System.

The optical system was the same as used by Hagenow (l. c.) with a few improvements. The light was more nearly homogeneous, two dispersing prisms being used in place of one and special precautions taken to eliminate extraneous light. Briefly stated, rays from the sun produced a pure spectrum in the plane of the slit S_2 (Fig. 1). By adjusting the prisms P_1P_2 any desired color passed through the slit and served as a monochromatic source for making the tests. The polariscope system consisted of a polarizing Nicol N_1 (set at 45° to the lines of force of the electric field in E); a thin mica strip H (with its optic axis at 45° to the plane of polarization) covering half the field; the cell E containing the liquid and the condenser plates; the mica compensator C (covering the whole field) mounted on a graduated circle; the analyzing Nicol N_2 (at right angles to N_1); and the telescope T focused on the half shade H. The electrical connections were made by means of screw contacts which was found to be very essential at low potentials. In some cases the effect of the order of the compensator was eliminated by following Hagenow's method of producing a wave-length retardation; in others the retardation produced by a definite potential difference was measured by the rotation of a compensator.

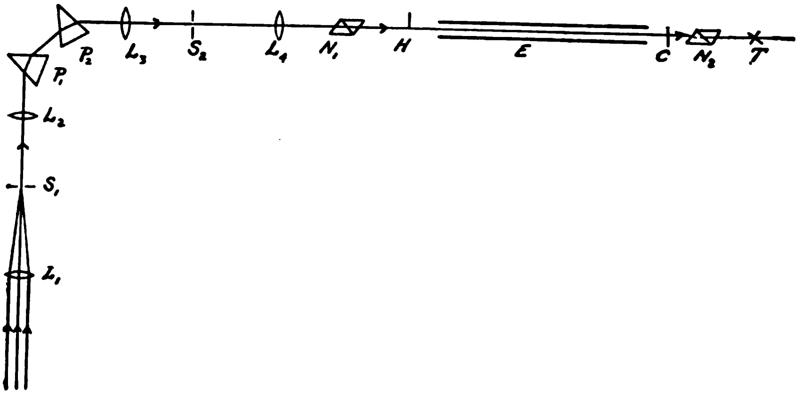

Fig. 1.

2. THE ELECTRICAL SYSTEM.

A Holtz machine supplied the potentials ranging from 10,000 to 15,000 volts. The potential difference between the condenser plates was regulated by means of a variable point discharge and measured by a Kaufmann absolute electrometer of the bi-filar type.[1] A mirror was attached to the movable plate and the deflections read by a telescope and scale. The scale was first calibrated by loading the plate with known weights and noting the deflection produced. From this the force of electrical attraction corresponding to an observed deflection was directly obtained and the corresponding potential difference calculated from the formula for an absolute electrometer.[2] The distance between the electrometer plates was adjusted for parallelism and at the same time measured by placing three glass cubes of equal thickness between them and adjusting for contact for all three points at the same time, after which the cubes were removed. This proved more satisfactory than making measurements at different points on the edges of the plates by means of a cathetometer microscope.

For the lower potentials, ranging from 1,400 to 6,000 volts, a series of dynamos was used, each capable of giving a maximum of 1,200 volts. The potential difference was measured by a Weston voltmeter with multipliers giving ranges: 0–150, 0–300, 0–450, 0–600, 0–750, 0–1,500, 0–3,000, 0–4,500, and 0–6,000 volts.

Potentials below 1,400 volts were supplied from a storage battery and also measured by the voltmeter. The storage battery and generators fluctuated so little during an entire set of observations that potentials as high as 6,000 volts were measured to an accuracy of at least .2 per cent.

3. CALIBRATION OF THE COMPENSATOR.

To measure very small retardations it was necessary to calibrate the compensator, for the various wave-lengths used. The method followed was to measure first the potential required to produce a wave-length retardation, then immediately drop down to relatively low potential differences and rotate the compensator for a "match."

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[1] Cf. J. E. Almy, Ann. d. Phys. (4), Vol. 1, p. 508, 1900.

[2] Gray's Absolute Measurements, Vol. 1, p. 264.

The potential difference for various angles of the compensator and for various colors was thus obtained. Kerr's law was then applied as follows: Let

$$\delta_1 = B \cdot \frac{V_1^2}{a^2} \cdot l$$

be the retardation produced by a definite low potential difference V_1, l being the length of the condenser plates, a their distance apart, and B the double refraction constant; and

$$\delta_2 = B \cdot \frac{V_2^2}{a^2} \cdot l = 1$$

the retardation produced by the high potential V_2.

Dividing gives

$$\delta_1 = \frac{V_1^2}{V_2^2} \qquad (1)$$

which gives in waves the retardation corresponding to the observed rotation of the compensator.

Representative observations on carbon bisulphide for six different wave-lengths are given in Table II. In the first column are given

TABLE II.

Calibration of Compensator. (CS$_2$, *Kahlbaum.*) $l = 50$ *cm.* $a = .175$ *cm.*

Wave-length.	One Wave-length Retardation.	Small Retardations.			Order of Compensator. (Degrees.)	Mean Order. (Degrees.)
	Potential Difference. (Electrostatic.)	Potential Difference. (Electrostatic.)	Compensator. (Degrees.)	Retardation. (Waves.)		
440	35.8	4.13	18.7	13.3×10^{-3}	7.50	7.48
		3.63	14.4	10.25	7.46	
460	36.7	2.57	6.94	4.9	7.32	7.34
		4.13	18.0	12.6	7.36	
500	39.75	3.92	15.5	9.71	6.54	6.58
		4.43	19.8	12.4	6.62	
540	41.96	2.61	6.1	3.87	6.16	6.37
		4.47	19.40	11.4	6.58	
600	45.5	2.58	6.05	3.21	5.54	5.58
		3.13	8.75	4.74	5.62	
660	48.5	2.58	5.65	2.82	5.24	5.11
		3.40	10.2	4.91	5.02	
		4.17	15.3	7.39	5.06	

the wave-lengths; in the second the potential differences (as measured by the absolute electrometer) required to produce a wave-length retardation. The third, fourth and fifth columns give respectively the potential differences (as measured by the voltmeter), the corresponding rotations of the compensator for a "match," and the retardations calculated from equation (1).

For this optical system it has been shown by Professor Tuckerman[1] that

$$tg2\pi(N_\lambda + \delta) = -\frac{\sin 2\pi N \sin 2\psi}{1 - 2\sin^2 \pi N \sin^2 2\psi}, \tag{2}$$

in which N_λ is one half the order of the half-shade mica strip; δ the retardation (in waves) produced by the liquid, N the order of the compensator; and ψ the angle between the principal axis of the compensator and the polarizing Nicol.

Then

$$tg2\pi N_\lambda = -\frac{\sin 2\pi N \sin 2\psi_0}{1 - 2\sin^2 \pi N \sin^2 2\psi_0},$$

in which ψ_0 is the value of ψ for $\delta = 0$. This is best obtained as $\frac{1}{2}$ (the rotation of the compensator between two consecutive "matches" on plane polarized light $- 90°$). For the system used $\psi_0 = 6.7°$ for all wave-lengths. Substituting this value of $2\pi N_\lambda$ in equation (2) gives after some reduction

$$tg2\pi\delta = -\sin 2\pi N(\sin 2\psi - \sin 2\psi_0)$$
$$\times \left[\frac{1 + 2\sin^2 \pi N \sin^2 2\psi_0 \sin 2\psi}{(1 - 2\sin^2 \pi N \sin^2 2\psi)(1 - 2\sin^2 \pi N \sin^2 2\psi_0) + \sin^2 2\pi N \sin 2\psi \sin 2\psi_0}\right].$$

For the compensator here used $\sin 2\pi N$ is small. Expanding to quantities of the second order in $\sin 2\pi N$ gives

$$\delta = -N(\sin 2\psi - \sin 2\psi_0)\{1 - \tfrac{1}{6}\sin^2 2\pi N[1 - (\sin 2\psi - \sin 2\psi_0)^2] + \text{etc.}\}.$$

Neglecting the quantity in the brackets introduces an error not greater than .3 per cent. for the compensator used, hence the equation in the following form may be used

$$\delta = N[\sin 2(\varphi - \varphi_0) + \sin 2\varphi_0], \tag{3}$$

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[1] L. B. Tuckerman, Jr., Univ. of Nebr. Studies, April, 1909.

in which $\varphi = -(\psi - \psi_0)$ is the rotation of the compensator between "matches" obtained respectively with potentials "on" and "off," and $\varphi_0 = \psi_0$.

Using this formula the value of N was calculated for the different wave-lengths from the results of the calibration given in column 5 of Table II. In column 6 are recorded these values of N and column 7 gives the means. In Fig. 2 is plotted the curve represented by equation (3) while the points located on it represent the corresponding values obtained by calibration. The equation is thus experimentally verified.

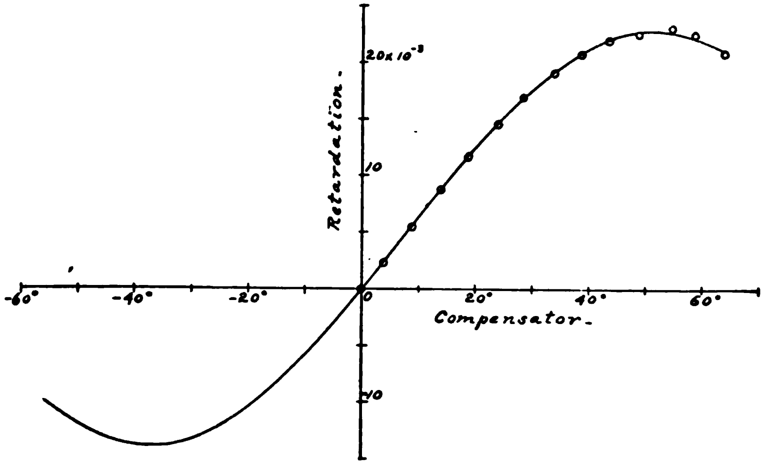

Fig. 2.

In addition to the values obtained with carbon bisulphide, which required a high potential difference to produce a wave-length retardation, observations were also made on nitro benzol in which the effect is about 100 times as large for the same potential difference. On account of the absorption in the violet a shorter cell was required so that a potential difference of about 2,400 volts gave a wave-length retardation. The potential was supplied by two of the dynamos in series and measured by the voltmeter. Observations made on a wave-length 500 $\mu\mu$ gave for the order of the compensator the value 6°.8 as against 6.6 in the other test. Each liquid has for the calibration its weak point as well as its strong. Carbon bisulphide is quite transparent and the compensator settings were very accurate even in the violet, but the high potentials could not be measured to an accuracy less than one per cent. With nitro benzol the potentials (measured by the voltmeter) were accurate in all cases to .2 per cent., but the depolarization in this liquid at potentials sufficient to produce a wave-length retardation reduced considerably the accuracy of the compensator settings. The two calibrations for the color tested agree fairly well but the results from the carbon bisulphide being regarded the better were used. The dispersion in the mica as given in column 7 of Table II. and plotted in Fig. 3 agrees very closely with that obtained in this laboratory by Mr. Tool who used an entirely different method.

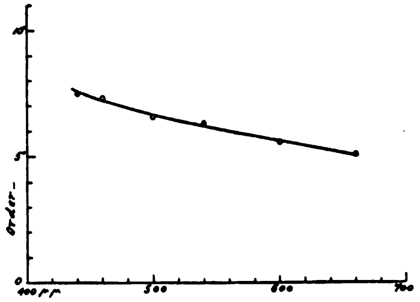

Fig. 3.

4. End Effects.

A correction for the effect at the ends of the condenser was necessary, especially when short ones were used. To determine this correction measurements were made on the same liquid with three different lengths (2, 15 and 50 cm.). It was found that the same small correction applied to each length gave consistent values for the double refraction constant. This correction was found to be approximately .18 cm.; it is therefore important when short condensers are used. The following table (III.) shows the value of this correction as obtained from different tests.

5. Refractive Indices and Absorption.

The refractive indices were determined by means of the spectrometer and are accurate to .0002. Observations were made on

TABLE III.

End Effects.

Liquid.	Length of Condenser. (cm.)	End Correction. (cm.)
Di-methyl aniline.	15 2	.19
Nitro toluol.	50 15	.17
Nitro toluol.	50 2	.19
	Mean	.18

exactly the same colors used in the double refraction measurements at practically the same temperature and usually directly afterwards.

The absorption region of the various liquids was also incidentally obtained. This was done by first photographing the spectrum of an iron arc produced by a Rowland concave diffraction grating. A photograph under the same conditions was then taken with the liquid (.4 cm. thick — in a quartz cell) placed between the arc and the slit of the spectroscope. The photographs through air gave lines ranging from 600 to 225 $\mu\mu$. All exposures were for the same length of time. An absorption region when present was found in all the liquids to be very abrupt.

6. EXPERIMENTAL RESULTS.

For the purpose of testing the reliability and accuracy of the system as a whole repeated observations were made on carbon bisulphide. In Table IV. are given the values of the constant B for commercial carbon bisulphide as obtained with potentials sufficient to produce a wave-length retardation. The four different columns represent tests with four different samples from the same bottle. The sixth and seventh columns give respectively the mean values of B as obtained from these results, and the corresponding values obtained from Hagenow's curve.[1] The close agreement with his furnishes a satisfactory check on the method. The last two columns give respectively the refractive indices and the value of B as calculated from Havelock's equation (page 544). The value of C given at the head of the column was the mean obtained by applying that equation to the experimental values in column 6, in which determination the unreliable values were omitted.

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[1] Though not stated in his paper, Mr. Hagenow informed me that he used commercial CS_2.

Table V. gives the values for Kahlbaum's carbon bisulphide under the same treatment as the commercial. The values of B are throughout the spectrum somewhat greater for Kahlbaum's than for the commercial while the refractive indices are practically the same. Havelock's equation fits the two equally well.

TABLE IV.

Carbon Bisulphide (CS_2). (*Commerical, distilled.*) $l = 50$ *cm.* $a = .175$ *cm.*

Wave-length. ($\mu\mu$)	B. (22° C.)				Mean. B	B (Hagenow.)	Refractive Index. (23°.5 C.)	B. Calculated. ($C = 1.024 \times 10^{-11}$.)
	1.	2.	3.	4.				
440	4.50×10^{-7}	4.56×10^{-7}	4.17×10^{-7}	4.36×10^{-7}	4.40×10^{-7}	4.35×10^{-7}	1.6672	4.42×10^{-7}
460	4.14	4.35	4.12	4.05	4.16	4.05	1.6586	4.12
480	3.88	4.09	3.91	3.83	3.92	3.78	1.6504	3.83
500	3.68	3.80	3.62	3.56	3.65	3.58	1.6439	3.61
520	3.45	3.54	3.43	3.67	3.52	3.39	1.6380	3.41
540	3.26	3.33	3.23	3.17	3.25	3.19	1.6330	3.23
560	3.10	3.17	3.07	3.03	3.09	3.04	1.6286	3.07
580	2.95	3.08	2.90	2.89	2.95	2.90	1.6250	2.93
600	2.81	2.83	2.79	2.78	2.80	2.76	1.6218	2.80
630	2.65	2.65	2.63	2.60	2.63	2.60	1.6172	2.62
660	2.51	2.51	2.47	2.47	2.49	2.47	1.6138	2.46

TABLE V.

Carbon Bisulphide (CS_2). (*Kahlbaum, distilled.*) $l = 50$ *cm.* $a = .175$ *cm.*

Wave-length. ($\mu\mu$)	B. (22°.5 C.)			Refractive Index. (23°.5 C.)	B Calculated. ($C = 1.114 \times 10^{-11}$.)
	1.	2.	Mean.		
440	4.87×10^{-7}	4.78×10^{-7}	4.82×10^{-7}	1.6672	4.81×10^{-7}
460	4.52	4.55	4.53	1.6586	4.48
480	4.28		4.28	1.6504	4.18
500	3.95	3.89	3.92	1.6439	3.93
520	3.69		3.69	1.6380	3.71
540	3.52	3.49	3.51	1.6330	3.51
560	3.36		3.36	1.6285	3.34
580	3.17		3.17	1.6250	3.19
600	3.03	2.96	3.00	1.6218	3.05
630	2.88		2.88	1.6172	2.85
660	2.66	2.61	2.64	1.6138	2.69

Tables VI. to XIII. give similar results for various liquids belonging to the benzene series in which as in all that follow the retardations were measured by rotating the compensator and applying equation (3). It was difficult to obtain accurate results on chlor- and brom-benzol on account of motions in the liquid produced by the electric strain. The constant for nitro benzol decreases gradually under electric strain so that it was found necessary to make corrections for this change during a series of observations.

TABLE VI.

Benzol (C_6H_6). *l* = 50 *cm.* *a* = .175 *cm.*

Wave-length. ($\mu\mu$)	Potential Difference. (Electrostatic.)	Compensator. (Degrees.)	Retardation. (Waves.)	*B* (23°.7 C.)	Refractive Index. (24° C.)	*B* Calculated. ($C = .329\times10^{-11}$.)
440	10.05	19.65	14.0×10^{-3}	$.845\times10^{-7}$	1.5191	$.842\times10^{-7}$
460	10.05	19.35	13.0	.780	1.5150	.792
480	10.10	19.40	12.7	.760	1.5113	.748
500	10.12	19.18	12.0	.710	1.5081	.708
520	10.05	18.22	11.1	.660	1.5054	.673
540	10.05	18.10	10.7	.640	1.5031	.644
560	10.05	17.60	10.1	.600	1.5009	.615
580	10.00	17.80	9.8	.595	1.4992	.582
600	10.00	18.10	9.7	.590	1.4974	.564
630	10.05	17.60	9.0	.540	1.4953	.533
660	10.05	17.40	8.5	.510	1.4935	.505

TABLE VII.

Nitro Benzol ($C_6H_5 \cdot NO_2$). *l* = 2.18 *cm.* *a* = .171 *cm.*

Wave-length. ($\mu\mu$)	Potential Difference. (Electrostatic.)	Compensator. (Degrees.)	Retardation. (Waves.)	*B* (22°.5 C.)	Refractive Index. (25° C.)	*B* Calculated. ($C = 117\times10^{-11}$.)
440	2.07	17.23	12.4×10^{-3}	388×10^{-7}	1.5829	$381\times10_{-}^{7}$
460	2.07	16.70	11.4	356	1.5759	355
480	2.07	16.12	10.7	333	1.5696	333
500	2.07	15.77	9.95	311	1.5645	314
520	2.07	15.60	9.60	296	1.5602	296
540	2.08	14.95	8.90	275	1.5566	283
560	2.08	15.00	8.65	269	1.5533	274
580	2.07	14.73	8.20	256	1.5506	255
600	2.07	14.47	7.80	242	1.5481	246
630	2.07	14.24	7.35	230	1.5449	231
660	2.07	14.10	6.90	216	1.5424	218

Table VIII.

Nitro Toluol ($C_6H_5 \cdot CH_3 \cdot NO_2$). $l = 2.18$ *cm.* $a = .171$ *cm.*

Wave-length. ($\mu\mu$)	Potential Difference. (Electrostatic.)	Compensator. (Degrees.)	Retardation. (Waves.)	B (24° C.)	Refractive Index. (22° C.)	B Calculated. ($C = 53.7 \times 10^{-11}$
440	3.10	(16.00)	11.6×10^{-3}	(161×10^{-7})	1.5757	171×10^{-7}
460	3.10	16.65	11.4	158	1.5695	160
480	3.09	15.78	10.5	147	1.5635	150
500	3.10	15.88	10.0	139	1.5589	141
520	3.11	15.92	9.75	134	1.5549	134
540	3.11	15.62	9.25	127	1.5516	127
560	3.12	15.46	8.9	122	1.5485	121
580	3.12	15.84	8.8	121	1.5463	116
600	3.13	15.31	8.25	112	1.5439	111
630	3.13	15.31	7.85	107	1.5409	104
660	3.13	14.76	7.25	99	1.5386	100

Observations in parenthesis are of questionable accuracy.

Table IX.

Di-methyl Aniline [$C_6H_5N(CH_3)_2$]. $l = 2.18$ *cm.* $a = .171$ *cm.*

Wave-length. ($\mu\mu$)	Potential Difference. (Electrostatic.)	Compensator. (Degrees.)	Retardation. (Waves.)	B (25° C.)	Refractive Index. (26° C.)	B Calculated. ($C = 4.34 \times 10^{-11}$.)
440	10.73	18.28	13.1×10^{-3}	15.4×10^{-7}	1.5875	14.4×10^{-7}
460	10.57	17.07	11.7	13.4	1.5808	13.4
480	10.53	16.10	10.7	12.8	1.5747	12.6
500	10.60	15.63	9.85	11.8	1.5696	11.9
520	10.58	15.95	9.80	11.7	1.5654	11.2
540	10.67	15.35	8.95	10.5	1.5620	10.7
560	10.67	15.15	8.75	10.3	1.5588	10.2
580	10.60	14.92	8.3	9.8	1.5562	9.7
600	10.60	14.43	7.8	9.3	1.5538	9.3
630	10.64	14.21	7.3	8.6	1.5506	8.8
660	10.60	13.81	6.8	8.1	1.5482	8.3

TABLE X.

Di-ethyl Aniline $[C_6H_5N(C_2H_5)_2]$. $l = 2.18$ *cm.* $a = .171$ *cm.*

Wave-length. ($\mu\mu$)	Potential Difference. (Electro-static.)	Compen-sator. (Degrees.)	Retardation. (Waves.)	B (25°.2 C.)	Refractive Index. (26° C.)	B Calculated. ($C=4.84\times10^{-11}$.)
440	10.25	16.60	12.0×10^{-8}	15.3×10^{-7}	1.5668	14.9×10^{-7}
460	10.23	15.81	10.8	13.8	1.5608	13.9
480	10.22	15.52	10.3	13.2	1.5555	13.1
500	10.22	(14.23)	(9.0)	(11.5)	1.5510	12.3
520	10.18	15.13	9.3	12.0	1.5473	11.7
540	10.20	14.70	8.7	11.2	1.5443	11.1
560	10.17	13.85	8.0	10.4	1.5414	10.6
580	10.13	13.50	7.5	9.8	1.5390	10.1
600	10.18	13.72	7.4	9.5	1.5369	9.75
630	10.15	13.52	7.0	9.1	1.5341	9.2
660	10.20	13.60	6.7	8.6	1.5319	8.7

Observations in parenthesis are of questionable accuracy.

TABLE XI.

a-mono-brom-naphtalene $(C_{10}H_7Br)$. $l = 2.18$ *cm.* $a = .171$ *cm.*

Wave-length. ($\mu\mu$)	Potential Difference. (Electrostatic.)	Compensator. (Degrees.)	Retardation. (Waves.)	B (21°.5 C.)	Refractive Index. (26°.3 C.)	B Calculated. ($C=2.83\times10^{-11}$.)
440	11.40	18.18	12.8×10^{-8}	13.2×10^{-7}	1.6970	13.4×10^{-7}
460	11.40	17.60	12.0	12.4	1.6885	12.5
480	11.40	16.90	11.2	11.5	1.6802	11.6
500	11.40	17.05	10.7	11.0	1.6738	11.0
520	11.45	16.50	9.85	10.2	1.6683	10.6
540	11.30	15.80	9.4	9.85	1.6638	9.87
560	11.30	15.70	9.1	9.5	1.6596	9.35
580	11.30	15.35	8.5	9.0	1.6560	8.9
600	11.30	15.43	8.3	8.7	1.6529	8.6
630	11.25	14.65	7.5	7.9	1.6487	8.05
660	11.25	(15.08)	(6.9)	(7.3)	1.6454	7.6

Observations in parenthesis are of questionable accuracy.

Table XII.

Chlor-Benzol (C_6H_5Cl). $l = 15.2$ *cm.* $a = .175$ *cm.*

Wave-length. ($\mu\mu$)	Potential Difference. (Electro-static.)	Compen-sator. (Degrees.)	Retardation. (Waves.)	B (24° C.)	Refrac-tive Index. (25°.5 C.)	B Calculated. ($C = 4.74\times10^{-11}$.)
440	4.08	15.26	11.0×10^{-3}	13.3×10^{-7}	1.5428	13.2×10^{-7}
460	4.08	15.10	10.4	12.6	1.5386	12.5
480	4.08	14.45	9.6	11.6	1.5348	11.8
500	4.08	14.69	9.25	11.2	1.5314	11.2
520	4.08	13.78	8.45	10.2	1.5286	10.7
540	4.08	13.75	8.20	9.9	1.5264	10.2
560	4.10	14.75	8.50	10.2	1.5241	9.7
580	4.10	14.63	8.10	9.8	1.5222	9.3
600	4.10	12.75	6.90	8.3	1.5207	8.9
630	4.10	13.62	7.0	8.6	1.5185	8.5
660	4.10	14.05	6.9	8.3	1.5164	8.0

Table XIII.

Brom-Benzol (C_6H_5Br). $l = 15.2$ *cm.* $a = .175$ *cm.*

Wave-length. ($\mu\mu$)	Potential Difference. (Electro-static.)	Compen-sator. (Degrees.)	Retardation. (Waves.)	B (24°.9 C.)	Refrac-tive Index. (25° C.)	B Calculated. ($C = 4.12\times10^{-11}$.)
440	4.27	18.40	13.2×10^{-3}	14.6×10^{-7}	1.5797	13.3×10^{-7}
460	4.30	18.06	12.3	13.4	1.5751	12.5
480	4.27	(16.00)	(9.8)	(10.9)	1.5708	11.8
500	4.27	16.10	10.2	11.3	1.5671	11.2
520	4.27	15.20	9.35	10.4	1.5639	10.6
540	4.27	15.20	9.00	10.0	1.5611	10.1
560	4.27	15.20	8.8	9.9	1.5585	9.65
580	4.27	(13.10)	(7.3)	(8.1)	1.5565	9.25
600	4.20	14.40	7.7	8.8	1.5547	8.9
630	4.27	(16.10)	(8.3)	(9.2)	1.5522	8.4
660	4.27	14.90	7.3	8.1	1.5501	7.9

Observations in parenthesis are of questionable accuracy.

Tables XIV., XV. and XVI. give the constants for liquids belonging to another series and of lower dispersion.

Table XVII. gives for comparison the dispersion of double refraction and the wave-length as shown by the photographs at which abrupt absorption begins. The dispersion was obtained by dividing the difference between the values of the constant B at 440 and 660 $\mu\mu$ by its value at 590. No relation between the

TABLE XIV.

Chloroform ($CHCl_3$). $l = 15.2$ *cm.* $a = .175$ *cm.*

Wave-length. ($\mu\mu$)	Potential Difference (Electrostatic.)	Compensator (Degrees.)	Retardation. (Waves.)	B (23° C.)	Refractive Index. (24°.5 C.)	B Calculated. ($C = -2.40 \times 10^{-11}$.)
440	6.60	−14.70	-9.30×10^{-8}	-4.32×10^{-7}	1.4536	-4.66×10^{-7}
460	6.60	−14.85	−9.05	−4.20	1.4514	−4.41
480	6.60	−14.90	−8.65	−4.01	1.4495	−4.19
500	6.61	−14.77	−8.20	−3.78	1.4479	−4.06
520	6.61	−15.90	−8.45	−3.90	1.4464	−3.80
540	6.61	−15.10	−7.82	−3.72	1.4452	−3.63
560	6.62	−15.20	−7.65	−3.53	1.4440	−3.48
580	6.65	−15.40	−7.47	−3.41	1.4430	−3.36
600	6.66	−14.90	−7.02	−3.21	1.4421	−3.24
630	6.60	−14.60	−6.60	−3.05	1.4412	−3.07
660	6.61	−14.65	−6.30	−2.91	1.4400	−2.91

TABLE XV.

Amyl Chloride [$CH_3(CH_2)_3CH_2Cl$]. $l = 15.2$ *cm.* $a = .175$ *cm.*

Wave-length. ($\mu\mu$)	Potential Difference. (Electrostatic.)	Compensator. (Degrees.)	Retardation. (Waves.)	B (23° C.)	Refractive Index. (24°.2 C.)	B Calculated. ($C = 2.77 \times 10^{-11}$.)
440	7.07	15.56	11.2×10^{-8}	4.52×10^{-7}	1.4155	4.49×10^{-7}.
460	7.07	15.18	10.4	4.21	1.4138	4.26
480	7.07	14.80	9.8	3.96	1.4123	4.03
500	7.07	15.01	9.5	3.85	1.4109	3.86
520	7.07	14.85	9.2	3.73	1.4097	3.69
540	7.07	14.83	8.8	3.56	1.4088	3.53
560	7.07	14.87	8.6	3.47	1.4077	3.39
580	7.07	14.48	8.1	3.27	1.4068	3.26
600	7.09	14.45	7.8	3.15	1.4062	3.14
630	7.09	14.15	7.3	2.94	1.4052	2.97
660	7.08	13.93	6.85	2.76	1.4044	2.83

magnitude of the dispersion and the position of the absorption region is apparent.

The theoretical equation given by Havelock is within experimental errors in agreement with the results obtained for the various liquids. This being true it is to be preferred to the empirical relation published in an abstract of this work a few months ago,[1] which stated: for a given liquid the slope of the B-curve is proportional

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[1] H. E. McComb, PHYS. REV., Vol. 27, p. 336, 1908.

TABLE XVI.

Ethylene Chloride ($C_2H_4Cl_2$). $l = 15.2$ *cm.* $a = .175$ *cm.*

Wave-length. ($\mu\mu$)	Potential Difference. (Electro-static.)	Compensator. (Degrees.)	Retardation. (Waves.)	*B* (18°.6 C.)	Refractive Index. (19°.4 C.)	*B* Calculated. ($C = 3.26 \times 10^{-11}$.)
440	6.40	17.98	12.9×10^{-3}	6.34×10^{-7}	1.4551	6.35×10^{-7}
460	6.37	17.09	11.7	5.82	1.4532	6.03
480	6.37	17.32	11.4	5.68	1.4512	5.72
500	6.40	17.47	10.9	5.42	1.4498	5.47
520	6.40	17.33	10.6	5.22	1.4484	5.21
540	6.41	17.30	10.2	5.02	1.4473	5.00
560	6.43	17.25	9.9	4.86	1.4461	4.79
580	6.45	17.52	9.75	4.78	1.4452	4.61
600	6.43	17.82	9.5	4.63	1.4444	4.43
630	6.43	17.01	8.7	4.30	1.4432	4.21
660	6.43	17.08	8.3	4.06	1.4424	4.00

TABLE XVII.

Absorption Region and Dispersion.

Liquid.	Dispersion of Electric Double Refraction.	Wave-length at which Total Absorption Begins.
Carbon bisulphide,	.682	372 $\mu\mu$
α-mono-brom naphtalene,	.662	345
Nitro benzol,	.652	420
Di-methyl aniline,	.641	346
Di-ethyl aniline,	.625	346
Nitro toluol,	.622	425
Brom benzol,	.593	340
Benzol,	.588	275
Chlor benzol,	.571	none.
Chloroform,	.530	none.
Ethylene chloride,	.520	340
Amyl chloride,	.518	none.

throughout the spectrum to the slope of the refractive index curve at the same wave-length.

In conclusion the writer wishes to thank Dr. C. A. Skinner, of this university, for the many kind and helpful suggestions received during this work.

THE BRACE LABORATORY OF PHYSICS,
UNIVERSITY OF NEBRASKA, LINCOLN, NEB.

DISPERSION OF MAGNETIC DOUBLE REFRACTION IN LIQUIDS COMPARED WITH THAT OF ELECTRIC DOUBLE REFRACTION

By C. A. Skinner.

THAT a number of liquids when placed in a magnetic field are plane doubly refracting to light propagated in a direction normal to the field and that this property follows Kerr's law for corresponding double refraction in an electric field has been discovered by Cotton and Mouton.[1] These physicists, investigating nitro benzol, also found that within experimental errors the dispersion of double refraction is the same in the magnetic as in the electric field.

In this paper are given the results of a simultaneous investigation of the double refraction of those liquids in a magnetic field which Mr. McComb has been studying in an electric.[2] In addition to the plane double refraction normal to the field the circular double refraction (rotation) along the lines of force was also incidentally measured. Differential absorption (dichroism) both along and across the lines of force was carefully sought for but nothing found.

The results corroborate in general the conclusions of Cotton and Mouton, and, as shown by McComb for electric double refraction, the form of the dispersion curve fits in most cases — within experimental errors — the theory of Havelock.[3]

Experimental Arrangement.

The apparatus was about the same as that recently used for measuring magnetic rotation and ellipticity produced by metal films.[4] For measuring rotation the arrangement was exactly the same. Light from the sun as source passed successively through a

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[1] A. Cotton and H. Mouton, Compt. Rend., July and Nov., 1907, and July, 1908.

[2] See preceding paper.

[3] T. H. Havelock, Proc. Roy. Soc., Vol. 80, p. 28, 1907. Phys. Rev., Vol. 28, p. 136, 1909.

[4] C. A. Skinner and A. Q. Tool, Phil. Mag., December, 1908.

Fuess spectral instrument (for giving the desired wave-length), a collimating lens, a Nicol prism, the liquid studied — placed between the pierced pole-pieces of the electromagnet — and a Lippich analyzing system mounted on a graduated circle.

For measuring the relative retardation normal to the field the electromagnet was placed in the proper position and provided with new pole pieces possessing faces 6 cm. long and 1 cm. wide — with an air gap of about 7 mm. The Nicol was set to give a vibration of the tra .smitted light at 45° to the lines of force and the Lippich system replaced by a Brace elliptic analyzing system consisting of a very thin strip of mica (with its optic axis at 45° to the plane of polarization) covering half the field of view, followed by a somewhat thicker sheet of mica (the compensator) covering the whole field and carried by a graduated circle. The relative retardation of the vibration perpendicular to the lines of force over that parallel to them was obtained from the formula[1]

$$\delta = N\left[\sin 2(\varphi - \varphi_0) + \sin 2\varphi_0\right], \tag{1}$$

in which δ = retardation in waves; N = the order of the compensator (.0181 wave for 560 $\mu\mu$); φ = the angle of rotation of compensator from position of "match" on plane polarized light to that on the light analyzed; φ_0 = the angle of rotation of the compensator from a "match" on the plane polarized light to the nearest position in which its optic axis is parallel (or perpendicular) to the plane of polarization. The magnitude of this angle φ_0 in the instrument used was 4.9°. Opposite rotations for φ and φ_0 give opposite signs to their magnitudes. The sign of φ serves to determine (from a previous test of the compensator) which of the two components of the light vibration is propagated with the slower velocity.

For tests normal to the field the liquid was placed in a cell made of glass tubing of rectangular cross-section (6 × 11 mm.). The length of the light path in the liquid was 6 cm. in all cases where the absorption was not too great.

The strength of the field in the space between the center of the pole faces was calculated for various magnetizing currents from the rotation (of light passing along the lines of force) produced by a given length of carbon bisulphide — the poles having been pierced at that point for the purpose. The mean field strength over the 6 cm. length of pole face was obtained therefrom by comparing the double refraction (normal to the field) of a 6 cm. cell with that of a 1.5 cm. cell placed at the center of the pole faces. One of the liquids producing the largest effect (α-mono-brom-naphthalene) was used for this purpose. It was found that the mean field strength sought was .97 that at the center of the pole faces.

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[1] The general theory of this instrument has recently been published by Professor Tuckerman, University of Nebraska Studies, April, 1909. The deduction of this formula from his general equation is given by McComb (*l. c.*).

Experimental Results.

In the following tables (I., II., III.) are given the observed relation between relative retardation and field strength (c.g.s.) for three different liquids showing the largest effects.

Table I.

Nitro Benzol. Wave-length, 500 $\mu\mu$.

Field (H)	9,050	10,100	10,900	11,750	13,750
Retardation (δ)	1.69×10^{-3}	2.11×10^{-3}	2.46×10^{-3}	2.93×10^{-3}	3.89×10^{-3}
δ/H^2	2.07×10^{-13}	2.07×10^{-13}	2.07×10^{-13}	2.11×10^{-13}	2.06×10^{-13}

Table II.

Nitro Toluol. Wave-length, 500 $\mu\mu$.

Field (H)	9,000	9,950	10,900	11,750	13,750
Retardation (δ)	1.15×10^{-3}	1.38×10^{-3}	1.63×10^{-3}	1.96×10^{-3}	2.57×10^{-3}
δ/H^2	1.42×10^{-13}	1.39×10^{-13}	1.39×10^{-13}	1.42×10^{-13}	1.36×10^{-13}

Table III.

α-mono-brom-naphthalene. Wave-length, 500 $\mu\mu$.

Field (H)	8,900	10,050	10,850	11,750	13,800
Retardation (δ)	1.71×10^{-3}	2.19×10^{-3}	2.60×10^{-3}	3.04×10^{-1}	4.15×10^{-1}
δ/H^2	2.17×10^{-13}	2.17×10^{-13}	2.21×100^{-13}	2.20×10^{-13}	2.18×10^{-13}

These results show that the fluctuation of the ratio of retardation to the square of the field strength is such that it is to be explained as arising from experimental errors, hence they corroborate the statement made by Cotton and Mouton that Kerr's law for electric double refraction is also applicable to the magnetic double refraction of these liquids.

With this law as a basis the double refraction constants of the different liquids at various wave-lengths were obtained. This law states that

$$\delta = B_m H^2 l, \qquad (2)$$

δ being the relative retardation in waves produced by a length l of the liquid in a field of intensity H (c.g.s.). B_m is then defined as the retardation produced by unit length of liquid in a unit field, or

$$B_m = \left(\frac{1}{\lambda_S} - \frac{1}{\lambda_P}\right)_{H=1} \qquad (3)$$

defines it so that a positive value of B_m indicates that the component vibration parallel to the direction of the field is propagated with a velocity greater than that of the perpendicular component, λ_S and λ_P representing the wave-lengths in the liquid of the perpendicular and parallel components respectively.

TABLE IV.

Benzol.

Wave-length. ($\mu\mu$)	Along Field (12,500).	Across Field (13,800 c.g.s.). Temperature 26.5° C.					
	Rotation. $\left(\frac{\text{degrees}}{\text{cm.} \times H}\right)$	Compensator. (Degrees.)	Retardation. (Waves.)	B_m.	B_e (McComb.)	B_m/B_e	B_m Calculated. ($C = 4.12 \times 10^{-10}$.)
440	1.00×10^{-3}	1.52	1.14×10^{-3}	10.0×10^{-13}	$.84 \times 10^{-7}$	11.9×10^{-6}	10.4×10^{-13}
460	.89	1.41	1.05	9.2	.78	11.8	9.9
480	.83	1.50	1.08	9.5	.76	12.4	9.5
500	.75	1.49	1.02	8.9	.71	12.5	8.8
520	.70	1.45	.95	8.3	.66	12.5	8.3
540	.63	1.39	.89	7.8	.64	12.2	8.0
560	.57	1.35	.86	7.5	.60	12.5	7.6
580	.53	1.43	.86	7.5	.59	12.7	7.4
600	.49	1.37	.83	7.3	.59	12.4	7.1
630	.44	1.41	.79	6.9	.54	12.8	6.6
660	.39	1.41	.76	6.7	.51	13.1	6.3

In Tables IV. to XVII. are incorporated the results of the investigation of dispersion, as indicated by the values of B_m, throughout the visible spectrum. From the observed rotation of the compen-

TABLE V.

Nitro Benzol.

Wave-length. ($\mu\mu$)	Along Field (13,200).	Across Field (13,600 c.g.s.). Temperature 24° C.					
	Rotation. $\left(\frac{\text{degrees}}{\text{cm.}\times H}\right)$	Compensator. (Degrees.)	Retardation. (Waves.)	B_m	B_e (McComb.)	B_m/B_e	B_m Calculated. ($C=13.1\times10^{-10}$.)
440	$.66\times10^{-3}$	5.90	4.49×10^{-3}	40.5×10^{-13}	388×10^{-7}	$.104\times10^{-6}$	42.1×10^{-13}
460	.60	5.82	4.30	38.7	356	.108	40.0
480	.56	5.82	4.15	37.5	333	.112	37.2
500	.51	5.64	3.86	34.9	311	.111	35.2
520	.48	5.58	3.71	33.5	296	.113	33.5
540	.445	5.52	3.56	32.1	275	.116	31.5
560	.415	5.41	3.41	30.8	269	.114	30.2
580	.38	5.29	3.22	29.0	256	.113	28.6
600	.36	5.21	3.06	27.6	242	.114	27.4
630	.325	5.14	2.91	26.3	230	.114	26.1
660	.295	5.10	2.74	24.6	216	.114	24.4

TABLE VI.

Nitro Toluol.

Wave-length. ($\mu\mu$)	Along Field (12,100).	Across Field (13,800 c.g.s.). Temperature 25° C.					
	Rotation. $\left(\frac{\text{degrees}}{\text{cm.}\times H}\right)$	Compensator. (Degrees.)	Retardation. (Waves.)	B_m	B_e (McComb.)	B_m/B_e	B_m Calculated. ($C=7.96\times10^{-10}$)
440	$.65\ \times10^{-3}$	3.70	2.83×10^{-3}	24.9×10^{-13}	161×10^{-7}	$.154\times10^{-6}$	25.1×10^{-13}
460	.61	3.56	2.65	23.3	158	.147	23.5
480	.57	3.46	2.50	22.0	147	.150	22.3
500	53	3.40	2.37	20.8	139	.150	21.0
520	.475	3.37	2.24	19.6	134	.147	19.6
540	.445	3.29	2.13	18.7	127	.147	18.8
560	.41	3.23	2.05	18.0	122	.147	17.9
580	.375	3.28	2.01	17.6	121	.145	17.3
600	.35	3.20	1.89	16.6	112	.148	16.5
630	.315	3.12	1.77	15.5	107	.145	15.6
660	.285	3.16	1.71	15.0	99	.151	14.6

sator the retardation δ for the wave-length considered was calculated by equation (1), and from this the value of B_m was obtained by the use of (2). The ratio of B_m to the corresponding constant of electric double refraction (B_e) as obtained by McComb is also given in those cases where the liquid furnished an appreciable or reliable value for the latter constant. Finally, from the refractive indices as determined by McComb (l. c.) the value of B_m was calculated from Havelock's formula

$$\frac{B_m \lambda n_0}{(n_0^2 - 1)^2} = C, \tag{4}$$

n_0 being the refractive index, λ the wave-length (*in vacuo*), and C a constant for the liquid. The value of C used in the calculation in each case is given at the top of the column.

TABLE VII.

Aniline.

Wave-length. ($\mu\mu$)	Along Field (14,200).	Across Field (13,800 c.g.s.). Temperature 25° C.				
	Rotation. $\left(\frac{\text{degrees}}{\text{cm.}\times H}\right)$	Compensator. (Degrees.)	Retardation. (Waves.)	B_m	Refractive Index.	B_m Calculated. ($C=2.04\times 10^{-10}$.)
440	1.49×10^{-8}					
460	1.31	1.04	$.77\times 10^{-3}$	6.7×10^{-13}	1.6114	7.1×10^{-13}
480	1.18	.97	.69	6.1	1.6056	6.5
500	1.05	1.04	.71	6.2	1.6008	6.1
520	.95	1.00	.65	5.7	1.5967	5.9
540	.86	.97	.63	5.5	1.5931	5.6
560	.78	.97	.60	5.3	1.5898	5.3
580	.71	.97	.58	5.1	1.5872	5.1
600	.65	1.03	.60	5.3	1.5847	4.9
630	.59	.98	.55	4.8	1.5815	4.6
660	.54	.99	.53	4.6	1.5789	4.4

Chloroform and bromoform give appreciable magnitudes for the electric double refraction — both negative. In the magnetic field they gave also an appreciable negative value for the double refraction constant, but too small to measure with any satisfactory accuracy. Amyl chloride which is positive in the electric field did not show an appreciable effect in the magnetic field. Ethylene chloride which, according to McComb, gives reliable values in the electric field was found to possess in the magnetic field a value which diminished so rapidly with time that no satisfactory measurements were made.

Aniline (Table VII.) mono-ethyl aniline (Table IX.) and toluidin (Table XVII.) gave measurable results in the magnetic field but no observable effect in the electric.

TABLE VIII.

Di-methyl Aniline.

Wave-length. ($\mu\mu$)	Along Field (12,900).	Across Field (13,800 c.g.s.). Temperature 26° C.					
	Rotation $\left(\frac{\text{degrees}}{\text{cm.} \times H}\right)$	Compensator. (Degrees.)	Retardation. (Waves.)	B_m	B_e (McComb.)	B_m/B_e	B_m Calculated. ($C=3.41\times10^{-10}$.)
440	1.64×10^{-3}	1.63	1.23×10^{-3}	10.8×10^{-13}	15.4×10^{-7}	$.70\times10^{-6}$	11.2×10^{-13}
460	1.46	1.53	1.14	10.0	13.4	.75	10.6
480	1.31	1.56	1.12	9.8	12.8	.76	9.9
500	1.16	1.51	1.10	9.6	11.8	.81	9.3
520	1.04	1.51	1.02	8.9	11.7	.76	8.8
540	.93	1.49	.96	8.4	10.5	.80	8.4
560	.85	1.48	.93	8.1	10.3	.79	7.9
580	.755	1.43	.87	7.6	9.8	.77	7.6
600	.70	1.45	.85	7.4	9.3	.79	7.3
630	.61	1.42	.79	6.9	8.6	.80	6.9
660	.56						

TABLE IX.

Mono-ethyl Aniline. Field, 13,700 *c.g.s. Temperature,* 27° *C.*

Wave-length. ($\mu\mu$)	Compensator. (Degrees.)	Retardation. (Waves.)	B_m	Refractive Index.	B_m Calculated. ($C=4.01\times10^{-10}$.)
520	1.63	1.08×10^{-3}	9.6×10^{-13}	1.5621	10.6×10^{-13}
540	1.60	1.04	9.2	1.5592	9.7
560	1.66	1.04	9.2	1.5563	9.3
580	1.68	1.02	9.0	1.5539	8.9
600	1.67	.98	8.7	1.5515	8.4
630	1.70	.95	8.4	1.5487	8.0
660	1.61	.89	7.9	1.5464	7.6

A remarkable difference in the magnitudes obtained from two different samples of brom-benzol (both from Kahlbaum) is shown in Tables XIV. and XV., the constant being over twice as large in the

TABLE X.

Di-ethyl Aniline.

Wave-length. ($\mu\mu$)	Along Field (12,900).	Across Field (13,800 c.g.s.). Temperature 25° C.					
	Rotation. $\left(\frac{\text{degrees}}{\text{cm.}\times H}\right)$	Compensator. (Degrees.)	Retardation. (Waves.)	B_m	B_e (McComb.)	B_m/B_e	B_m Calculated. ($C=2.62\times10^{-10}$.)
440	1.40×10^{-3}	1.18	$.91\times10^{-8}$	8.0×10^{-13}	15.3×10^{-7}	$.52\times10^{-6}$	7.9×10^{-13}
460	1.22	1.17	.87	7.6	13.8	.55	7.5
480	1.10	1.18	.84	7.4	13.2	.56	7.1
500	.97	1.13	.78	6.8	—	—	6.7
520	.87	1.14	.75	6.6	12.0	.55	6.3
540	.78	1.09	.70	6.1	11.2	.54	6.0
560	.69	1.03	.64	5.6	10.4	.54	5.7
580	.625	1.07	.64	5.6	9.8	.57	5.4
600	.575	1.00	.59	5.2	9.5	.55	5.2
630	.51	.95	.53	4.6	9.1	.51	4.9
660	.46	.96	.51	4.5	8.6	.52	4.7

TABLE XI.

α-mono-brom-naphthalene.

Wave-length. ($\mu\mu$)	Along Field (14,200).	Across Field (13,800 c.g.s.). Temperature 25° C.					
	Rotation. $\left(\frac{\text{degrees}}{\text{cm.}\times H}\right)$	Compensator. (Degrees.)	Retardation. (Waves.)	B_m	B_e (McComb.)	B_m/B_e	B_m Calculated. ($C=8.7\times10^{-10}$.)
440	1.80×10^{-3}	5.94	4.60×10^{-8}	40.5×10^{-13}	13.2×10^{-7}	3.06×10^{-6}	41.0×10^{-13}
460	1.57	5.80	4.35	38.0	12.4	3.05	38.0
480	1.42	5.68	4.10	36.0	11.5	3.13	35.8
500	1.27	5.42	3.80	33.4	11.0	3.03	33.7
520	1.14	5.44	3.65	32.0	10.2	3.14	31.8
540	1.03	5.31	3.45	30.3	9.8	3.09	30.1
560	.94	5.23	3.30	29.0	9.5	3.05	28.6
580	.86	5.10	3.10	27.2	9.0	3.02	27.5
600	.79	5.14	3.00	26.3	8.7	3.02	26.3
630	.71	4.96	2.80	24.5	7.9	3.10	24.5
660	.66	5.01	2.70	23.7			23.1

TABLE XII.

Chlor-Benzol. (*Sample No. 1.*) *Field*, 13,800 *c.g.s.* *Temperature* 25° C.

Wave-length. ($\mu\mu$)	Compensator. (Degrees.)	Retardation. (Waves.)	B_m	B_e (McComb.)	B_m/B_e
440	1.81	1.39×10^{-3}	12.2×10^{-13}	19.5×10^{-7}	$.65\times10^{-6}$
460	1.85	1.38	12.1	17.8	.68
480	1.87	1.34	11.8	16.5	.71
500	1.76	1.22	10.7	15.6	.69
520	1.67	1.12	9.8	15.0	.65
540	1.67	1.08	9.5	14.1	.67
560	1.54	.96	8.4	13.9	.60
580	1.62	.98	8.6	12.9	.66
600	1.59	.93	8.2	12.3	.67
630	1.66	.93	8.2	11.4	.72
660	1.56	.85	7.5	10.8	.69

TABLE XIII.

Chlor-Benzol. (*Sample No.* 2.)

Wave-length. ($\mu\mu$)	Along Field (14,200).	Across Field (13,200 c.g.s.). Temperature 25° C.				
	Rotation. $\left(\frac{\text{degrees}}{\text{cm.}\times H}\right)$	Compensator. (Degrees.)	Retardation. (Waves.)	B_m	B_e (McComb.)	B_m/B_e
440	$.95\times10^{-8}$	2.00	1.54×10^{-8}	13.8×10^{-13}	13.2×10^{-7}	1.04×10^{-6}
460	.85	2.00	1.49	13.4	12.6	1.06
480	.78	1.95	1.39	12.5	11.6	1.08
500	.70	1.85	1.28	11.5	11.2	1.03
520	.64	1.80	1.19	10.7	10.2	1.05
540	.58	1.87	1.21	10.9	9.9	1.10
560	.52	1.91	1.19	10.7	10.2	1.05
580	.50	1.85	1.12	10.1	9.8	1.03
600	.46	1.90	1.13	10.2	8.3	1.22
630	.42	1.88	1.05	9.5	8.6	1.10
660	.38	1.79	1.02	9.2	8.3	1.11

one as in the other. A test by McComb of the same samples showed no correspondingly large difference in the electric field. Two samples of chlor-benzol were also tested (Tables XII. and XIII.) which showed an appreciable difference in magnitude of B_m though not so large as brom-benzol. These two liquids were found to be unsatisfactory for making optical investigations owing to the field of view being periodically disturbed apparently by some motions within the liquid. The other liquids investigated did not reveal any appreciable variations in the constant with different samples.

TABLE XIV.

Brom-Benzol. (*Sample No.* 1.) *Field*, 13,800 *c.g.s.* *Temperature*, 25° *C.*

Wave-length. ($\mu\mu$)	Compensator. (Degrees.)	Retardation. (Waves.)	B_m
440	1.80	1.39×10^{-3}	12.2×10^{13}
460	1.82	1.36	11.9
480	1.80	1.30	11.4
500	1.71	1.19	10.4
520	1.69	1.13	9.9
540	1.72	1.12	9.8
560	1.64	1.04	9.1
580	1.70	1.04	9.1
600	1.70	1.00	8.8
630	1.74	.98	8.6
660	1.72	.93	8.2

TABLE XV.

Brom-Benzol. (*Sample No.* 2.)

Wave-length. ($\mu\mu$)	Along Field (14,200).	Across Field (13,600 c.g.s.). Temperature 25° C.				
	Rotation. $\left(\frac{\text{degrees}}{\text{cm.}\times H}\right)$	Compensator. (Degrees.)	Retardation. (Waves.)	B_m	B_e (McComb.)	B_m/B_e
440	1.04×10^{-3}	4.00	3.06×10^{-3}	27.0×10^{-13}	14.0×10^{-7}	1.93×10^{-6}
460	.95	4.02	2.99	26.3	13.4	1.96
480	.86	4.03	2.90	25.5	10.9	2.33
500	.78	4.10	2.85	25.0	11.3	2.20
520	.71	4.20	2.78	24.5	10.4	2.15
540	.64	4.14	2.68	23.6	10.0	2.36
560	.60	4.10	2.60	22.9	9.9	2.30
580	.55	4.07	2.50	22.0	8.1	2.70
600	.51	4.09	2.41	21.2	8.8	2.30
630	.46					
660	.43					

Carbon bisulphide (Table XVI.) was the only liquid tested in which the constant in the magnetic field was negative to that in the electric.

An inspection of the tables reveals that the ratio B_m/B_e is, for those liquids giving reliable results, practically constant throughout the spectrum, the fluctuation in the value being such as would be ascribed to experimental errors. The values of B_m as calculated from Havelock's formula agree equally well with the measured quantities. There seems to be a general tendency of the calculated

TABLE XVI.

Carbon Bisulphide.

Wave-length. ($\mu\mu$)	Along Field (13,200).	Across Field (13,800 c.g.s.). Temperature 28° C.					
	Rotation. $\left(\frac{\text{degrees}}{\text{cm.}\times H}\right)$	Compensator. (Degree.)	Retardation. (Waves.)	B_m	B_e (McComb.)	B_m/B_e	B_m Calculated. ($C=-1.44\times10^{-10}$).
440	1.38×10^{-8}	$-.95$	$-.74\times10^{-8}$	-6.5×10^{-13}	4.17×10^{-7}	-1.56×10^{-6}	-6.2×10^{-13}
460	1.23	—	—	—	—	—	—
480	1.13	$-.90$	$-.65$	-5.7	3.91	-1.46	-5.4
500	1.03	$-.86$	$-.60$	-5.7	3.62	-1.47	-5.1
520	.95	$-.79$	$-.53$	-4.7	3.43	-1.37	-4.7
540	.86	$-.75$	$-.49$	-4.3	3.23	-1.33	-4.5
560	.79	$-.81$	$-.51$	-4.5	3.07	-1.46	-4.3
580	.73	$-.74$	$-.46$	-4.0	2.90	-1.38	-4.1
600	.67	$-.70$	$-.42$	-3.7	2.79	-1.33	-3.9
630	.60	$-.68$	$-.39$	-3.4	2.63	-1.31	-3.6
660	.55	$-.73$	$-.40$	-3.5	2.45	-1.43	-3.4

TABLE XVII.

Toluidin. Length of Cell, 4 cm.

Wave-length. ($\mu\mu$)	Along Field (14,200).	Across Field (13,800 c.g.s.).				
	Rotation. $\left(\frac{\text{degrees}}{\text{cm.}\times H}\right)$	Compensator. (Degrees.)	Retardation. (Waves.)	B_m	Refractive Index.	B_m Calculated. ($C=2.39\times10^{-10}$.)
440	1.31×10^{-8}					
460	1.17					
480	1.06					
500	.94	.71	$.50\times10^{-8}$	6.6×10^{-13}	1.5814	6.8×10^{-13}
520	.85	.73	.49	6.4	1.5778	6.4
540	.77	.73	.48	6.3	1.5746	6.1
560	.71	.70	.44	5.8	1.5721	5.8
580	.64	.67	.42	5.5	1.5696	5.6
600	.59	.69	.41	5.4	1.5670	5.4
630	.54	.69	.40	5.3	1.5645	5.1
660	.50					

value at 440 $\mu\mu$ to run above the observed; but since the ratio B_m/B_e for this wave-length is in some cases lower, in others higher than the rest, we are inclined to ascribe the divergence to the difficulty of making accurate observations in this region of the spectrum.

THE BRACE LABORATORY OF PHYSICS,
UNIVERSITY OF NEBRASKA.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE FORTY-SEVENTH MEETING.

A REGULAR meeting of the Physical Society was held in the Palmer Laboratory of Princeton University on Saturday, October 23, 1909. President Henry Crew presided.

The following papers were presented:

1. The Relationship Between Entropy and Time. W. S. FRANKLIN.
2. The Lorentz Shortening: an Apparent Paradox. GILBERT N. LEWIS.
3. Ionization Produced by Entladungsstrahlen and Experiments on the Nature of the Radiation. ELIZABETH R. LAIRD.
4. The Effect of NO_3 and Al on the Uranyl Bands and of Ca and Temperature on Neodymium Bands. W. W. STRONG.
5. The Octave Overtone from Tuning Forks. D. C. MILLER.
6. The Electrostatic Effect of a Changing Magnetic Field. J. M. KUEHNE.
7. A Modification of the Thomson-Wilson Method of Determining the Elementary Electrical Charge, and the Most Probable Value of that Charge. R. A. MILLIKAN.
8. Polarization of Röntgen Rays. WM. R. HAM.
9. A New Radiant Emission from the Spark. R. W. WOOD.
10. The Dependence of the Photo-electric Current on the Wave-length of Incident Light. F. K. RICHTMYER.
11. Kinetic Energy of Thermions. O. W. RICHARDSON.
12. Contact Difference of Potential in the Magnetic Field. E. P. ADAMS.
13. A Neglected Form of Relativity. D. F. COMSTOCK.
14. The Relation between the Velocity of Light and the Velocity of its Source. R. C. TOLMAN.
15. The Radiation from Platinum. (By title.) A. TROWBRIDGE.
16. A Characteristic of Spectral Energy Curves. (By title.) W. W. COBLENTZ.
17. The Depth of Complete Scattering of Kathode Rays in Lead and the Variation of the Depth with Velocity of the Rays. WM. R. HAM.

18. Landscapes and Still Life in Infra-red and Ultra-violet. R. W. Wood.

19. Some Laboratory Applications of the Photo-electric Current. F. K. Richtmyer.

20. Reflection of Slow-Speed Electrons. O. W. Richardson.

21. The Temperature Radiation of Transparent Bodies. R. W. Wood.

22. A Thermo-luminescent Glass. W. W. Coblentz.

Ernest Merritt,
Secretary.

A Characteristic of Spectral Energy Curves.[1]

By W. W. Coblentz.

Using the Wien spectral energy equation it can be shown that the wave-length of maximum emission can be computed from the observed spectral energy curve by taking two wave-lengths, λ_1 and λ_2, corresponding to any two points where the emissivity $E_1 = E_2$. Paschen observed this property before the establishment of the theoretical equation. In the subsequent work of Paschen, and of Lummer and Pringsheim, this point is passed over without discussion. In fact the latter observers do not (to my knowledge) say how they obtain the point of maximum emission from the spectral energy curve.

From the beginning of my work on this subject it was observed that in computing the $\lambda_{\max}$ by the aforesaid method the value usually decreases uniformly from a high one (several per cent. higher) when λ_1 and λ_2 are far apart, to a fairly constant value for λ_1 and λ_2 taken close together. In the spectra of metals the $\lambda_{\max}$ was therefore computed by taking values of λ_1, and λ_2 close together, assuming that the discrepancy was due to experimental error. In looking over previous work by Paschen it was found that his data indicate a similar condition. Unfortunately Lummer and Pringsheim never published a full report of their observations.

From the complete solution of Planck's equation

$$\lambda_{\max} = \frac{(\log\lambda_2 - \log\lambda_1)\lambda_1\lambda_2}{(\lambda_2-\lambda_1)\log\epsilon} - \left(\frac{\log\left[\left(e^{\frac{a\lambda_m}{\lambda_1}} - 1\right)e^{\frac{\lambda_m}{\lambda_2}}\right] - \log\left[\left(e^{\frac{a\lambda_m}{\lambda_2}} - 1\right)e^{\frac{a\lambda_m}{\lambda_1}}\right]}{\frac{a(\lambda_2-\lambda_1)\log\epsilon}{\lambda_1\lambda_2}}\right).$$

The first member on the right hand side of the equation is the usual solution for $\lambda_{\max}$, from Wien's equation, while the second member enters as a correction factor. This correction is never negligible even when

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[1] Abstract of a paper presented at the Princeton meeting of the Physical Society, October 23, 1909.

$\lambda_1 = \lambda_2 = \lambda_{\max}$ (data by extrapolation, by taking λ_1 close to $\lambda_{\max}$), where it is a minimum. This minimum correction to $\lambda_{\max}$ amounts to 3.1 per cent. taken at a temperature of 465° C., and decreases to 1.2 per cent. at 1430° C.

From this it appears that, for a black body, $\lambda_{\max} T =$ const. should not be found constant for all temperatures, as heretofore observed, before applying this correction.

POLARIZATION OF RÖNTGEN RAYS.[1]

BY WM. R. HAM.

A DETERMINATION, by means of electroscopes, of the relative intensity of Röntgen rays in various directions from the target, using tubes especially constructed to prevent error due to absorption of glass walls, absorption in target, and secondary rays. The results show a change indicating a polarization of the order of magnitude found by Barkla by indirect methods. The effect on this polarization of absorbing sheets of various materials is such as can be accounted for by assuming the Röntgen ray beams to be made up of rays of two types, one showing very little, if any, polarization.

DEPTH OF COMPLETE SCATTERING OF KATHODE RAYS IN LEAD AND VARIATION OF THE DEPTH WITH VELOCITY OF THE RAYS.[1]

BY WM. R. HAM.

SINCE Röntgen rays originate at a finite depth in the target, there is a certain amount of absorption in the target that varies in different directions. With an adjustable target this variation together with data concerning the coefficient of absorption for the material of the target for rays of the hardness under consideration, make possible the determination of the mean depth at which the Röntgen rays originate. With a sufficiently high velocity, the kathode rays are completely scattered at this depth as is known from the fact that the Röntgen rays then show practically no polarization. This depth of complete scattering is found to vary directly with the potential across the tube. At the velocity of beta rays of uranium, the depth in lead is found to be of the same order of magnitude as that found by Crowther for gold.

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[1] Abstract of a paper presented at the Princeton meeting of the Physical Society, October 23, 1909.

THE EFFECT OF NO_3 AND AL ON THE URANYL BANDS AND OF CA AND TEMPERATURE ON NEODYMIUM BANDS.[1]

BY W. W. STRONG.

A CONTINUATION of the work upon absorption spectra has brought out some new phenomena. The absorption spectra of uranyl salts consist of twelve bands in the blue and violet. These bands are about 50 Ångström units wide, have hazy edges and are about 100 Ångström units apart. Starting from the blue end of the series the bands will be designated by the letters *a*, *b*, *c*, etc. In water solutions it is found that the bands of uranyl nitrate have shorter wave-lengths than the bands of the other salts. The following table gives the approximate wave-lengths of the uranyl bands for aqueous solutions.

Uranyl Bands.

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*
Nitrate.	4870	4705	4550	4390	—	4155	4030	3905	3815	3710	3600
Bromide.	4880	4720	4560	4450	4280	4160	—	—	—	—	—
Sulphate.	4900	4740	4580	4460	4332	4200	4070	3970	3850	3740	—
Acetate.	4910	4740	4595	4455	4310	4160	4070	3970	3865	—	—
Chloride.	4920	4740	4560	4460	4315	4170	4020				

Some of the bands appear especially diffuse (chloride for example) and can only be measured approximately. The *a* and *b* bands come out best in the photographs.

Uranous salts are green and have an entirely different absorption spectra compared with that of the uranyl salts. Following are the wave-lengths of a few of the uranous bands for aqueous solutions.

Uranous Bands.

Nitrate.	6700	6470	—
Sulphate.	6730	6550	5500
Chloride.	6730	6530	5510

Becquerel (Commun. Phys. Lab. Univ. Leiden, Nos. 110, 111) gives the following wave-lengths of the phosphorescent bands of some uranyl salts at a temperatures of 80°(K.).

In the case of the phosphorescent bands there are several series of bands. From these three tables it seems that the NO_3 group possesses an influence upon the frequency of vibration of the absorbing or emitting vibrators of the uranium compound. If it were possible to find the value of e/m for these vibrators it might be possible to find the force exerted by the NO_3 group upon each vibrator. It is quite remarkable that within widely different concentrations the wave-length of the uranyl bands do not change. Probably only a small percentage of the NO_3 groups take part in a light absorption or emission and these may possess properties that are quite different from those of the greater number of NO_3 groups.

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[1] Abstract of a paper presented at the Princeton meeting of the Physical Society, October 23, 1909.

Uranyl Phosphorescent Bands.

Nitrate.	—	5069.6	5301.5	5554.6	5832.9	—
Sulphate.	4918.3	5133.9	5369.4	5626.8	5910.1	6219.5
Double acetate, uranyl and sodium.	4932.5	5148.6	5384.5	5642.7	—	—

The addition of sufficient aluminium chloride to a water solution of uranyl chloride or of calcium chloride to a methyl alcohol solution of uranyl chloride is found to cause the *d* and *e* bands to come together, so as to form a single wide band and to cause the other uranyl bands to change their positions so that the resulting series of bands is almost the same as that of uranyl chloride in ethyl alcohol The general effect of the presence of calcium and aluminium is to shift the first bands (*a*, *b*, etc.) of the series towards the red.

Rise of temperature (0° to 90° C.) causes very little if any change in the wave-length of the neodymium bands. The bands at the higher temperatures are invariably broader. When calcium chloride is present many of the neodymium bands narrow with rise of temperature and are shifted towards the red. It may be possible that many of the temperature and Zeeman effects found by Becquerel are due to the presence of foreign bodies in the neodymium and erbium compounds with which he worked. It is also possible that the presence of calcium in the sun might alter the Zeeman effect so that the strength of field required to produce a given separation would be different from that required in the laboratory where only pure substances are used.

THE KINETIC ENERGY OF THE THERMIONS.[1]

BY O. W. RICHARDSON.

THE present investigation is a continuation of one published in the *Philosophical Magazine* (Dec., 1908) under the title of "The Kinetic Energy of the Ions Emitted by Hot Bodies." The apparatus used involves the same principles as that already employed but it has been greatly improved in detail. The increased accuracy of measurement and the greater extent to which the theoretical conditions are satisfied enable the author's previous conclusions to be subjected to a more rigorous test. In particular the spreading out of the negative electrons emitted by a narrow strip in the absence of an electric field has been examined and is found to agree very satisfactorily with the distribution which was predicted in the earlier paper, on the assumption that the distribution of velocity among the electrons is in accordance with Maxwell's law.

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[1] Abstract of a paper presented at the Princeton meeting of the Physical Society, October 23, 1909.

It is found however that at points at a considerable distance from the center of the system the current through the slit is in excess of the theoretical requirements. The discrepancy is found to be much greater when an electric field is applied between the two plates than when the field is absent. It appears to be due to the reflection of the electrons at the surface of the receiving plates. This difficulty does not appear to arise when the "sideways kinetic energy" of the positive ions is being measured.

The Reflection of Low-Speed Electrons by Metal Surfaces.[1]

By O. W. Richardson.

In the author's investigations of the properties of the electrons emitted by hot bodies phenomena have frequently arisen which have only seemed capable of explanation on the view that the slow-speed electrons dealt with gave rise to a reflected or secondary radiation when they struck a metal surface.[2] The present communication describes an experiment in which the result of this reflection is exhibited using electrons whose speed corresponds to a fall of potential of 8 volts. Indirect evidence is also adduced in favor of the view that the unaccelerated electrons also give rise to a similar effect. Their speed corresponds to a fall of potential of about .03 volt. It appears that this reflection has previously been observed by von Baeyer[3] who concluded that the effect occurred with speeds corresponding to 5 volts and upwards.

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[1] Abstract of a paper presented at the Princeton meeting of the Physical Society, October 23, 1909.

[2] See for example Phil. Mag. [6], vol. 16, 761, 1908, and later papers in the same journal.

[3] Ber. Deutsch. Phys. Gesell., 10, 4, p. 96, 1908.

On the Electrostatic Effect of a Changing Magnetic Field.[1]

By J. M. Kuehne.

A CHARGED body suspended in a changing magnetic field, or better in a field swept by magnetic lines of force, should, according to the electro-magnetic theory of Maxwell and others, be subjected to a force at right angles to the lines of magnetic force as well as to the direction of their motion. The magnitude of this force will depend on the product $Q \cdot dN/dt$, where Q is the quantity of the charge and dN/dt the rate at which magnetic lines are moving across the space occupied by the charge.

The apparatus used in the experiment consists of a parallel plate condenser having twenty pairs of ring-shaped plates — the one set of plates is suspended free to rotate, the oppositely charged plates are stationary. Through the centers of both sets of plates passes one limb of the closed magnetic circuit of a transformer. The magnetic circuit is excited by an alternating current, and the condenser is charged from the secondary of a potential transformer, whose primary is connected in parallel with the magnetic circuit. The charged plates are thus subjected to a torque whose value, according to theory, is $L = \frac{1}{2}nNCE$, where n is the frequency of the alternating magnetic field, N the maximum magnetic flux through the transformer core, C the capacity of the condenser, and E the maximum charging e.m.f.

E_1	E_2	L	L'
600	.345	.33	.26
606	.342	.33	.35
606	.490	.47	.45
600	.492	.47	.44
956	.345	.52	.48
956	.348	.52	.46
806	.444	.56	.53
772	.485	.59	.57
828	.490	.64	.63
840	.494	.65	.64
848	.495	.66	.67
864	.485	.66	.73
916	.490	.71	.66
956	.474	.72	.70
956	.490	.74	.72
956	.492	.74	.72
1,294	.480	.98	1.05
1,314	.498	1.03	1.04

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[1] Abstract of a paper presented at the Princeton meeting of the Physical Society October 23, 1909.

In the experiment N was varied between 127,000 and 190,000 lines, and E between 850 and 1,860 volts, the other factors remaining constant. The effect was thus tested over a range of variation of more than 3 : 1, and throughout this range the observed torque was in satisfactory agreement with that demanded by theory, both in direction and magnitude. The observed torque was measured by rotation of the torsion head through a known angle, the moment of torsion of the suspending fiber being known.

In the following table of results E_1 indicates the "effective" charging E.M.F., E_2 the effective E.M.F. induced by the magnetic field in each turn of a coil surrounding the magnet core, L the torque in dyne cm. calculated from theory, and L' the observed torque.

IONIZATION PRODUCED BY ENTLADUNGSSTRAHLEN AND EXPERIMENTS ON THE NATURE OF THE RADIATION.[1]

BY ELIZABETH R. LAIRD.

THIS work is a continuation of that previously reported. Experiments have been made to show that the effect of Entladungsstrahlen cannot be due to dust driven from the electrodes, nor to electrostatic induction, nor to a mechanical pressure wave, nor to light of wave-length greater than 1,000.

Other experiments show that the same radiation ionizes the gas through which it passes, since the ionization is increased in proportion to the thermoluminescence, and is transmitted or not by substances according as they transmit or cut off thermoluminescence, and remains unchanged under conditions in which the thermoluminescence is unchanged. In particular the ionization is cut off by thin quartz. It is much increased by the presence of certain vapors, notably turpentine, ammonia, ether, methyl iodide. The radiation appears to be reflected very slightly, but is reflected in about the same degree by such various substances as paper, lead, copper, glass. Some experiments seem to show that the penetrating power of the radiation differs according to the source used. A fuller report will appear shortly in this journal.

MOUNT HOLYOKE COLLEGE.

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[1] Abstract of a paper presented at the Princeton meeting of the Physical Society, October 23, 1909.

A NEW MODIFICATION OF THE CLOUD METHOD OF MEASURING THE ELEMENTARY ELECTRICAL CHARGE, AND THE MOST PROBABLE VALUE OF THAT CHARGE.[1]

BY R. A. MILLIKAN.

THIS modification of the cloud method of determining e consists:

1. In making observations, not upon the surface of a cloud, but upon single isolated drops carrying multiple charges.

2. In exactly balancing gravity upon these single charged drops by an electrical field.

3. In observing the rate of fall of these same drops under gravity after the electrical field has been thrown off.

4. In eliminating any possible error due to evaporation by first obtaining stationary, *i. e.*, balanced, drops, and then measuring the times of passage of these same drops across equal spaces in the field of the reading telescope.

5. In directly measuring the temperature of the cloud chamber instead of computing it. (It is this computation which was found to have introduced the chief error into preceding determinations of e by the cloud method.[2]

In obtaining the results herewith reported the distances through which the falling drops were timed varied from 1 mm. to 1.55 mm.; the times of fall through these spaces from 3 to 5 seconds: the distances between the plates to which the potentials were applied from 4.63 to 5.45 mm.: the intensities of the electrical fields from 14 to 17 absolute electrostatic units: the liquids used for forming drops were alcohol and water.

The method compares favorably in directness and precision with any which has thus far been used for determining e. The results of 7 very concordant observations on water drops carrying triple positive charges gave $e = 4.59 \times 10^{-10}$: of 11 concordant observations on alcohol drops carrying double positive charges gave $e = 4.64 \times 10^{-10}$: of 10 observations on water drops carrying quadruple positive charges gave $e = 4.56 \times 10^{-10}$: of 5 observations on water drops carrying quintuple positive charges gave $e = 4.83 \times 10^{-10}$: of three observations on water drops carrying sextuple positive charges gave $e = 4.69 \times 10^{-10}$: of two observations on water drops carrying double positive charges gave $e = 4.87 \times 10^{-10}$. The weighted mean of these results is 4.65×10^{-10}. The error in this determination is estimated as not more than 2 per cent. Mr. Begeman has just completed in this laboratory a very long and careful series of observations by the regular Wilson method timing however the layers corresponding to multiple charges, and obtains as his final mean $e = 4.66 \times 10^{-10}$.

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[1] Abstract of a paper presented at the Princeton meeting of the Physical Society, October 23, 1909.

[2] Millikan and Begeman, PHYS. REV., 26, p. 197, 1908.

The mean of all the recent determinations of e by methods which seem least open to question is given below.

Planck	4.69
Rutherford and Geiger	4.65
Regener	4.79
Millikan	4.65
Begeman	4.66

Mean $e = 4.69 \times 10^{-10}$

A Thermoluminescent Glass.[1]

By Wm. W. Coblentz.

An experimental demonstration of a thermoluminescent glass made of feldspar, formed into a perfectly transparent rod which was heated electrically. The luminiscence appears as a faint white cloud, at a temperature of about 800° C., which increases in intensity with rise in temperature. The important point illustrated was that the light is white at all temperatures, *i. e.*, it does not pass through the usual chromatic changes found in the radiation from opaque substances which would appear red at these temperatures. The absorption and behavior in a magnetic field will require further investigation.

The Dependence of Photo-Electric Current on Wave-Length of Incident Light.[1]

By F. K. Richtmyer.

Using the apparatus previously described[2] the author has investigated the relation between wave-length of the incident light and the resulting photo-electric current from a sensitive sodium surface. The spectrum of a Nernst glower was thrown on the sensitive surface by means of a spectrometer with a slit substituted for the eyepiece and the resulting photo-electric currents measured by the rate of drift of an electrometer from $\lambda = .65\,\mu$ to $\lambda = .42\,\mu$.

More satisfactory results were obtained by use of the spectrum of acetylene. In this case the observed currents were corrected for (1) variable dispersion of the prism and (2) energy distribution in the spectrum of acetylene. The curve plotted between corrected photo-electric currents and wave-lengths is similar to the visibility curve but has a maximum at $\lambda = .46\,\mu$, suggesting resonance in some form with the free period of the electrons in the atom of sodium in its solid state corresponding to the period of $\lambda = .46\,\mu$.

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[1] Abstract of a paper presented at the Princeton meeting of the Physical Society, October 23, 1909.

[2] Phys. Rev., Vol. XXIX., pp. 71–80.

Computation of the energy changes involved shows that, in a certain case, while approximately 100 ergs per second of energy were absorbed from the incident light, the energy of the expelled electrons was less than one erg per second.

LABORATORY APPLICATIONS OF THE PHOTO-ELECTRIC CURRENT.[1]

BY F. K. RICHTMYER.

THE production of measurable photo-electric currents from a sodium surface by extremely feeble illuminations recommends this phenomenon for use in making photometric measurements of monochromatic or of isochromatic light sources where observations by eye are extremely difficult. For example the author has studied the decay of phosphorescence of a sample of Balmain's paint for a period of over one hour following a half-minute excitation by sunlight. The curve is of the usual form and when plotted with $1/\sqrt{I}$ as ordinates, shows three distinct linear branches.

It is also pointed out that this combination of photo-electric cell and electrometer acts as a time integrator for light intensity in much the same way that an electrolytic cell would measure total quantity of electricity. In this connection the cell might be used to study short decay phosphorescence or, as has been done by the author, to measure the *total* quantity of light passing through a photographic shutter on exposure.

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[1] Abstract of a paper presented at the Princeton meeting of the Physical Society, October 23, 1909.

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Page 286, the upper figure is Fig. 7; the lower figure is Fig. 6.

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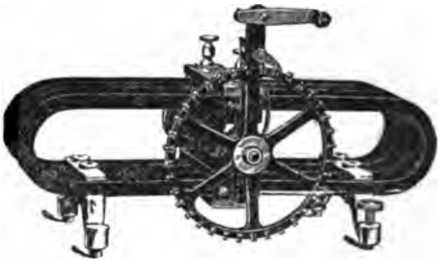

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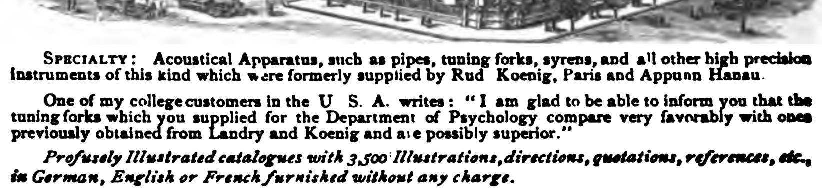

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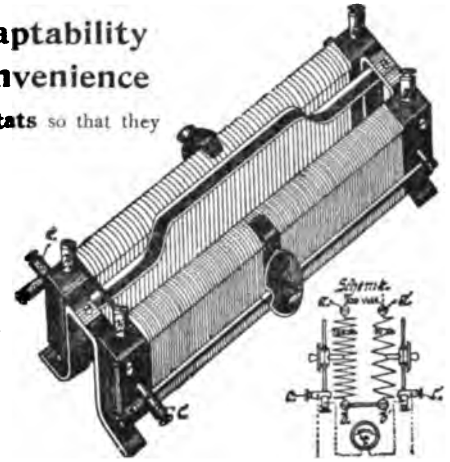

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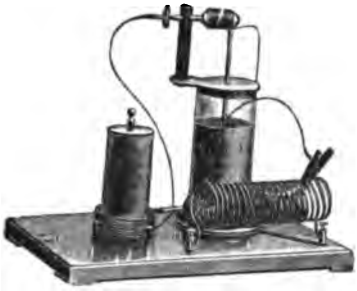

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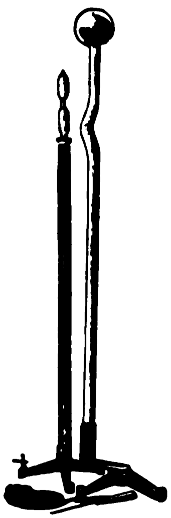

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